Phase Transitions in an Expanding Universe: Stochastic Gravitational Waves in Standard and Non-Standard Histories

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Aug 11, 2020

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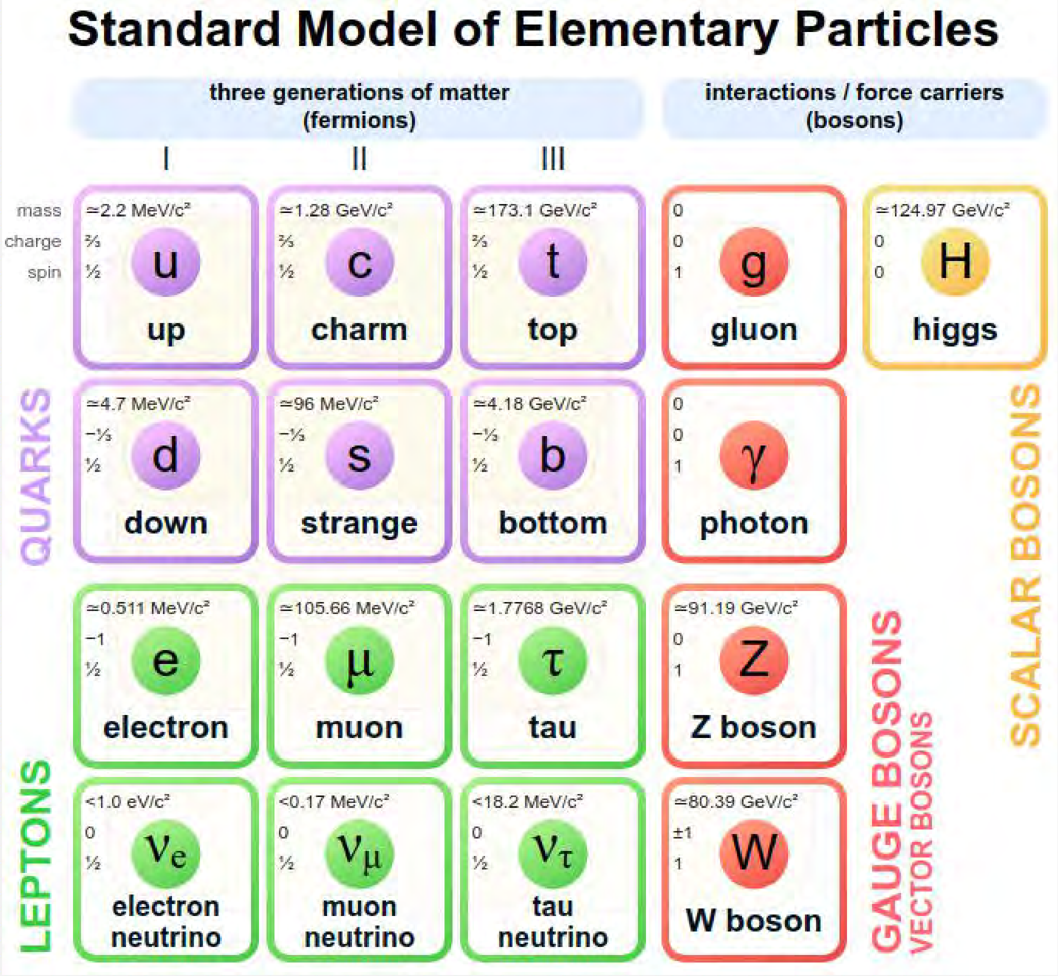
Institute of High Energy Physics Chinese Academy of Sciences

talk based on arxiv:hep-ph/2007.08537
in collaboration with Kuver Sinha, Daniel Vagie, Graham White

Triumphs of our Age

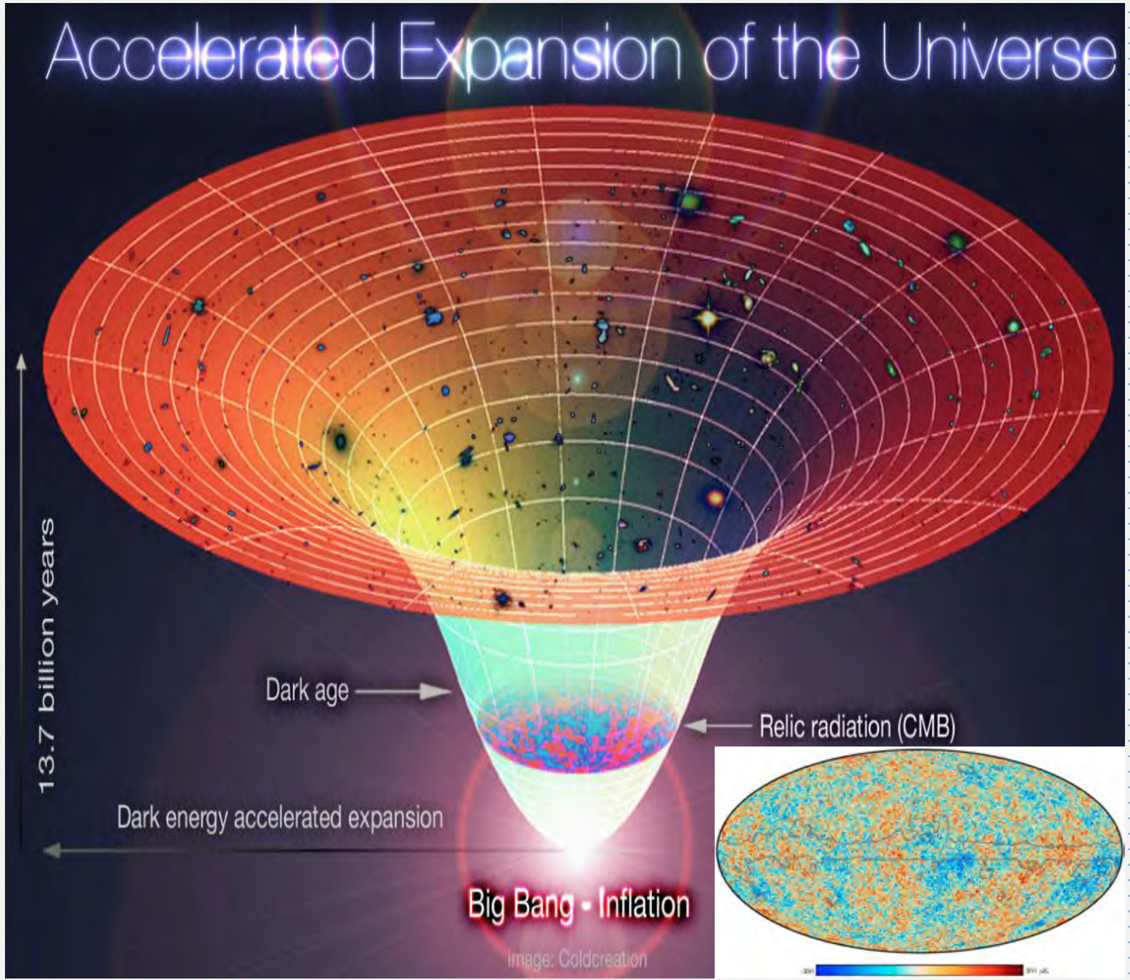
Particle Physics



Standard Model of Elementary Particles

		three generations of matter (fermions)			interactions / force carriers (bosons)	
		I	II	III		
QUARKS	mass charge spin	≃2.2 MeV/c^2 ⅔ ½ u up	≃1.28 GeV/c^2 ⅔ ½ c charm	≃173.1 GeV/c^2 ⅔ ½ t top	0 0 1 g gluon	≃124.97 GeV/c^2 0 0 H higgs
		≃4.7 MeV/c^2 −⅓ ½ d down	≃96 MeV/c^2 −⅓ ½ s strange	≃4.18 GeV/c^2 −⅓ ½ b bottom	0 0 1 γ photon	SCALAR BOSONS
LEPTONS		≃0.511 MeV/c^2 −1 ½ e electron	≃105.66 MeV/c^2 −1 ½ μ muon	≃1.7768 GeV/c^2 −1 ½ τ tau	≃91.19 GeV/c^2 0 1 Z Z boson	GAUGE BOSONS VECTOR BOSONS
		<1.0 eV/c^2 0 ½ ν_e electron neutrino	<0.17 MeV/c^2 0 ½ ν_μ muon neutrino	<18.2 MeV/c^2 0 ½ ν_τ tau neutrino	≃80.39 GeV/c^2 ±1 1 W W boson	

Cosmology



Planck 2018

Triumphs of our Age

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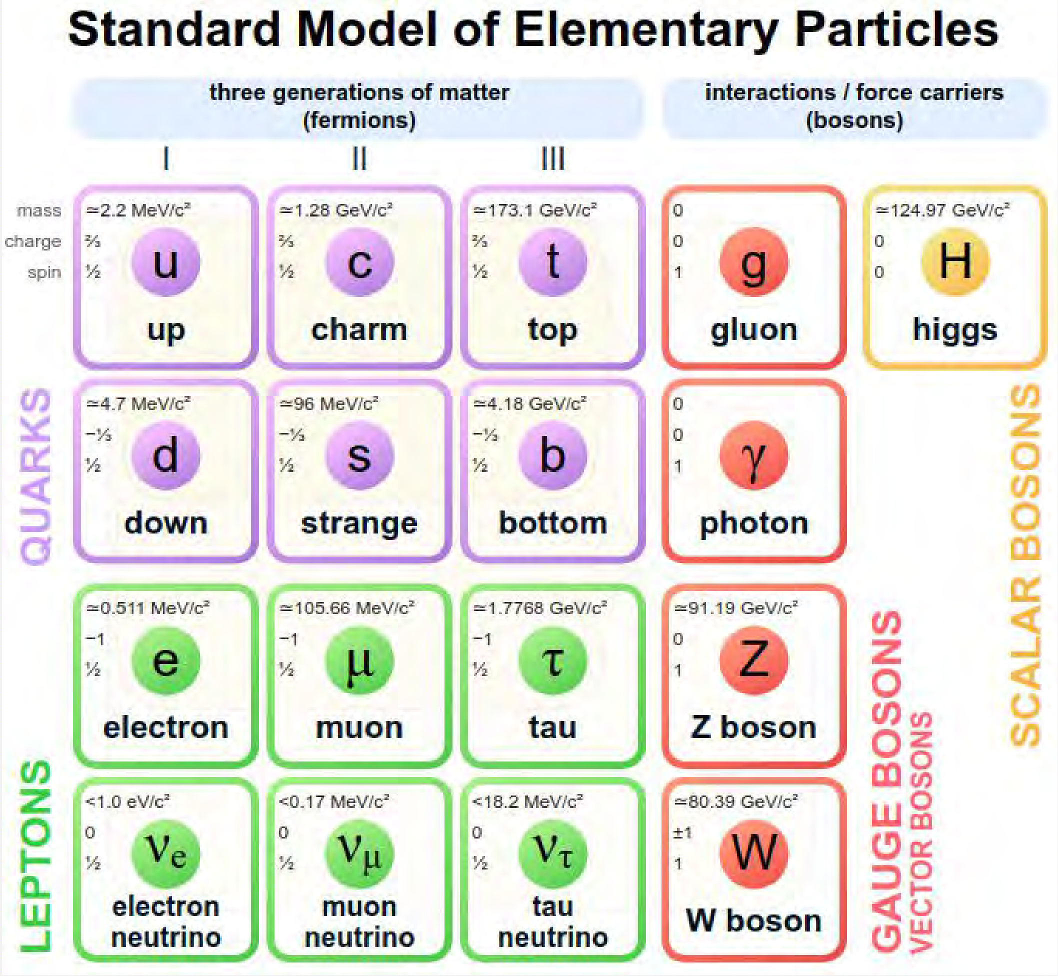

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QUARKS	≃4.7 MeV/c^2 −⅓ ½ d **down**	≃96 MeV/c^2 −⅓ ½ s **strange**	≃4.18 GeV/c^2 −⅓ ½ b **bottom**	0 0 1 γ **photon**	SCALAR BOSONS
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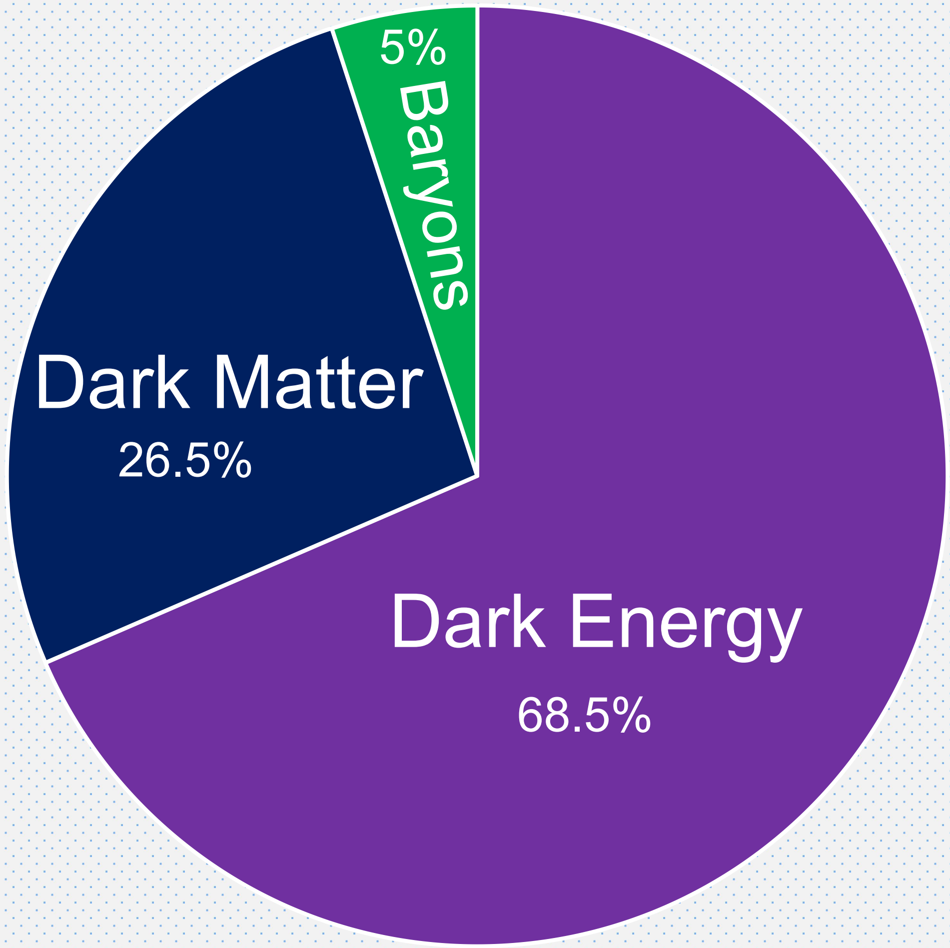

Triumphs of our Age

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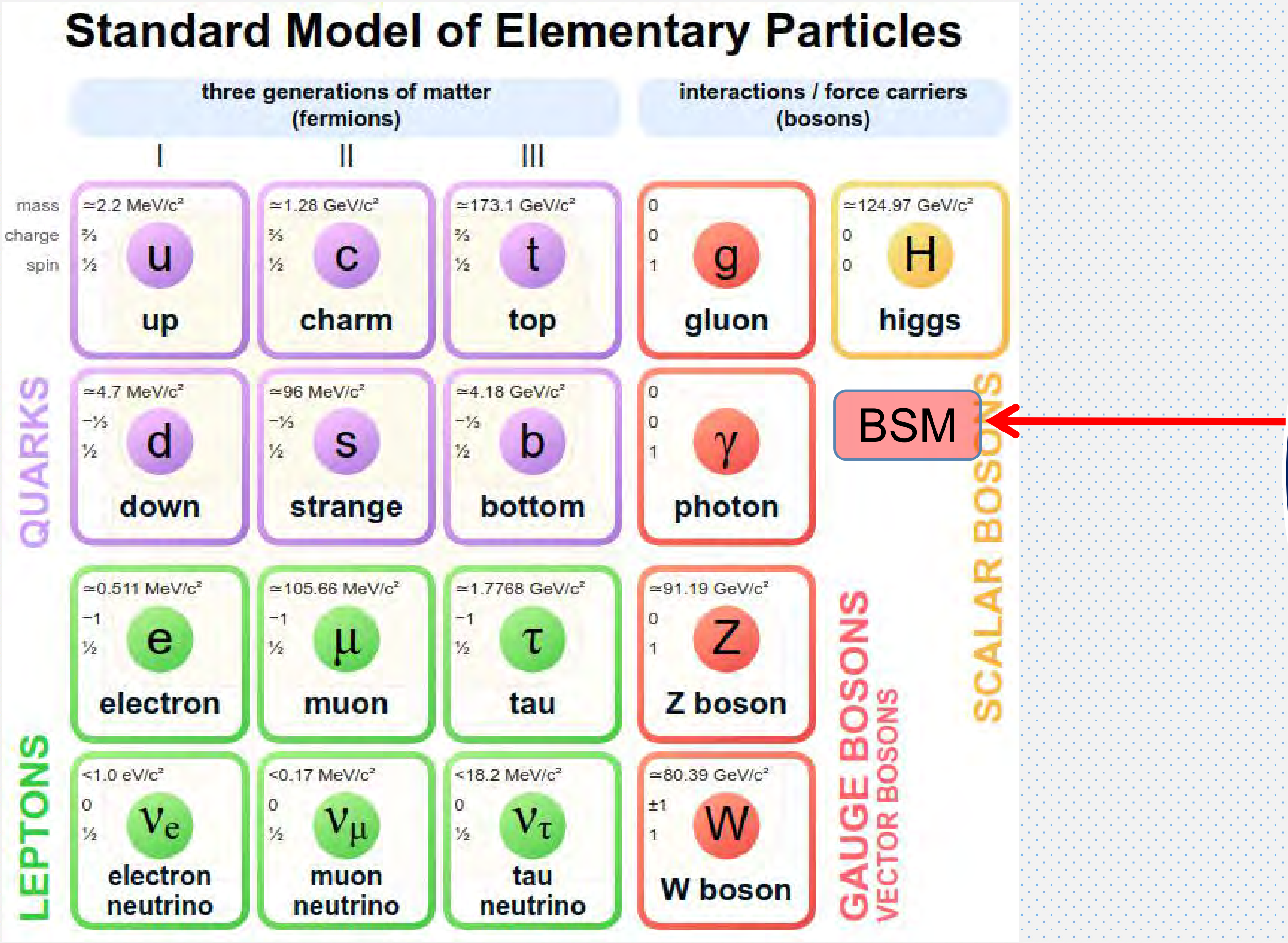

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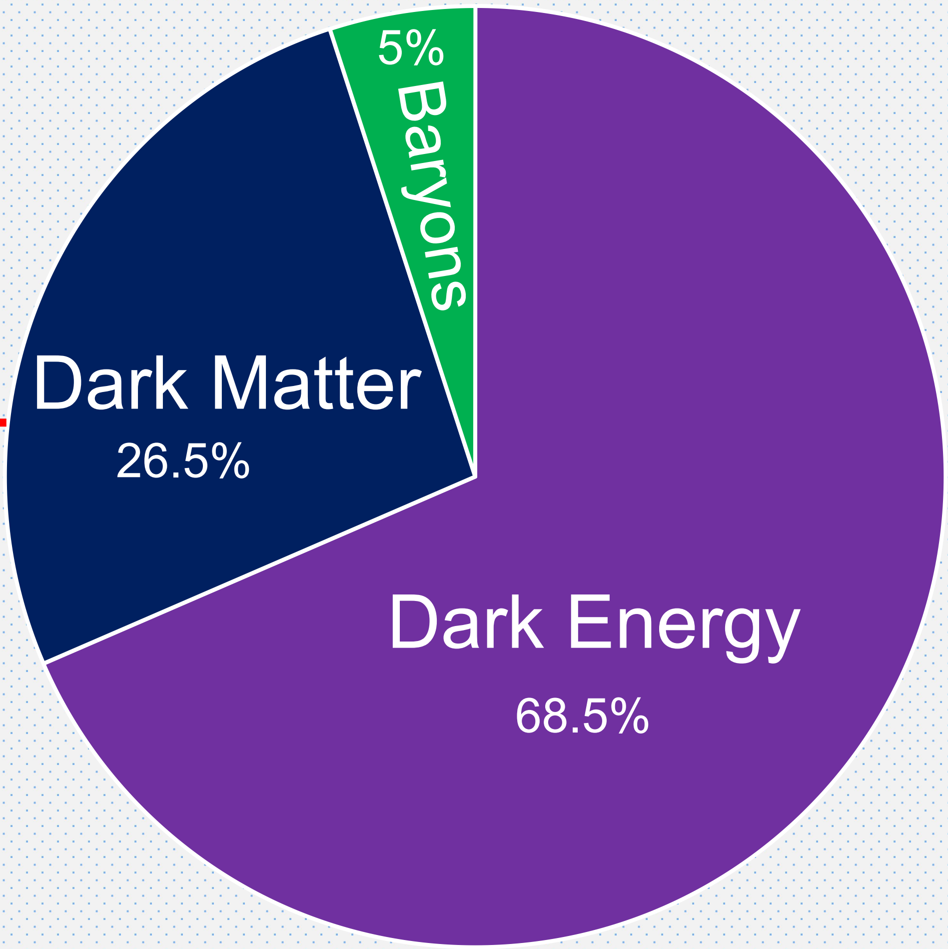
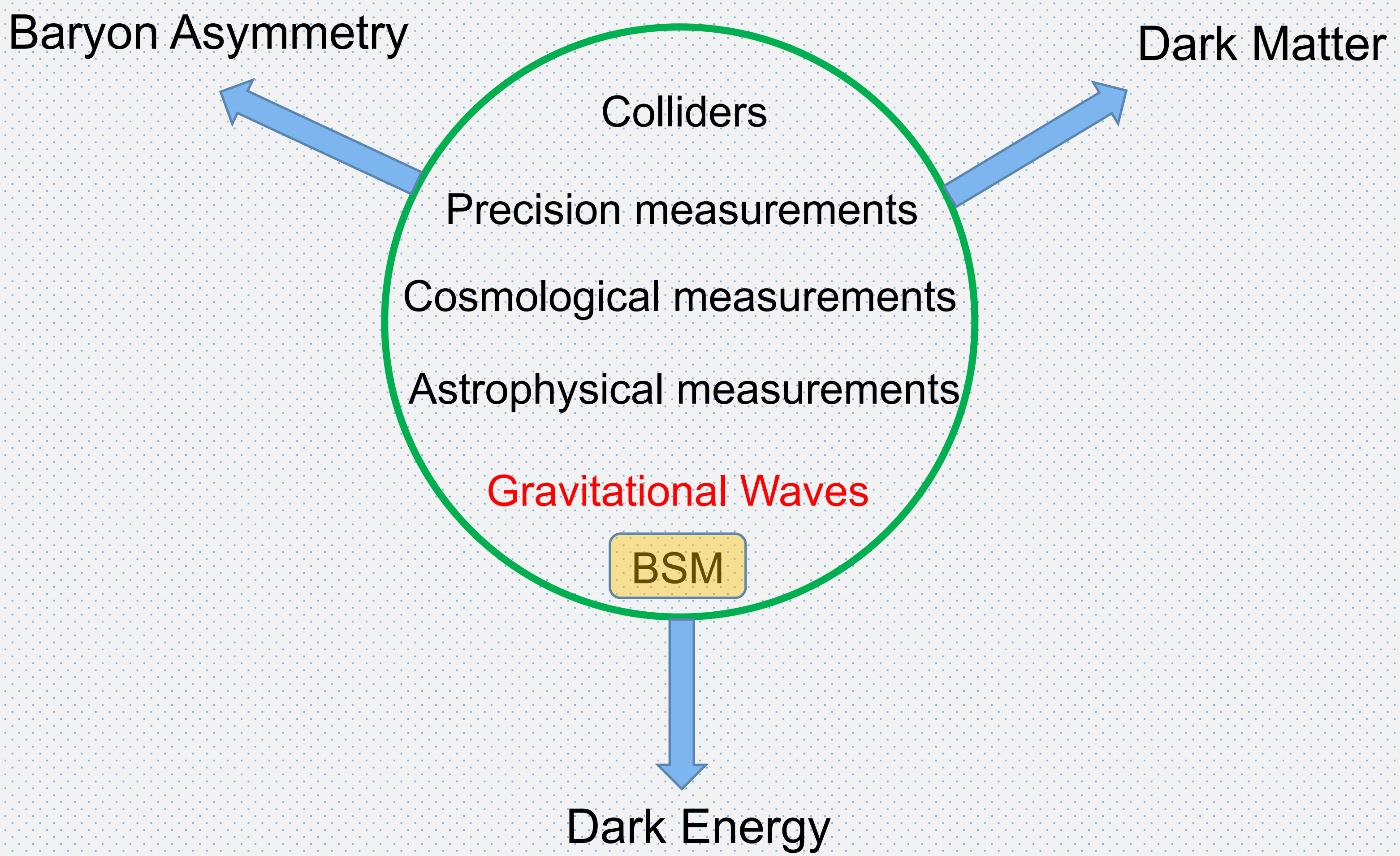


The Problems



Baryon Asymmetry

Dark Matter

Colliders

Precision measurements

Cosmological measurements

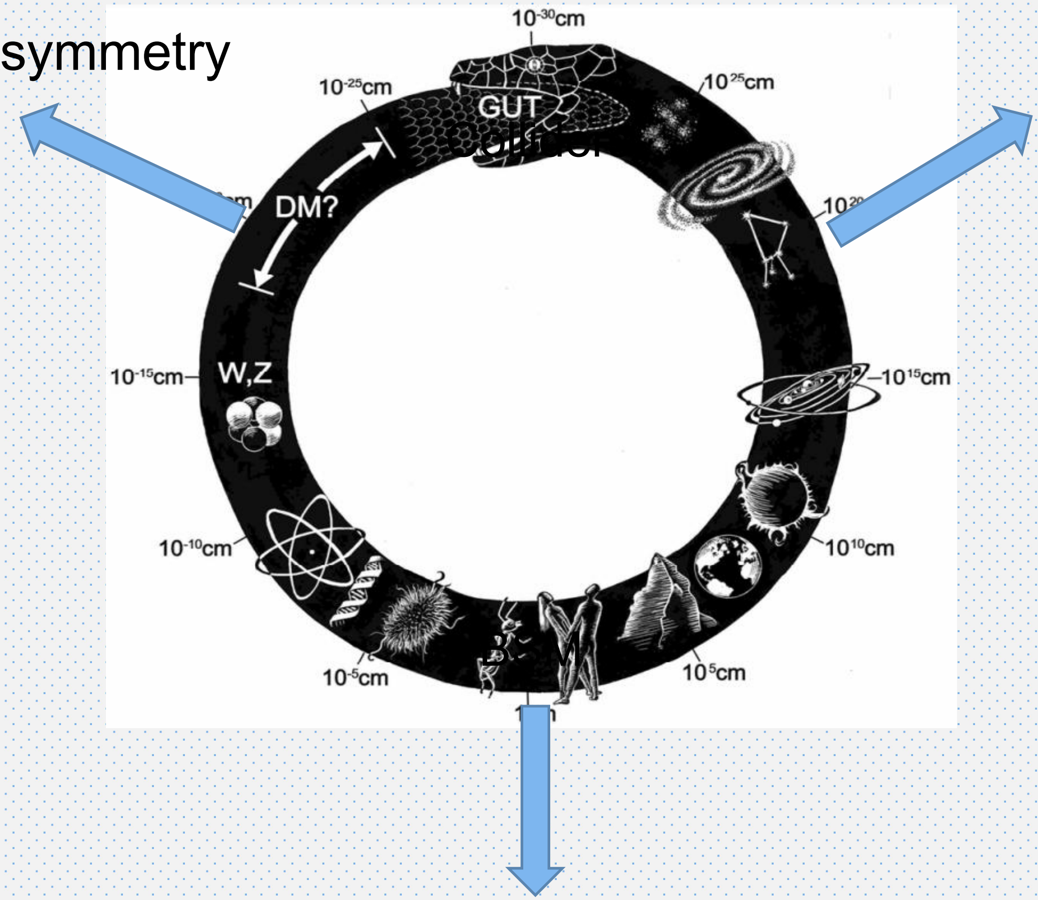
Astrophysical measurements

Gravitational Waves

BSM

Dark Energy

The Problems



Baryon Asymmetry

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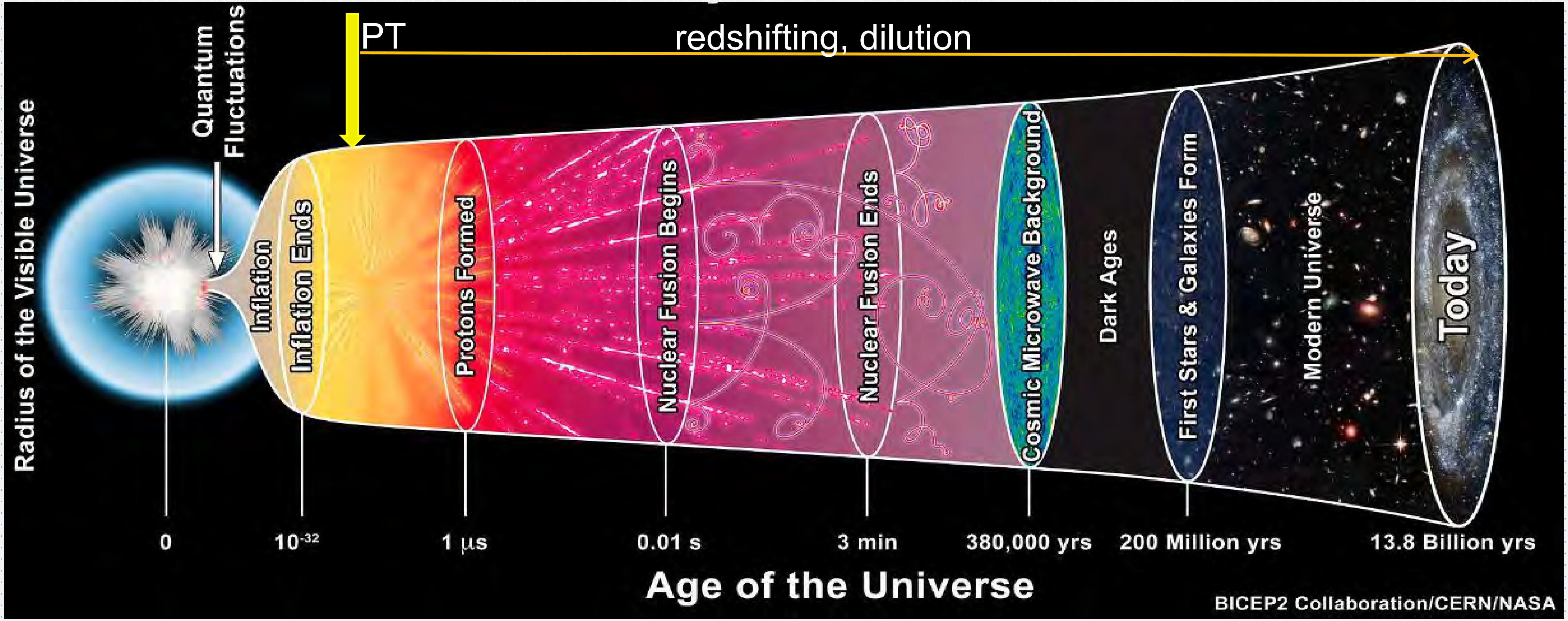
Cosmological Phase Transitions

- Realization of symmetry breakings (Higgs potential, Baryogenesis)
- Gravitational waves as a clean relic, and a cosmic witness

BSM for particle physics: Peccei-Quin, SUSY, Extra Dimensions

BSM for cosmology: Early matter domination(string moduli), Kination, Intermediate Inflationary stage, etc



BICEP2 Collaboration/CERN/NASA

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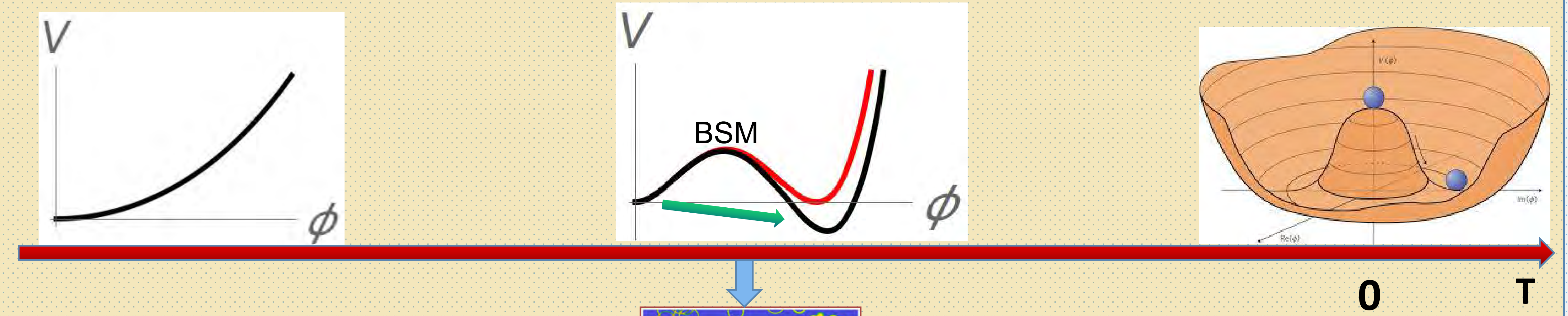

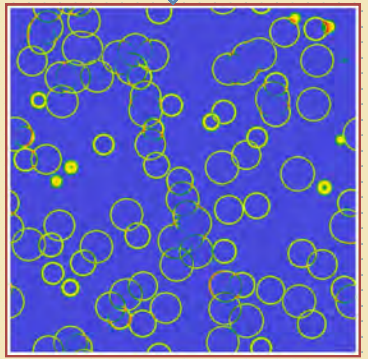

Hindmarsh, et al, 2015

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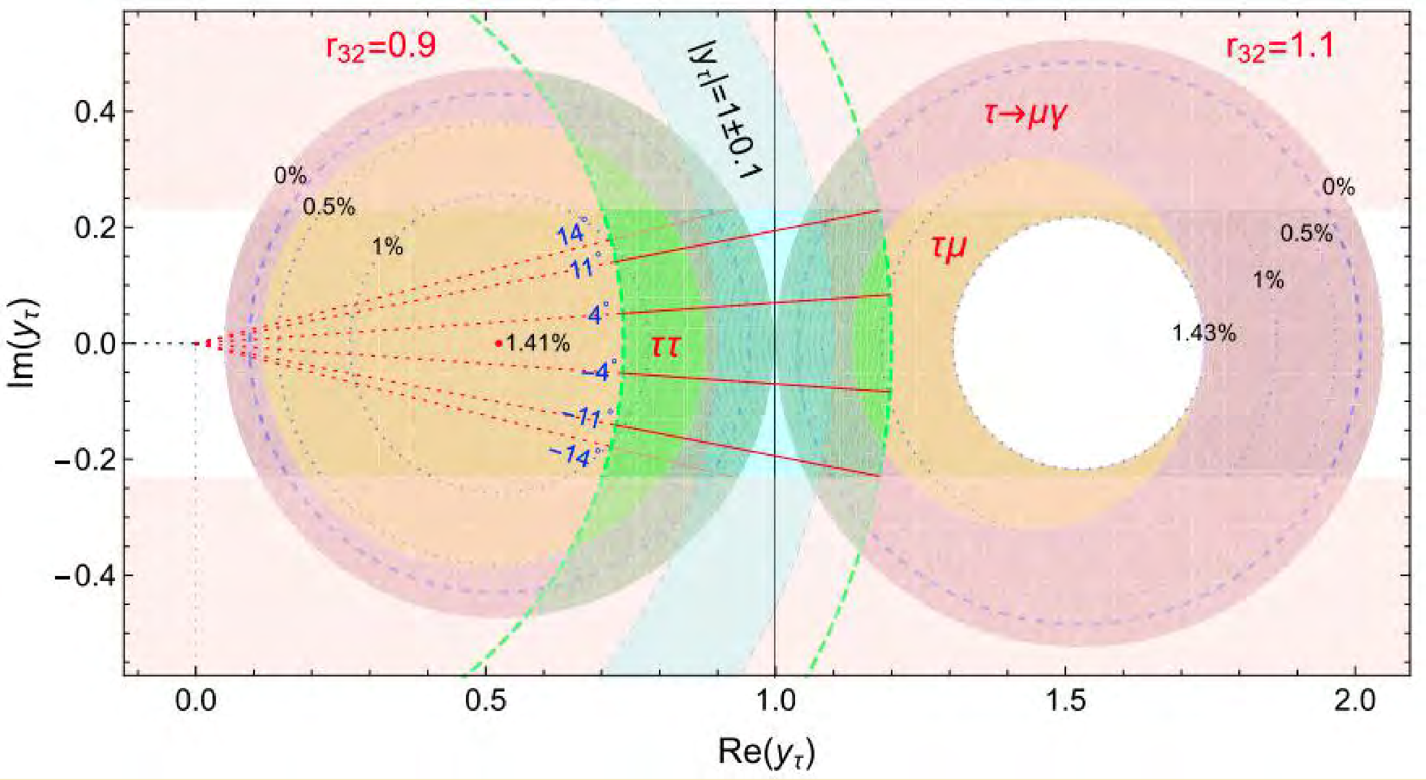
Type III 2HDM

$$\mathcal{L}_{\text{Yukawa}}^{\text{Lepton}} = -\overline{L^i}[Y_{1,ij}\Phi_1 + Y_{2,ij}\Phi_2]e_R^j + \text{H.c.},$$

Jarlskog Invariant

$$J_A = \frac{1}{v^2 \mu_{12}^{\text{HB}}} \sum_{a,b,c=1}^{2} v_a v_b^* \mu_{bc} \text{Tr}[Y_c Y_a^\dagger]$$



Guo, Li, Liu, Ramsey-Musolf, Shu, PRD96,115034(2017)

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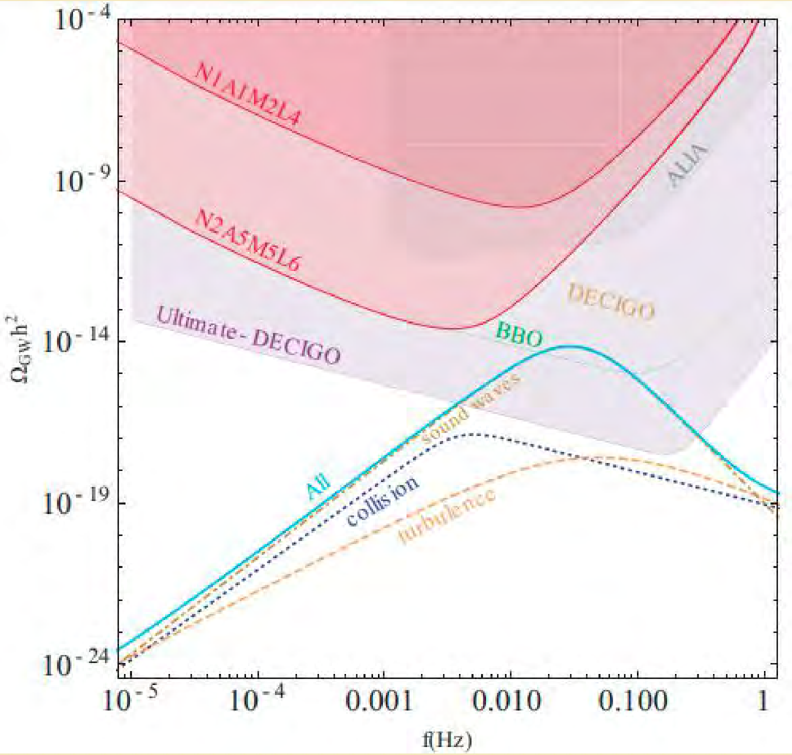
NMSSM

$$W_{\rm Higgs}=\lambda\widehat{S}\,\widehat{H}_{\rm u}\cdot\widehat{H}_{\rm d}+\frac{\kappa}{3}\widehat{S}^3,$$

$$-\Delta\mathcal{L}_{\rm soft}\supset\lambda A_\lambda H_{\rm u}\cdot H_{\rm d}S+\frac{1}{3}\kappa A_\kappa S^3+{\rm h.c.}$$

Jarlskog Invariant

$$\mathcal{I}=|\lambda||\kappa|\sin(\phi'_\lambda-\phi'_\kappa)$$



Bian, Guo, Shu, CPC42,093106(2018)

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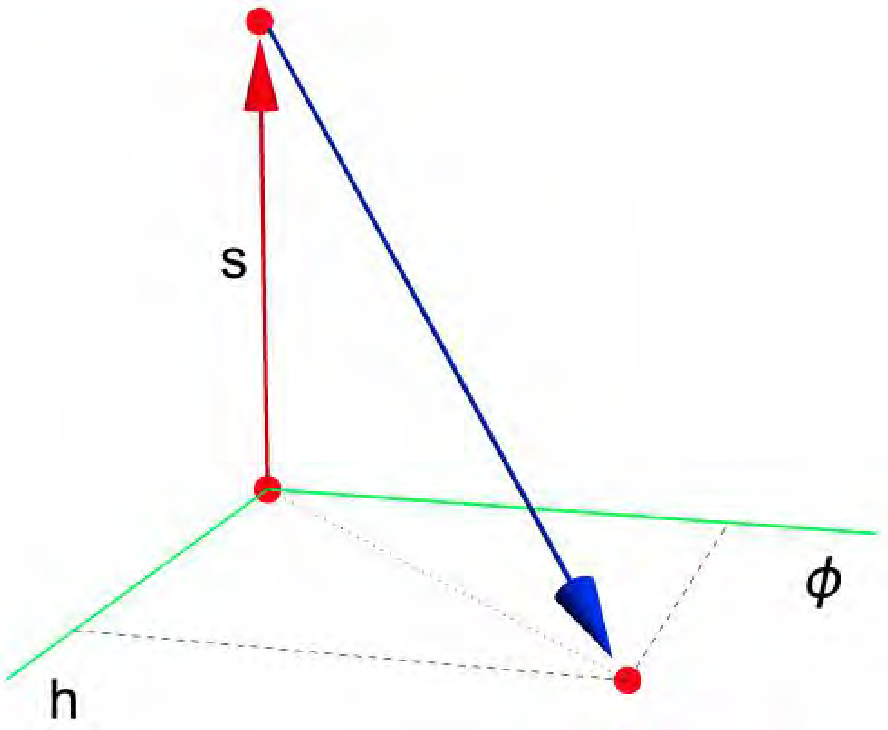
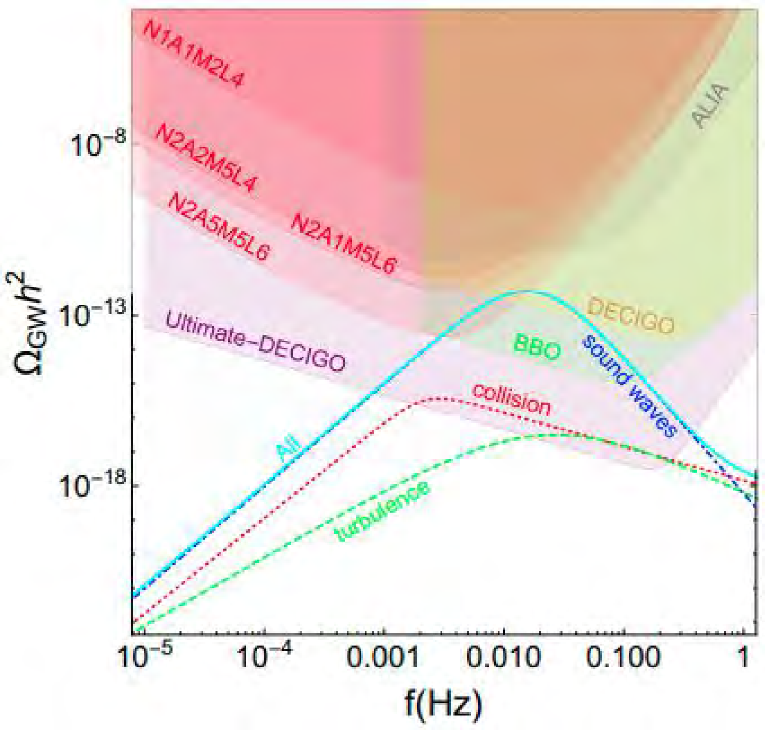
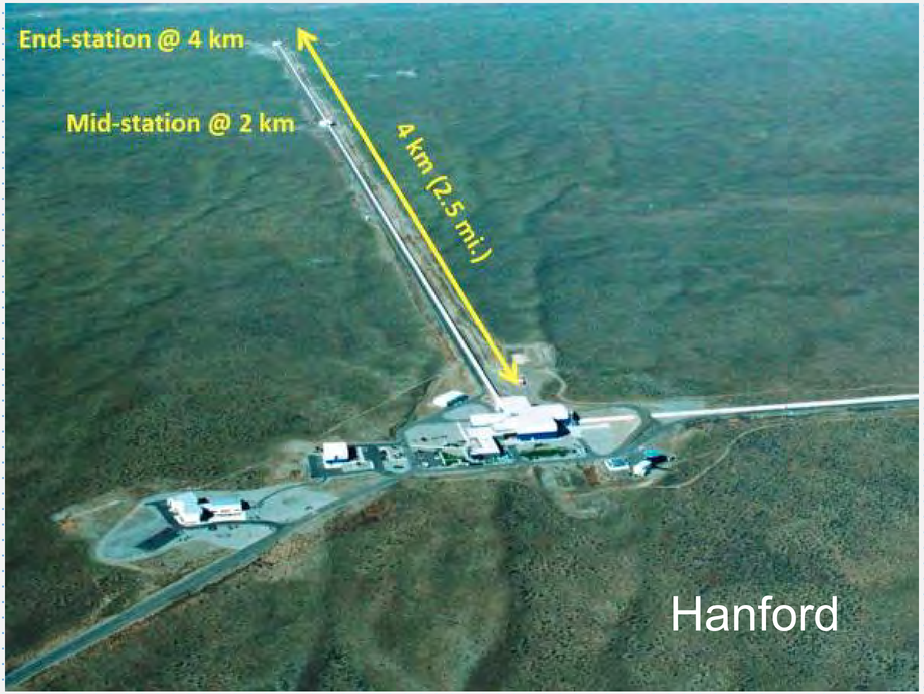
Probe the Dark Matter



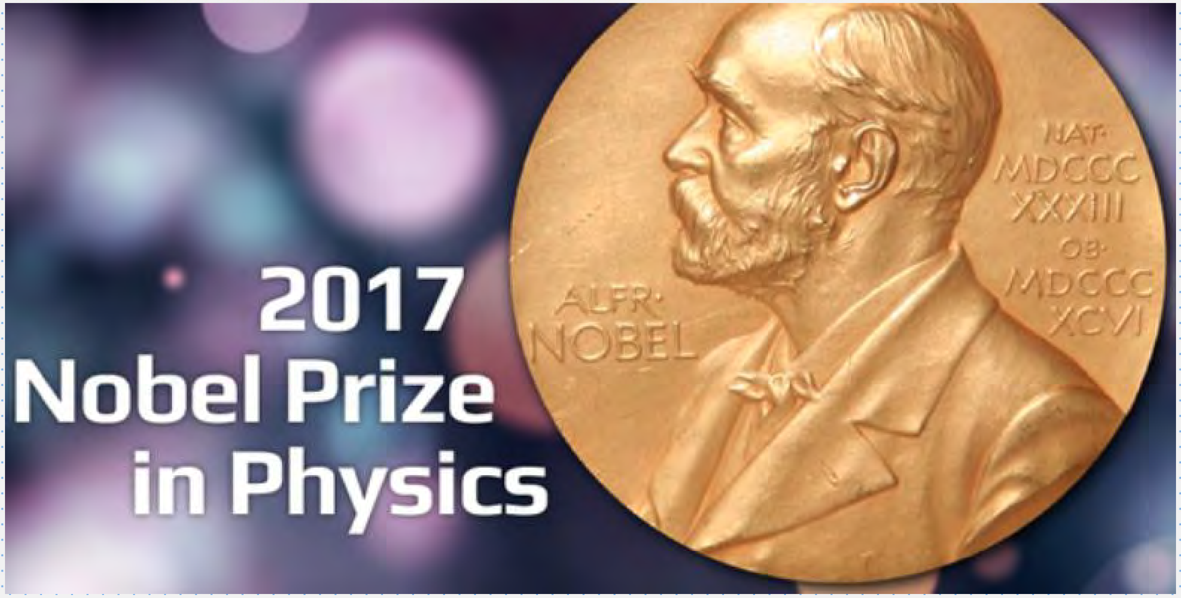


Chao, Guo, Shu, JCAP09(2017)009

LIGO and Gravitational Wave Detection



https://www.ligo.caltech.edu/



https://nobelprize.org/

A new era for Gravitational Wave Astronomy

Also new tools for particle physics and cosmology

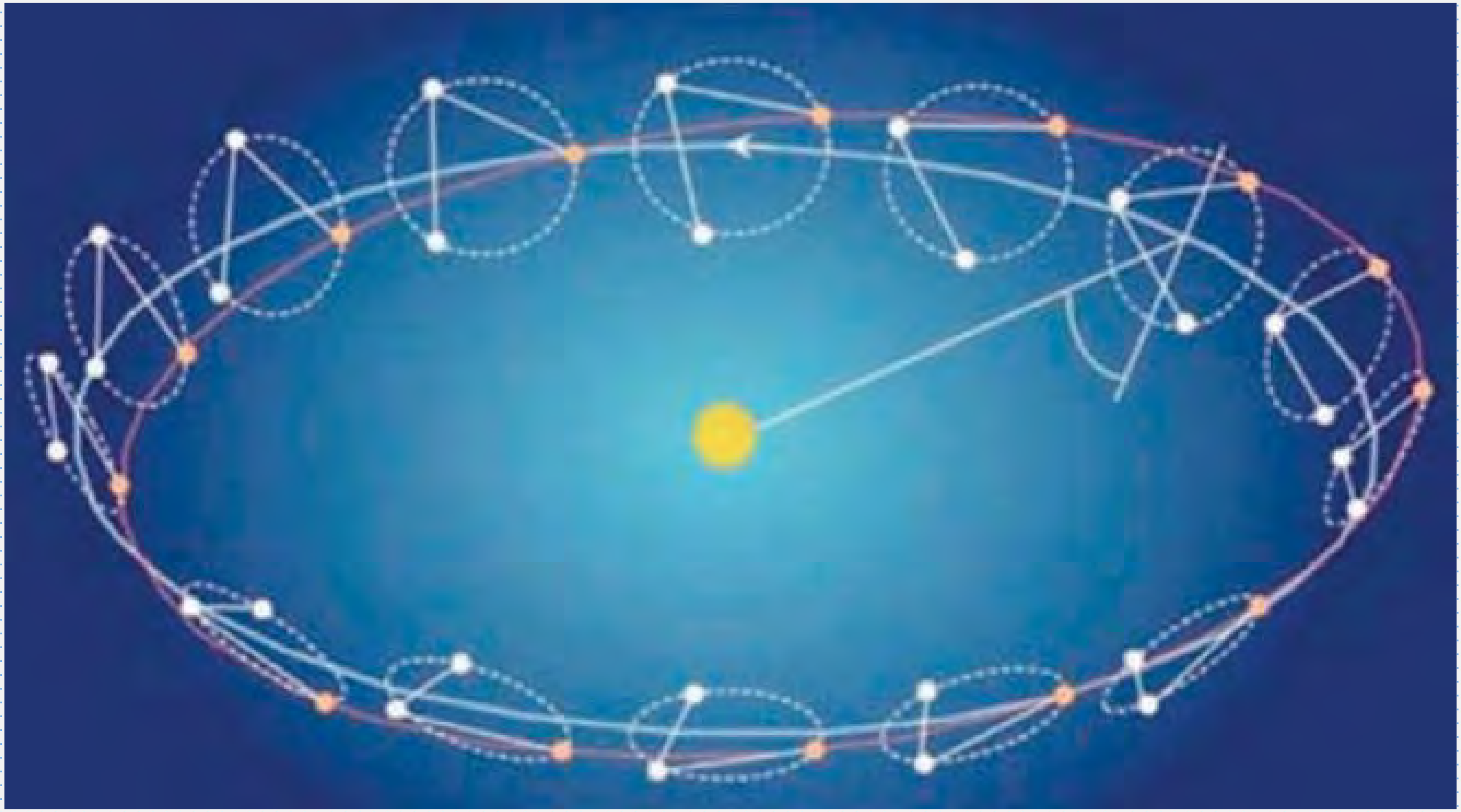
Space-based Gravitational Wave Detectors

Supermassive Black Hole Binary

Extreme Mass Ratio Inspiral

Stochastic Gravitational Waves

arm length: several million-km
mili-hz GW signal



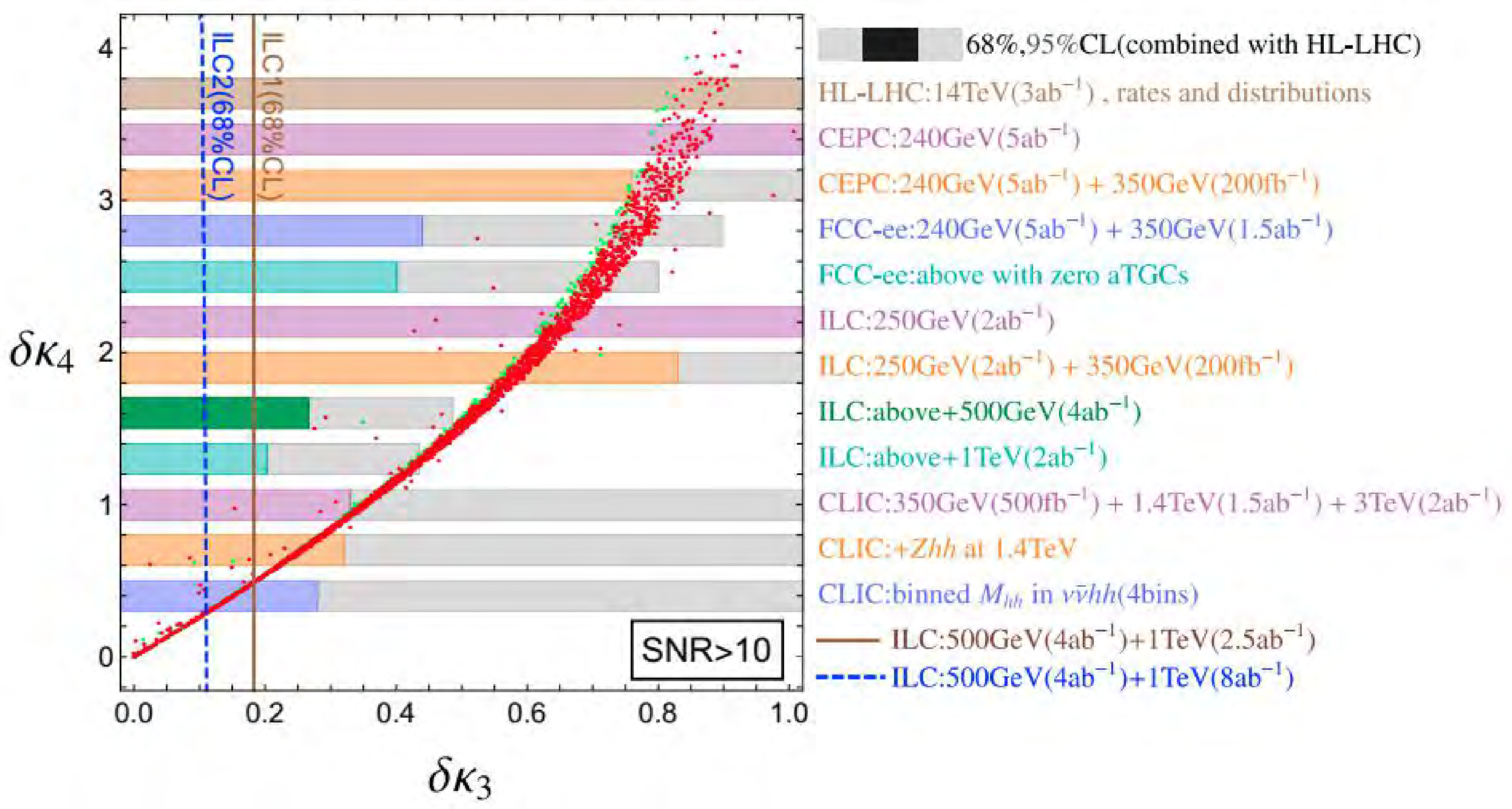
https://lisa.nasa.gov/

天琴、太极、LISA、BBO、DECIGO

Collider and GW Correlation and Complementarity

Higgs self-couplings

$$\Delta\mathcal{L} = -\frac{1}{2}\frac{m_{h_1}^2}{v}(1+\delta\kappa_3)h_1^3 - \frac{1}{8}\frac{m_{h_1}^2}{v^2}(1+\delta\kappa_4)h_1^4$$



Alves, Ghosh, Guo, Sinha, Vagie, JHEP04(2019)052

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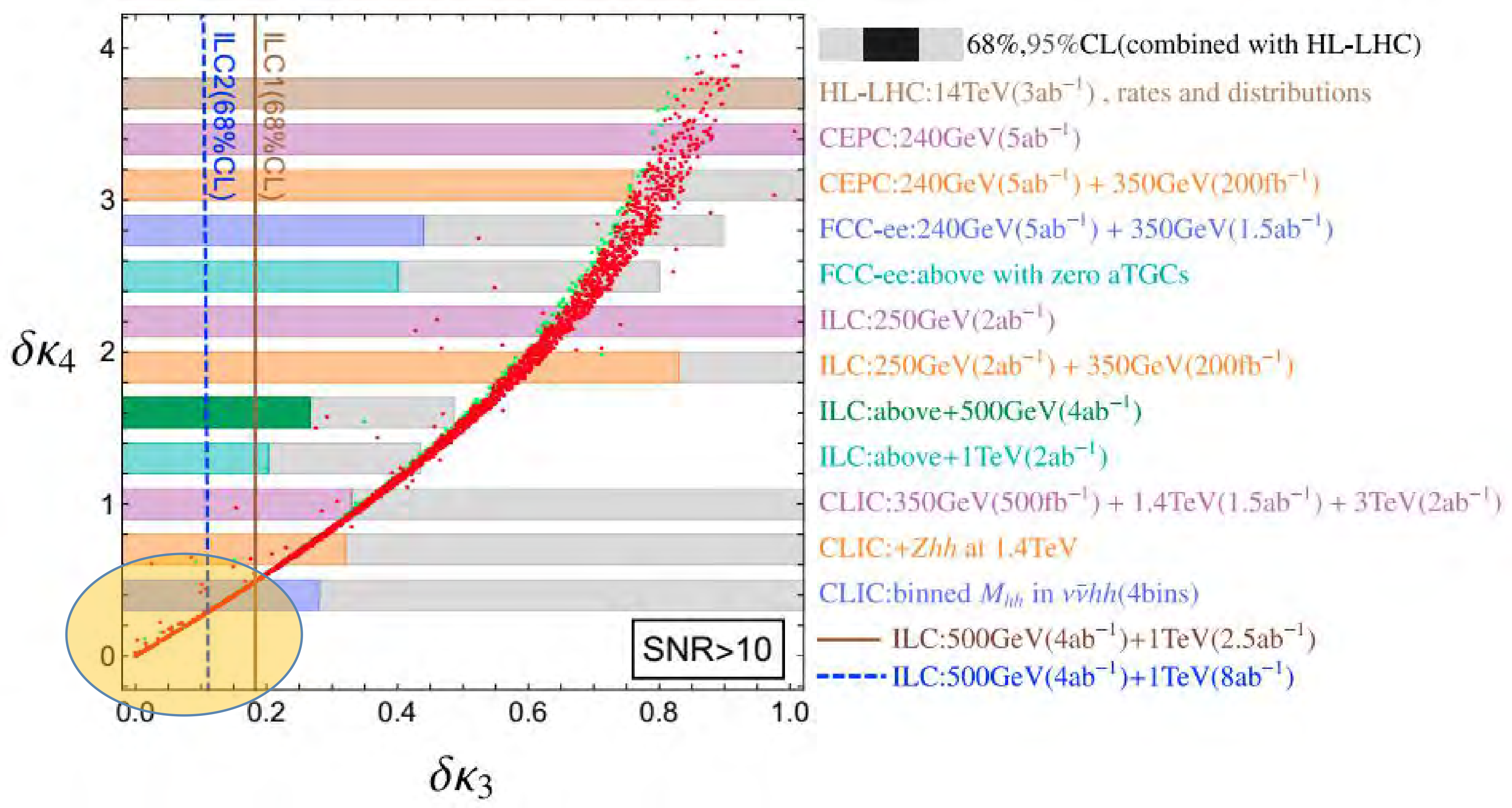
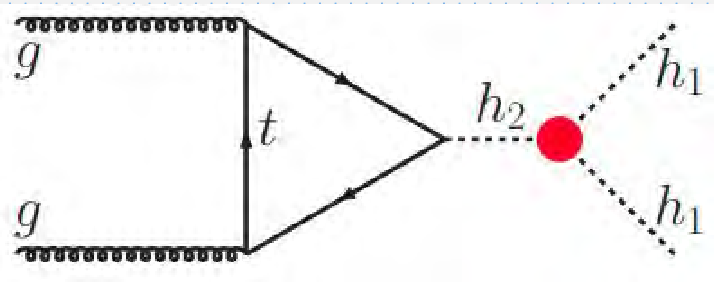
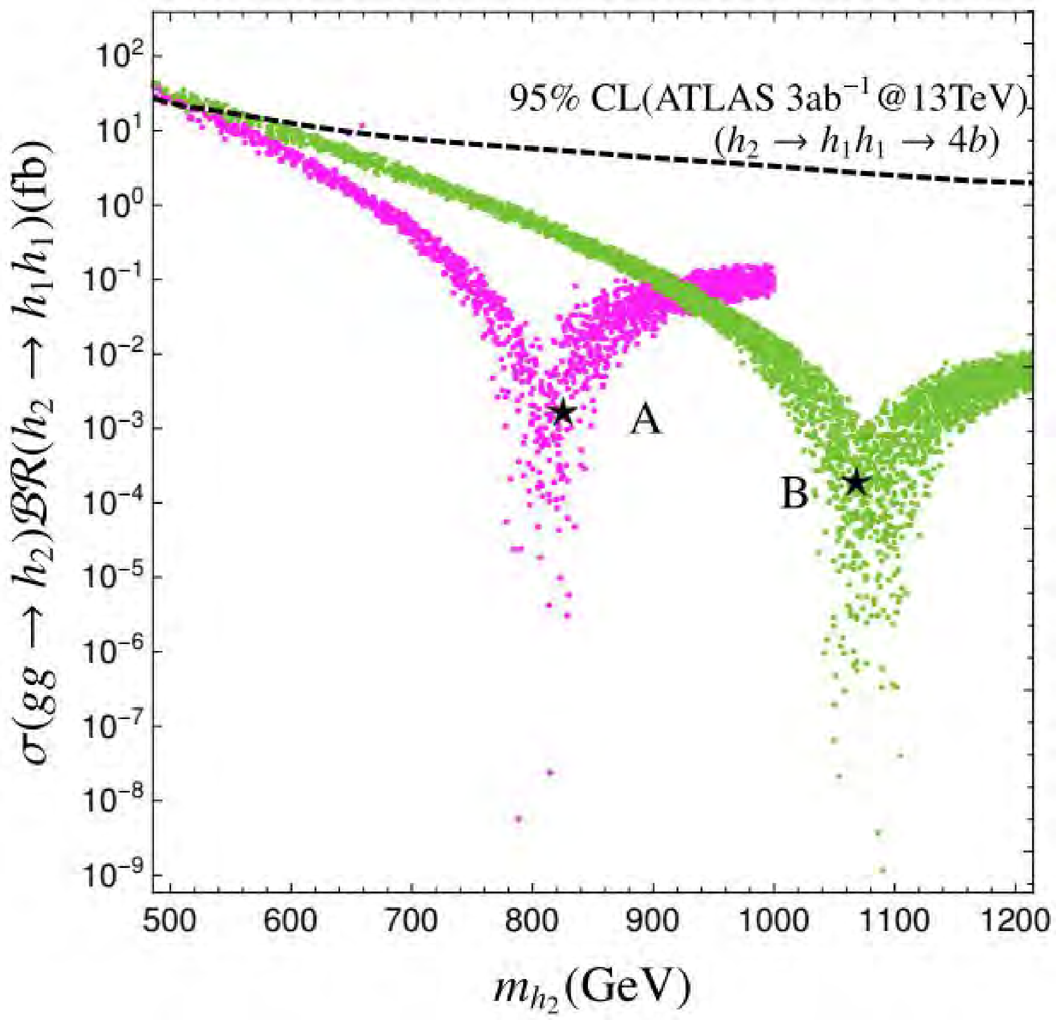


Alves, Ghosh, Guo, Sinha, Vagie, JHEP04(2019)052

Blind Spots of Collider Measurements

Most striking complmentarity
blind spot in Di-Higgs, but strong GWs





Alves, Goncalves, Ghosh, Guo, Sinha, Vagie, arxiv:2007.15654

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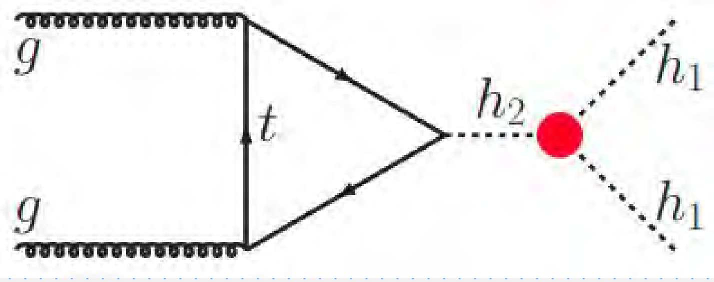

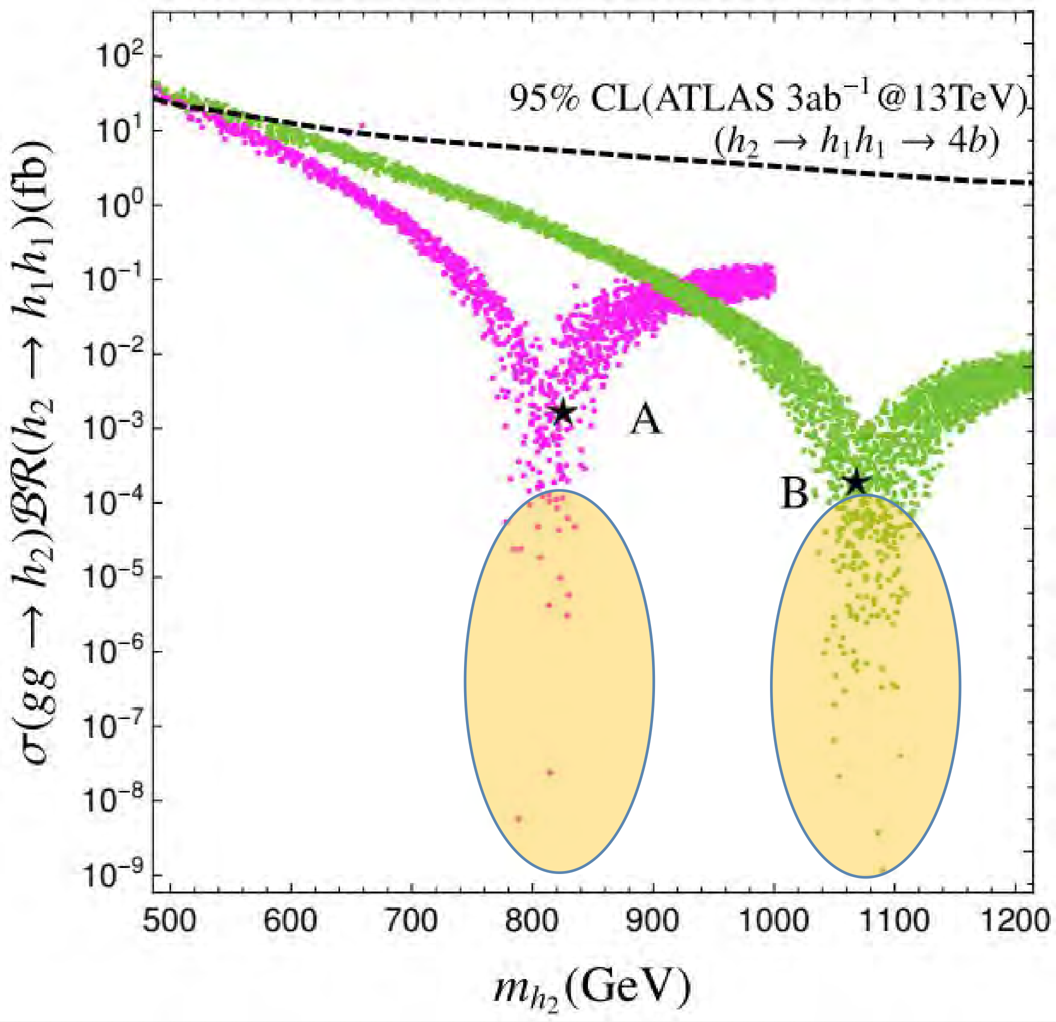

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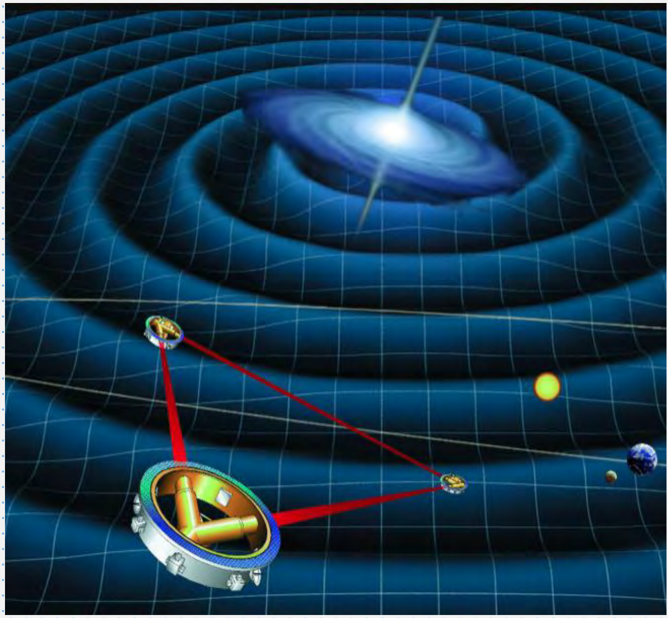
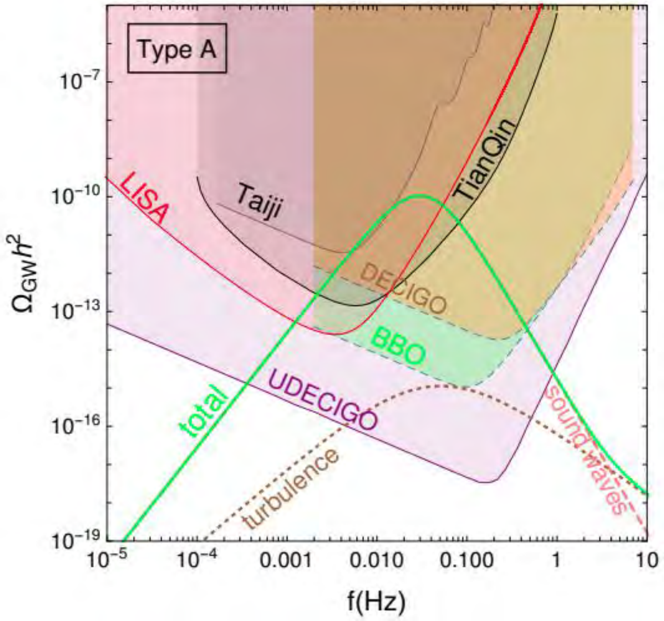
The Roadmap





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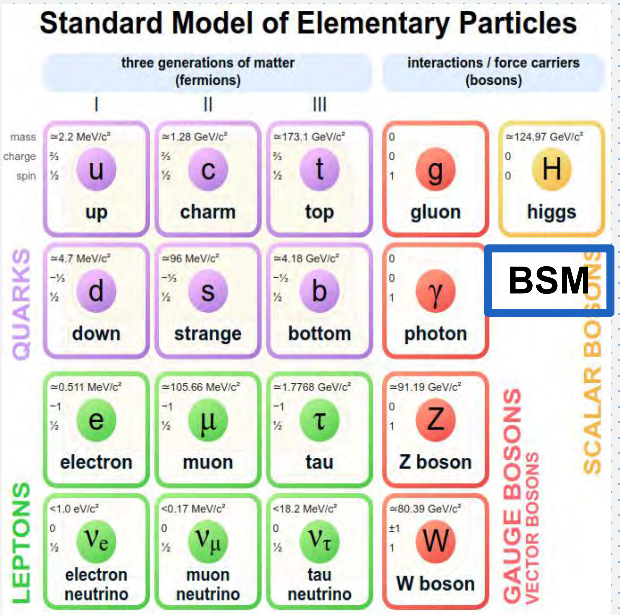

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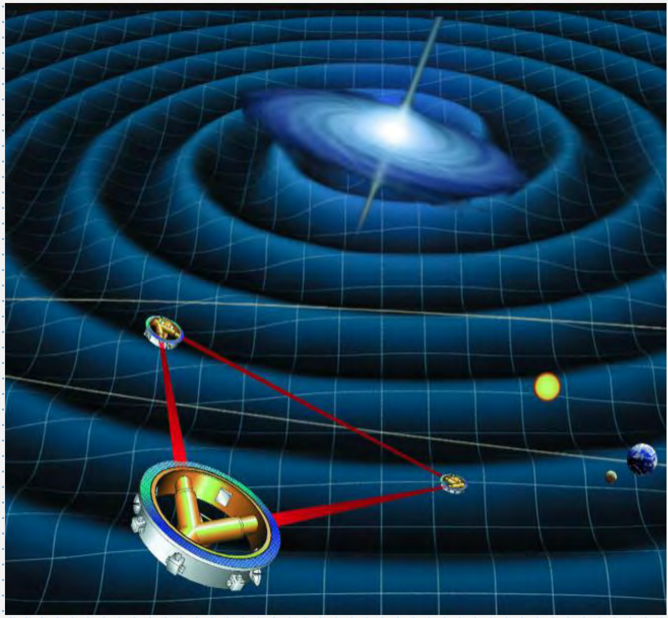
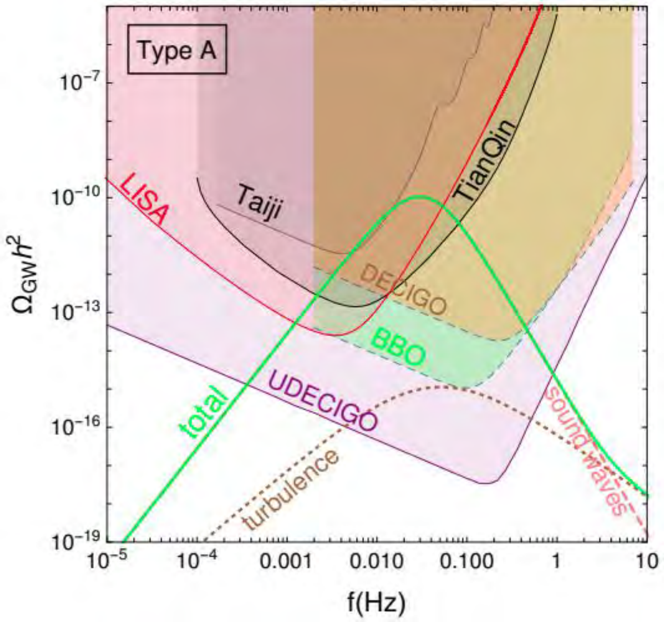
The Roadmap

theoretical prediction of power spectrum and simulation





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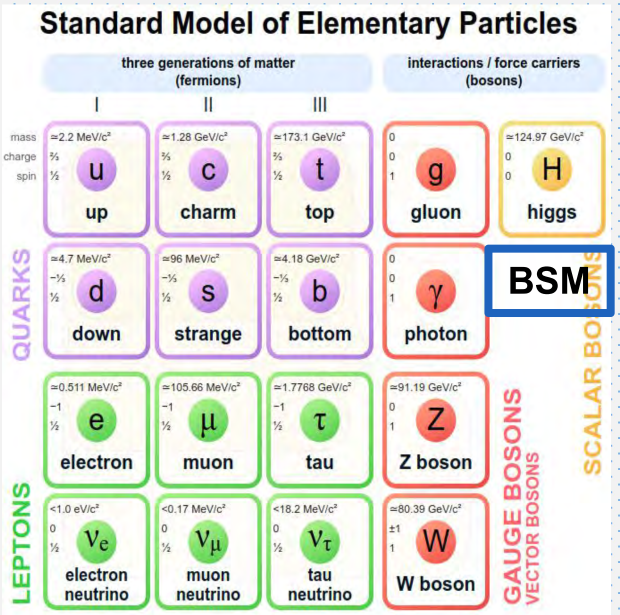

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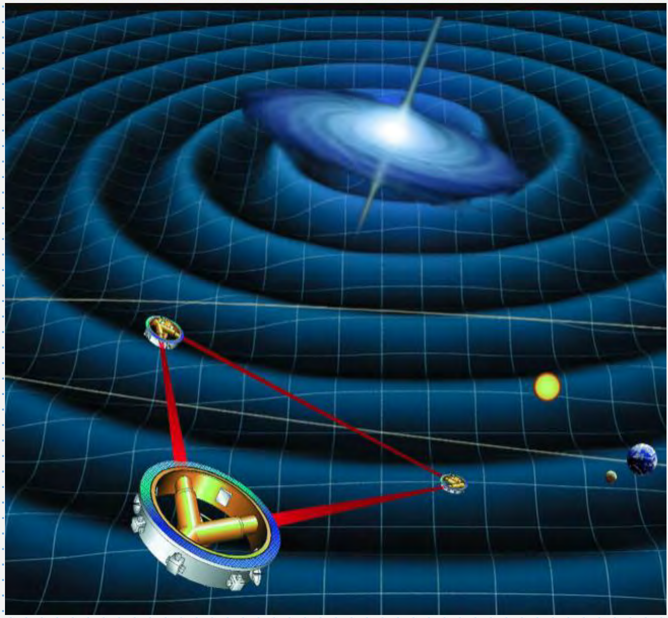

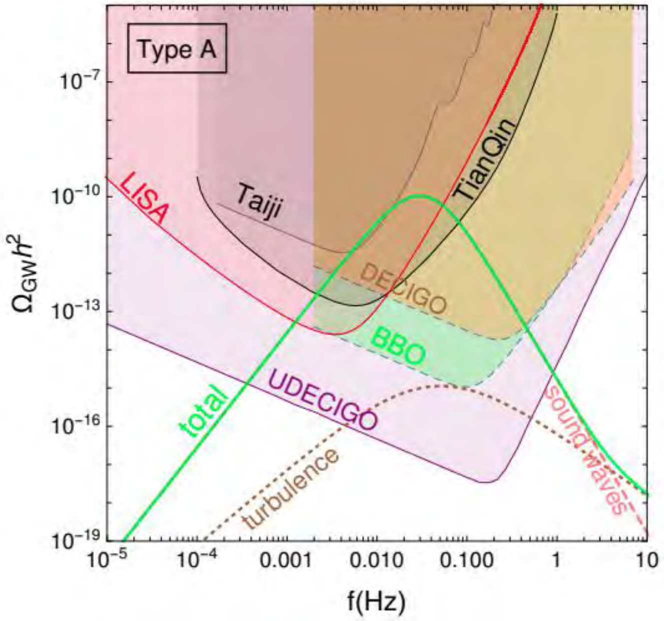

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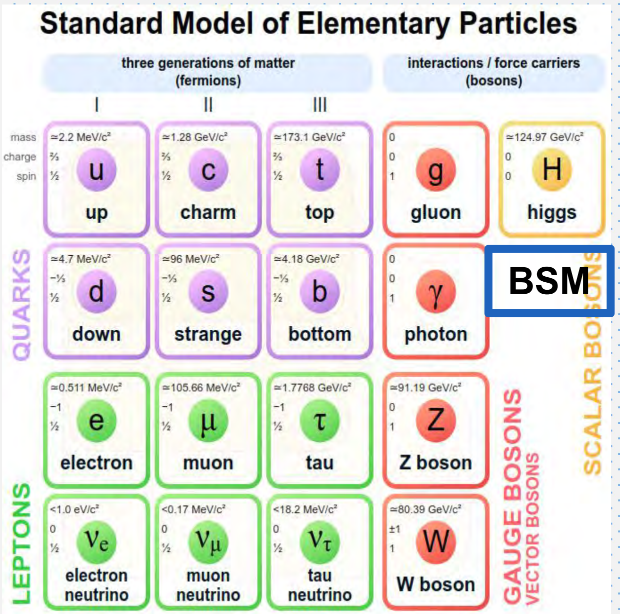


data analysis, constraints or discovery(parameter estimation)

e.g., LIGO O1, O2 results

Focus of the Following Talk

- Precision calculation of the gravitational wave spectrum

  Lay out the framework for modelling GW production in an expanding universe

  Any changes to the spectrum?

- Scrutize for hints of non-standard expansion histories

  Early matter domination(string moduli), Kination, Intermediate Inflationary stage(supercooling), etc

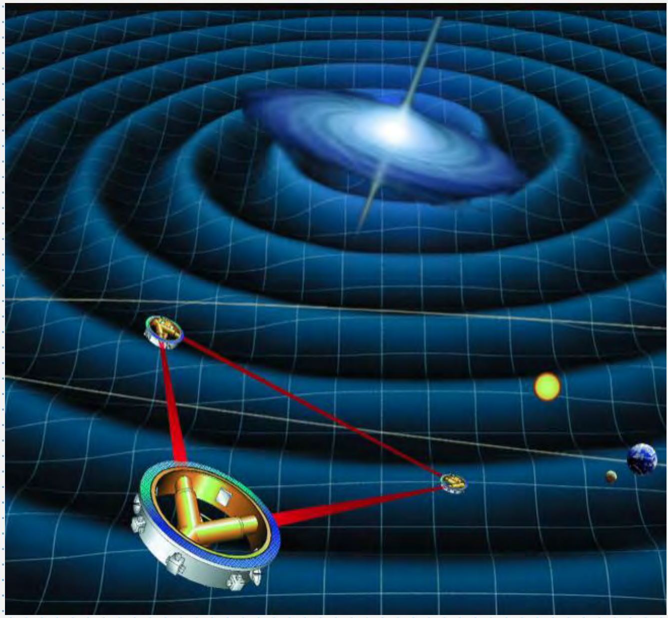
- Do we need a new simulation?

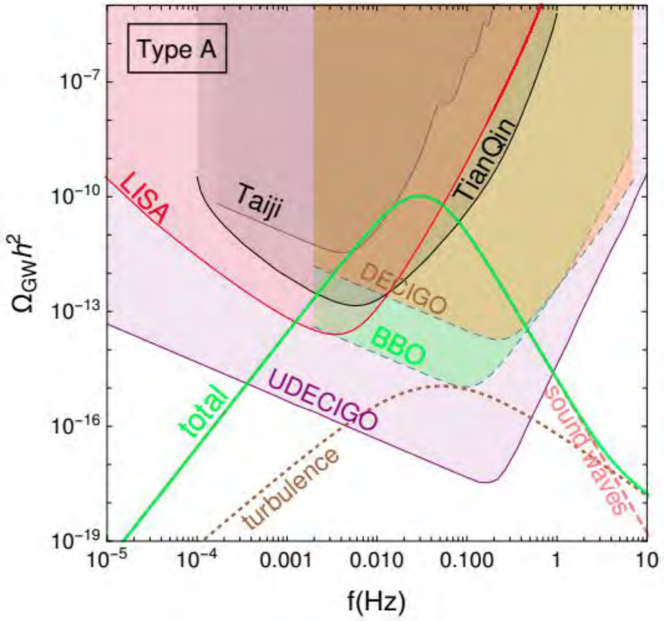
  Simulation is costly, and may not be necessary.

  Analytical insight of the underlying physics

The Roadmap

theoretical prediction of power spectrum and simulation





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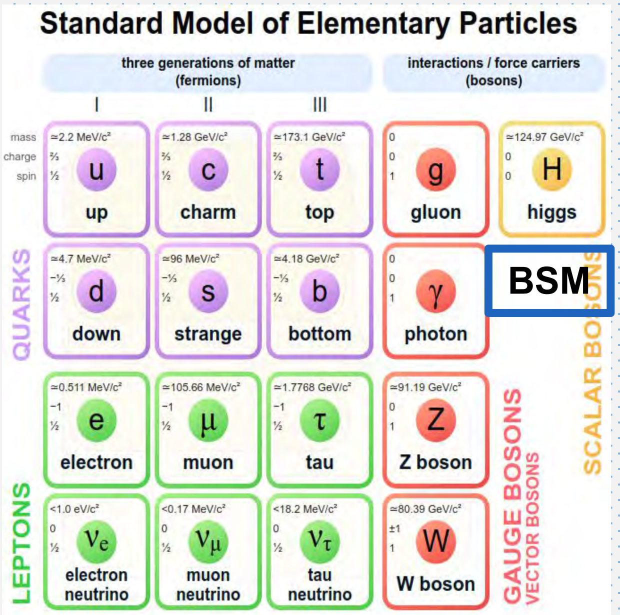

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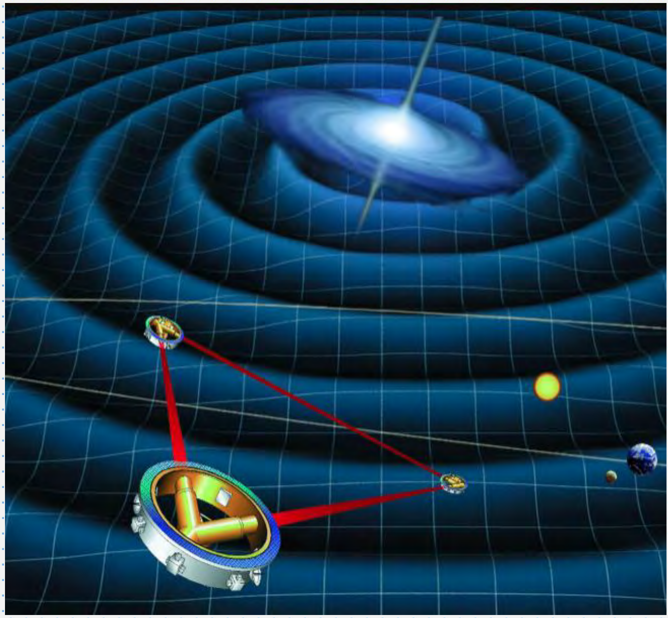

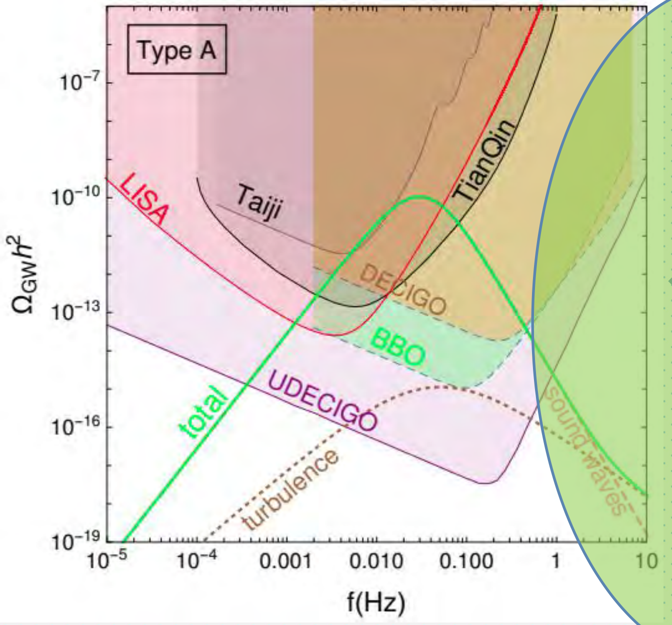

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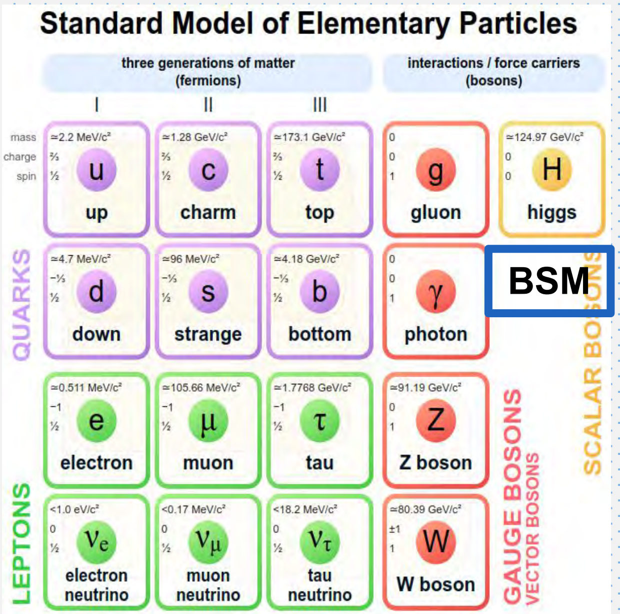

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three generations of matter (fermions) | interactions / force carriers (bosons)

	I	II	III		
mass charge spin	≈2.2 MeV/c² ⅔ ½ u up	≈1.28 GeV/c² ⅔ ½ c charm	≈173.1 GeV/c² ⅔ ½ t top	0 0 1 g gluon	≈124.97 GeV/c² 0 0 H higgs
QUARKS	≈4.7 MeV/c² −⅓ ½ d down	≈96 MeV/c² −⅓ ½ s strange	≈4.18 GeV/c² −⅓ ½ b bottom	0 0 1 γ photon	BSM
LEPTONS	≈0.511 MeV/c² −1 ½ e electron	≈105.66 MeV/c² −1 ½ μ muon	≈1.7768 GeV/c² −1 ½ τ tau	≈91.19 GeV/c² 0 1 Z Z boson	
	<1.0 eV/c² 0 ½ ν_e electron neutrino	<0.17 MeV/c² 0 ½ ν_μ muon neutrino	<18.2 MeV/c² 0 ½ ν_τ tau neutrino	≈80.39 GeV/c² ±1 1 W W boson	

GAUGE BOSONS
VECTOR BOSONS
SCALAR BOSONS

data analysis, constraints or discovery(parameter estimation)

e.g., LIGO O1, O2 results

How to Calculate Gravitational Waves?

$$ds^2 = -dt^2 + a^2(\delta_{ij} + \boxed{h_{ij}(\mathbf{x})})d\mathbf{x}^2$$

Tensor Mode

$$\Downarrow$$

$$\langle \dot{h}_{ij}(t,\mathbf{q})\dot{h}_{ij}(t,\mathbf{k})\rangle = (2\pi)^{-3}\delta^3(\mathbf{k}+\mathbf{q})P_{\dot{h}}(k,t)$$

$$\Downarrow$$

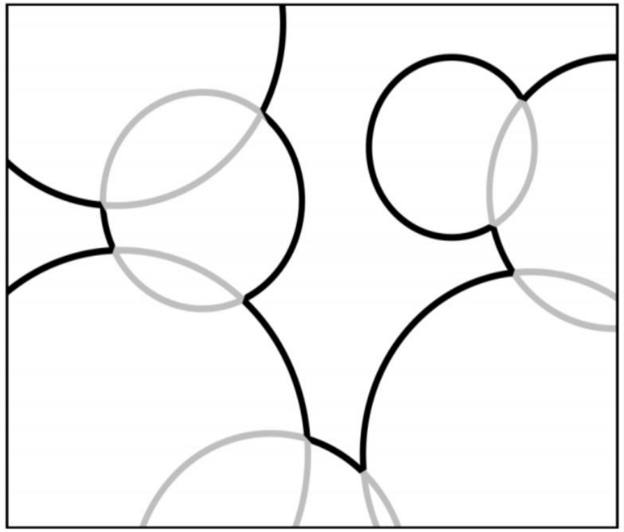
$$\frac{d\rho_{\rm GW}(t)}{d\ln k} = \frac{1}{64\pi^3 G}k^3 P_{\dot{h}}(t,k) \xrightarrow{\text{dilution, redshifting}} \text{GW Spectrum}$$

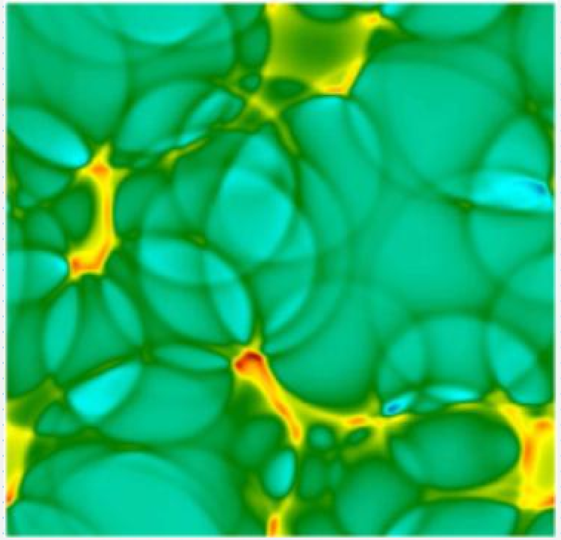
Einstein equation $$h_q'' + 2\frac{a'}{a}h_q' + q^2 h_q = 16\pi G a^2 \pi_q^T$$

Source evolutions Plasma(relativistic species), Matter(non-relativistic), Scalar field, EM
Energy-momentum conservation (hydrodynamic limit)

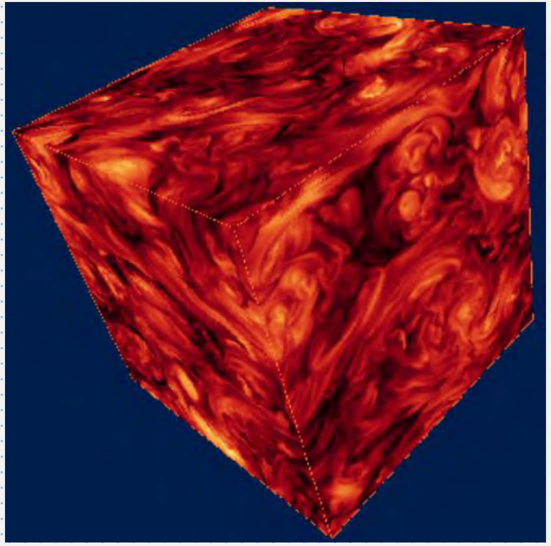
Sources for Gravitational Wave Production

- Bubble Collisions
- Sound Waves in Plasma → dominant in a thermal plasma
- MagnetoHydrodynamic Turbulence



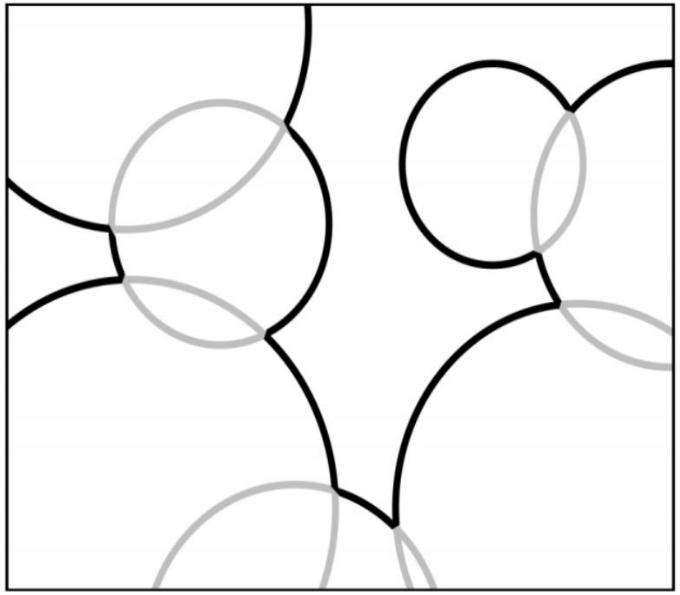


Hindmarsh, et al,PRL112,041301(2013)



https://home.mpcdf.mpg.de/~wcm/projects/homog-mhd/mhd.html

Bubble Collisions



Envelope Approximation

Simulations:
Kosowsky, Turner, Watkins, Kamionkowski
PRL69,2026(1992), PRD45,4514(1992), PRD47,4372(1993), PRD49,2837(1994)
Huber, Konstandin, JCAP09(2008)022
Analytical Modelling:
Jinno, Takimoto, PRD95,024009(2017)

Beyond the Envelope Approximation

Bulk flow model: Konstandin, JCAP03(2018)047, Jinno, Takimoto, JCAP01(2019)060
Direct large scalar lattice simulations: Cutting, Escartin, Hindmarsh, Weir, PRD97,123513(2018), arXiv:2005.13537:

$$\boxed{h^2\Omega_{\rm coll}(f) = 1.67\times 10^{-5}\Delta(v_w)\left(\frac{H_n}{\beta}\right)^2\left(\frac{\kappa_\phi\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{1/3} S_{\rm env}(f)}$$

LISA Cosmology Workinggroup, JCAP04(2016)001

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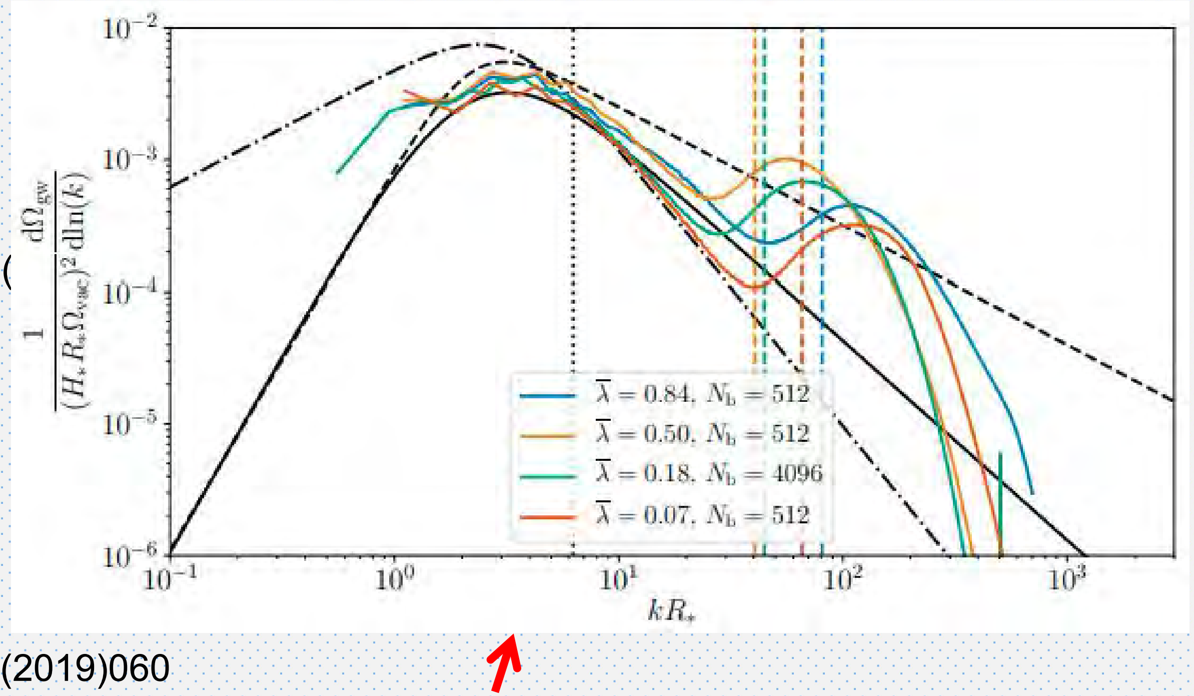

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LISA Cosmology Workinggroup, JCAP04(2016)001

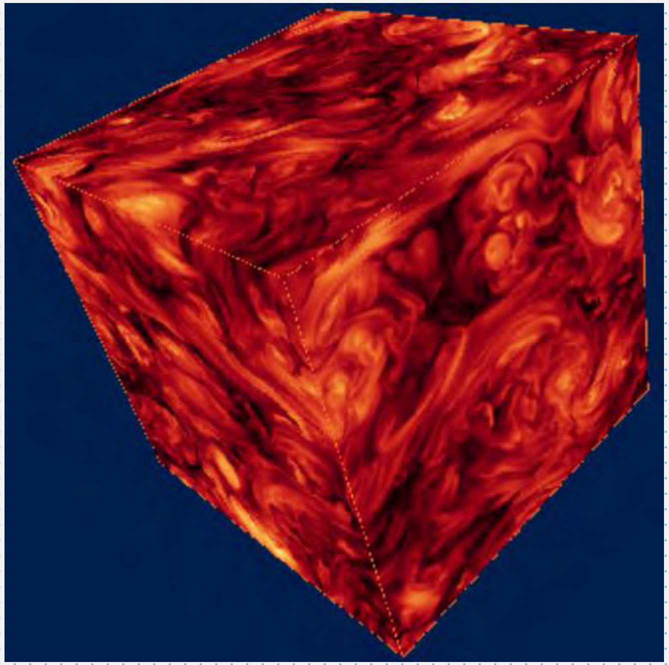
Magnetohydrodynamic Turbulence

Analytical Modelling

Kolmogorov spectrum:
Kosowsky, Mack,Kahniashvili, PRD66,024030(2002)
Gogoberidze, Kahniashvili, Kosowsky, PRD76,083002(2007)
Caprini, Durrer, Servant, JCAP12(2009)024

Numerical Simulations

Pol, Mandal, Brandenburg, Kahniashvili, Kosowsky, arxiv:1903.08585



https://home.mpcdf.mpg.de/~wcm/projects/homog-mhd/mhd.html

$$\boxed{h^2\Omega_{\text{turb}}(f) = 3.35 \times 10^{-4} \left(\frac{H_*}{\beta}\right) \left(\frac{\overset{\text{unknown}}{\kappa_{\text{turb}}}\,\alpha}{1+\alpha}\right)^{\frac{3}{2}} \left(\frac{100}{g_*}\right)^{1/3} v_w\, S_{\text{turb}}(f)}$$

LISA Cosmology Workinggroup, JCAP04(2016)001

Sound Waves

Numerical Simulations:

Hindmarsh, Huber, Rummukainen, Weir,
PRL112, 041301 (2014), PRD92, 123009 (2015), PRD96, 103520 (2017)
Reduction found: Cutting, Hindmarsh, Weir, PRL125, 021302 (2020)

Analytical Modelling(sound shell model)

Hindmarsh, 120, 071301 (2018)
Hindmarsh, Hijazi, JCAP12(2019)062

$$h^2\Omega_{\rm sw}(f) = 2.65 \times 10^{-6} \left(\frac{H_n}{\beta}\right)\left(\frac{\kappa_v \alpha}{1+\alpha}\right)^2 \left(\frac{100}{g_*}\right)^{\frac{1}{3}} \times v_w S_{\rm sw}(f)$$

LISA Cosmology Workinggroup, JCAP04(2016)001

The dominant source in a thermal plasma.

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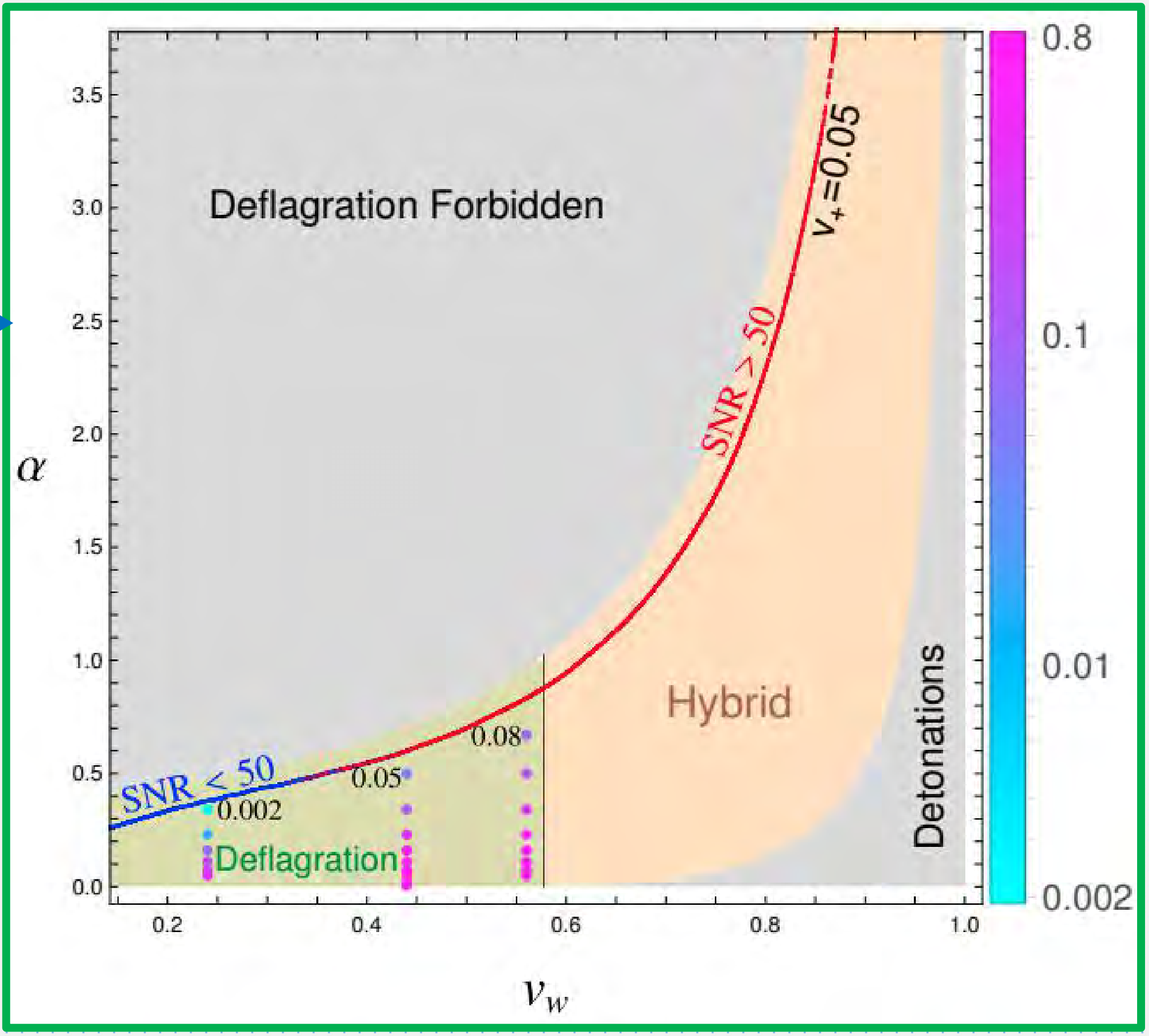

Alves, Goncalves, Ghost, Guo, Sinha, JHEP03(2020)053

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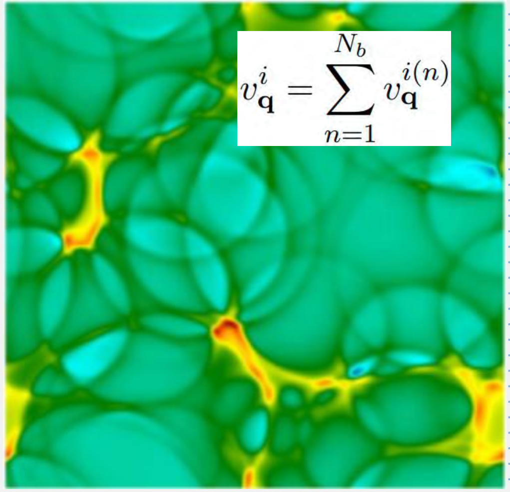
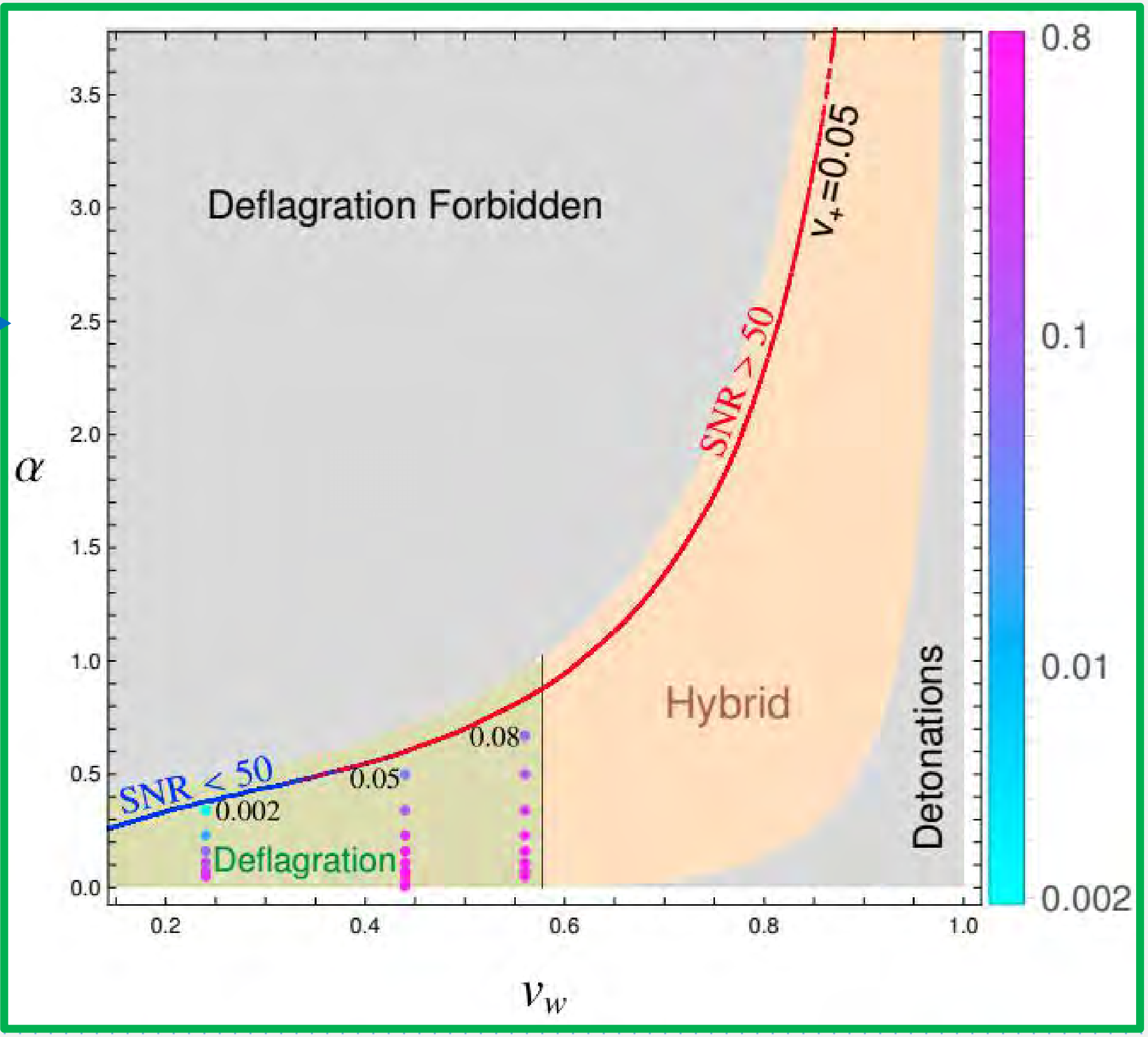


$$v^i_{\mathbf{q}} = \sum_{n=1}^{N_b} v^{i(n)}_{\mathbf{q}}$$



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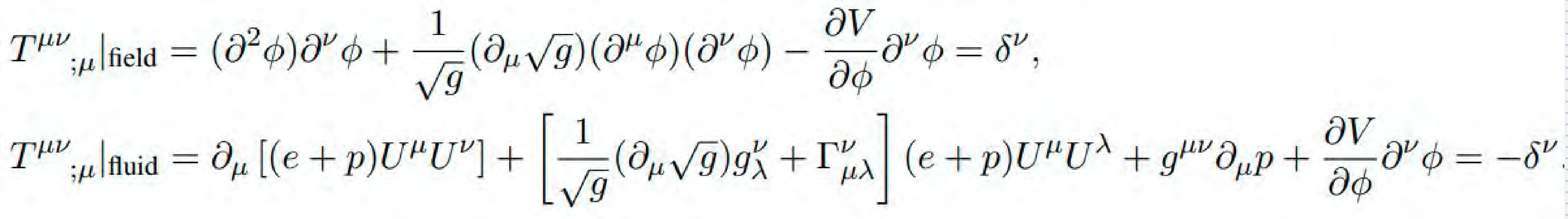
Hindmarsh, 120, 071301 (2018)
Hindmarsh, Hijazi, JCAP12(2019)062

Expanding Universe Analysis(Guo,Sinha,Vagie,White,arxiv:2007.08537):
Numerical simulations: equations in an expanding universe
Analytical modelling in an expanding universe(sound shell model)
Found an additional effect not captured in previous spectrum

How numerical simulations are performed?

- Realized only several years ago (Hindmarsh, et al,PRL112,041301,2013)



$$T^{\mu\nu}{}_{;\mu}|_{\text{field}} = (\partial^2\phi)\partial^\nu\phi + \frac{1}{\sqrt{g}}(\partial_\mu\sqrt{g})(\partial^\mu\phi)(\partial^\nu\phi) - \frac{\partial V}{\partial\phi}\partial^\nu\phi = \delta^\nu,$$

$$T^{\mu\nu}{}_{;\mu}|_{\text{fluid}} = \partial_\mu\left[(e+p)U^\mu U^\nu\right] + \left[\frac{1}{\sqrt{g}}(\partial_\mu\sqrt{g})g^\nu_\lambda + \Gamma^\nu_{\mu\lambda}\right](e+p)U^\mu U^\lambda + g^{\mu\nu}\partial_\mu p + \frac{\partial V}{\partial\phi}\partial^\nu\phi = -\delta^\nu$$

bubble generation → bubble, fluid, metric evolution → gravitational wave measurement

↓

partial differential equation solving on a lattice, difficult

Equations of Motion in an Expanding Universe

Scalar field:

(Klein-Gordon equation but with friction)

$$-\ddot{\phi} + \frac{1}{a^2}\nabla^2\phi - \frac{\partial V}{\partial \phi} - 3\frac{\dot{a}}{a}\dot{\phi} = \eta\gamma\left(\dot{\phi} + \frac{1}{a}\mathbf{v}\cdot\nabla\phi\right)$$

Parallel projection:

(Energy equation)

$$\dot{E} + p\left[\dot{\gamma} + \frac{1}{a}\nabla\cdot(\gamma\mathbf{v})\right] + \frac{1}{a}\nabla\cdot(E\mathbf{v}) - \gamma\frac{\partial V}{\partial\phi}\left(\dot{\phi} + \frac{1}{a}\mathbf{v}\cdot\nabla\phi\right) + 3\frac{\dot{a}}{a}\gamma(e+p) = \eta\gamma^2\left(\dot{\phi} + \frac{1}{a}\mathbf{v}\cdot\nabla\phi\right)^2,$$

$$\boxed{E \equiv e\gamma}$$

Vector equation:

(Euler equation)

$$\dot{Z}^i + \frac{1}{a}\nabla\cdot(\mathbf{v}Z^i) + 5\frac{\dot{a}}{a}Z^i + \frac{1}{a^2}\partial_i p + \frac{1}{a^2}\frac{\partial V}{\partial\phi}\partial_i\phi = -\frac{1}{a^2}\eta\gamma\left(\dot{\phi} + \frac{1}{a}\mathbf{v}\cdot\nabla\phi\right)\partial_i\phi,$$

$$\boxed{Z^i \equiv \gamma(e+p)U^i = \gamma^2(e+p)v^i/a.}$$

Can not simply reduce to Minkowski form.

But collision phase is short and the expansion effect should be small. New numerical simulations not needed.

Sound Waves when bubbles have all disappeared

- Equations of motion can be obtained by simply rescaling of Minkowski conterpart
- Sound waves(fluctuations of energy, pressure, velocity)

$$\boxed{\begin{aligned}&(a^4S^i)' + \nabla\cdot(a^4S^i\mathbf{v}) + \partial_i(a^4p) = 0, \quad \boxed{S^i = \gamma^2(\epsilon+p)v^i}\\&(a^4\epsilon\gamma)' + [\gamma' + \nabla\cdot(\gamma\mathbf{v})](a^4p) + \nabla\cdot(a^4\epsilon\gamma\mathbf{v}) = 0,\\&\gamma^2\left(v' + \frac{1}{2}\hat{\mathbf{v}}\cdot\nabla v^2\right)[a^4(\epsilon+p)] + v(a^4p)' + \hat{\mathbf{v}}\cdot\nabla(a^4p) = 0\end{aligned}}$$

conformal time

reduces to special relativistic Hydrodynamics when using rescaled quantities

How to Calculate Gravitational Waves Analytically?

$$ds^2 = -dt^2 + a^2(\delta_{ij} + h_{ij}(\mathbf{x}))d\mathbf{x}^2$$

Tensor Mode

$$\langle \dot{h}_{ij}(t,\mathbf{q})\dot{h}_{ij}(t,\mathbf{k})\rangle = (2\pi)^{-3}\delta^3(\mathbf{k}+\mathbf{q})P_{\dot{h}}(k,t)$$

$$\frac{d\rho_{\mathrm{GW}}(t)}{d\ln k} = \frac{1}{64\pi^3 G}k^3 P_{\dot{h}}(t,k)$$

GW Spectrum

Einstein equation

$$h_q'' + 2\frac{a'}{a}h_q' + q^2 h_q = 16\pi G a^2 \pi_q^T$$

neglect backreaction
solve with Green's function

Source evolutions

Plasma(relativistic species), Matter(non-relativistic), Scalar field, EM
Energy-momentum conservation (hydrodynamic limit)

The flow of calculations

$$T_{ij} = a^2 \left[p\delta_{ij} + (p+e)\gamma^2 v^i v^j \right]$$

$$\langle \pi^T_{ij}(\eta_1, \mathbf{k}) \pi^T_{ij}(\eta_2, \mathbf{q}) \rangle$$

$$\pi^T_{ij}(\mathbf{q}, \eta) = \frac{a_s^4}{a^4(\eta)} \tilde{\pi}^T_{ij}(\mathbf{q}, \eta)$$

$$v^i = dx^i / d\eta$$

$$\langle \tilde{v}^i_{\mathbf{q}}(\eta_1) \tilde{v}^{j*}_{\mathbf{k}}(\eta_2) \rangle = \delta^3(\mathbf{q} - \mathbf{k}) \hat{q}^i \hat{k}^j G(q, \eta_1, \eta_2)$$

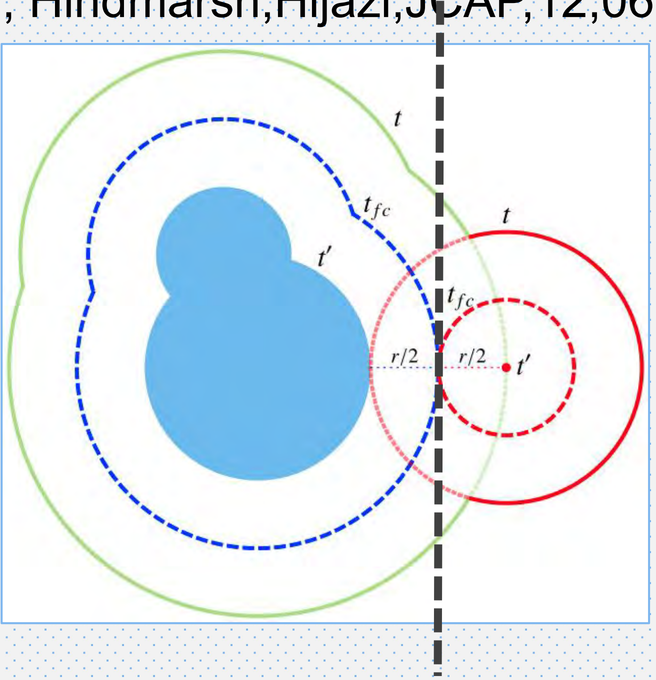
the key part

The Sound Shell Model

- The velocity field is a linear superposition of the contributions from all the bubbles

Hindmarsh, PRL,120,071301,2018, Hindmarsh,Hijazi,JCAP,12,062,2019

contribution from the red bubble



before collision: velocity profile

after collision: sound waves

$$v^i(\eta < \eta_{fc}, \mathbf{x}) = \frac{1}{2}\int \frac{d^3q}{(2\pi)^3}\left[\tilde{v}^i_{\mathbf{q}}(\eta)e^{i\mathbf{q}\cdot\mathbf{x}} + \tilde{v}^{i*}_{\mathbf{q}}(\eta)e^{-i\mathbf{q}\cdot\mathbf{x}}\right]$$

$$v^i(\eta, \mathbf{x}) = \int \frac{d^3q}{(2\pi)^3}\left[v^i_{\mathbf{q}}e^{-i\omega\eta+i\mathbf{q}\cdot\mathbf{x}} + v^{i*}_{\mathbf{q}}e^{i\omega\eta-i\mathbf{q}\cdot\mathbf{x}}\right]$$

$$v^i_{\mathbf{q}} = \sum_{n=1}^{N_b} v^{i(n)}_{\mathbf{q}}$$

Velocity Profile Around a Single Bubble

- Equations in the bag equation of state model can be reduced to Minkowski form

  vacuum energy cancels out, junction condition at the wall maintains the same form

- Velocity profile remain unchanged (time → conformal time)

Energy fluctuation:

$$(\xi - v)\partial_\xi \tilde{e} = \tilde{w}\left[2\frac{v}{\xi} + \gamma^2(1 - \xi v)\partial_\xi v\right]$$

Pressure fluctuation:

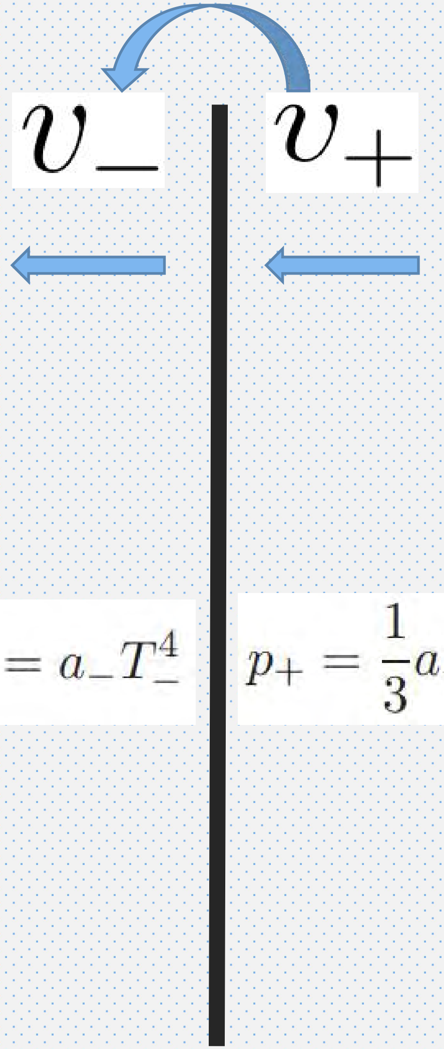
$$(1 - v\xi)\partial_\xi \tilde{p} = \tilde{w}\gamma^2(\xi - v)\partial_\xi v,$$

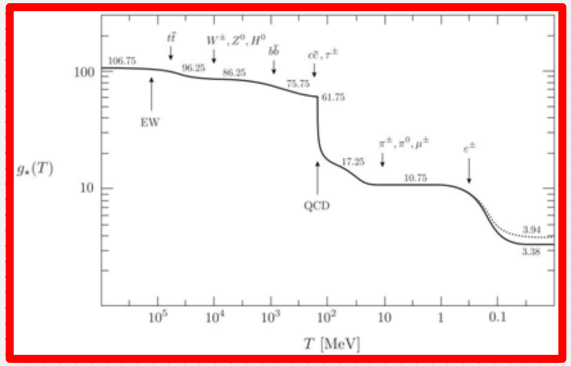
Velocity equation:

$$2\frac{v}{\xi} = \gamma^2(1 - v\xi)\left[\frac{\mu^2}{c_s^2} - 1\right]\partial_\xi v$$

self-similar coordinate: $r/\Delta\eta$

Velocity Profile Around a Single Bubble





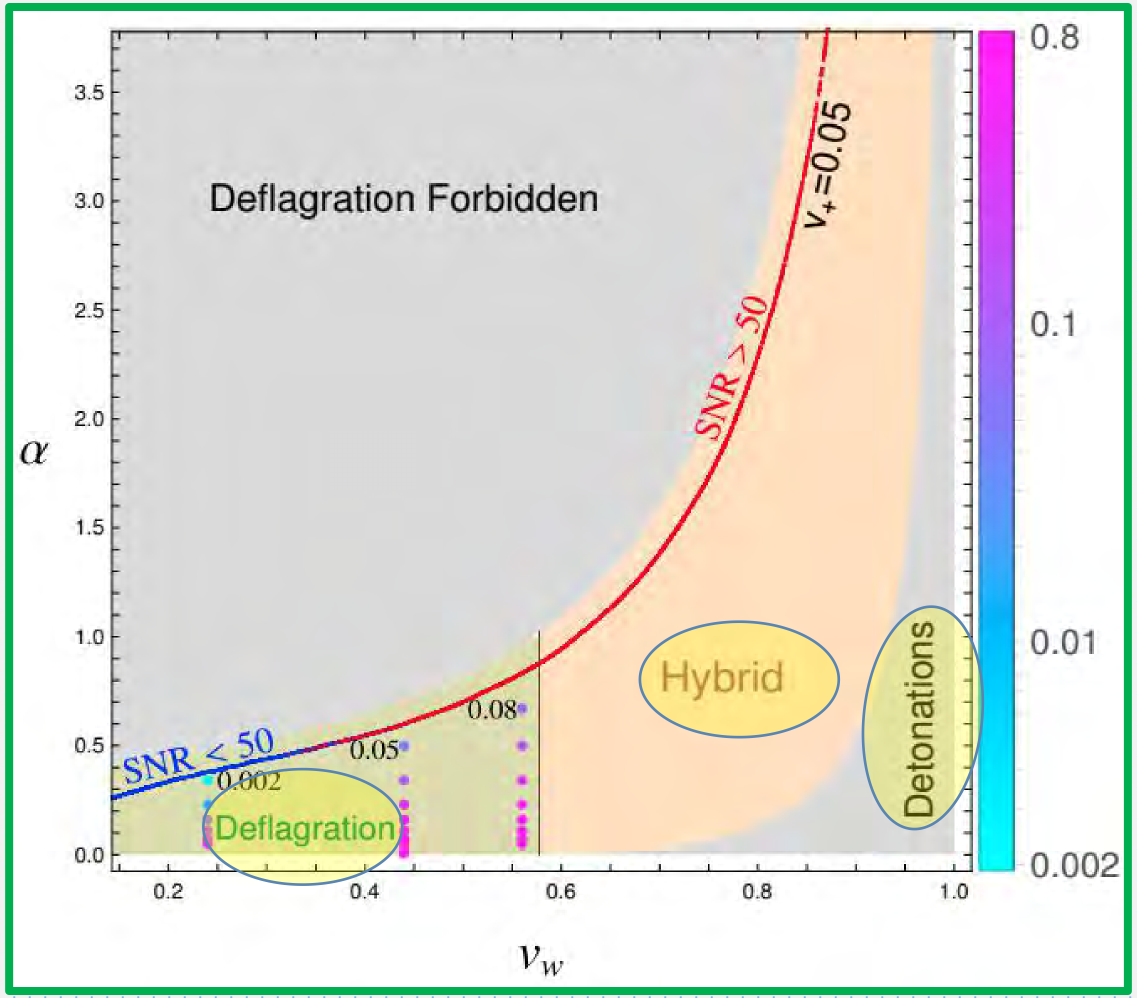
$$p_- = \frac{1}{3}a_- T_-^4 \,, \quad e_- = a_- T_-^4$$

$$p_+ = \frac{1}{3}a_+ T_+^4 - \epsilon \,, \quad e_+ = a_+ T_+^4 + \epsilon$$

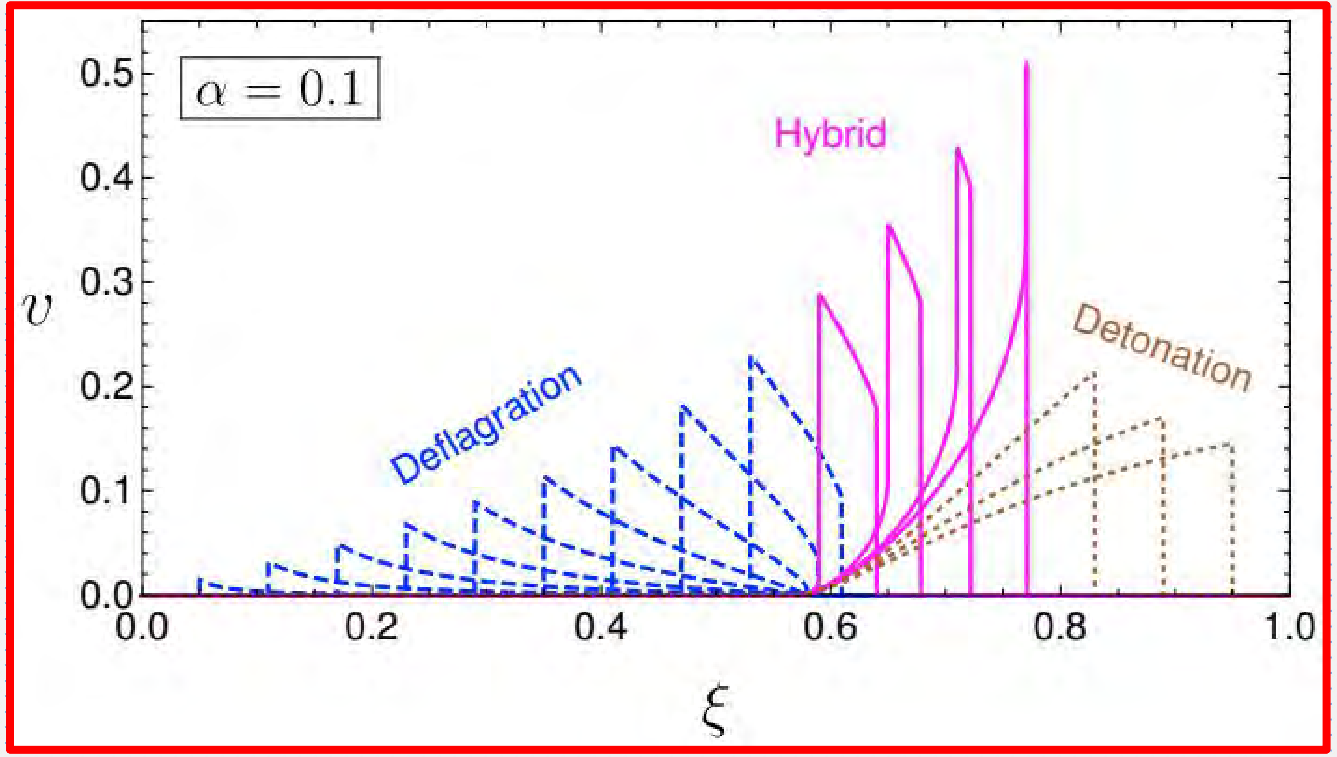
Wall Frame

Velocity Profile Around a Single Bubble

3 Fluid Modes



Velocity Profiles



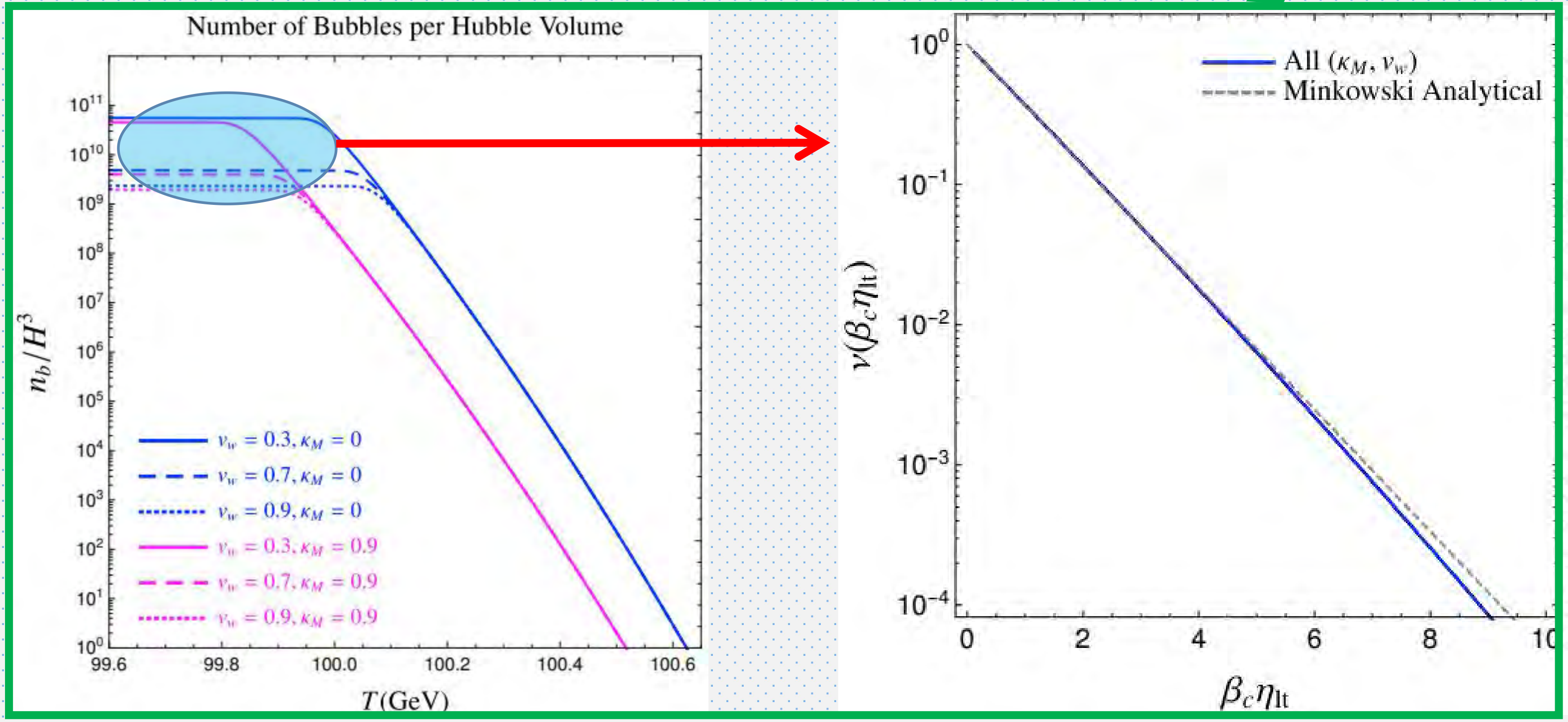
Alves, Ghost, Guo, Sinha, Vagie, JHEP04(2019)052

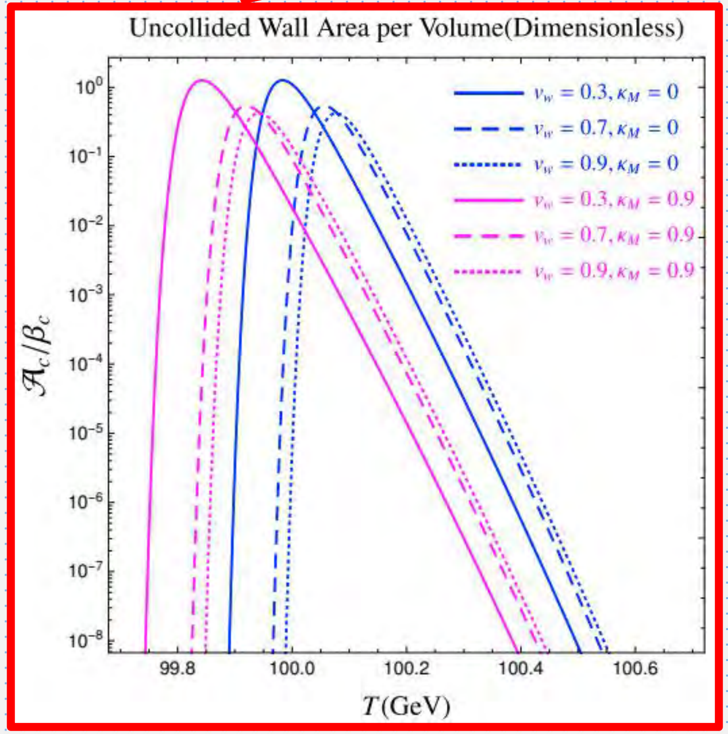
Velocity Field Power Spectrum

stochastic field: bubble position, formation time, collision time(final size)

after averaging, depends only on conformal lifetime distribution of the bubbles

$$\langle v^i_{\mathbf{q_1}} v^{j*}_{\mathbf{q_2}}\rangle = \hat{q}^i_1\hat{q}^j_2(2\pi)^3\delta^3(\mathbf{q}_1-\mathbf{q}_2)\underbrace{\frac{1}{R^3_{*c}\beta_c^6}\int d\tilde{T}\tilde{T}^6\nu(\tilde{T})|A(\frac{q\tilde{T}}{\beta_c})|^2}_{\equiv P_v(q)},$$





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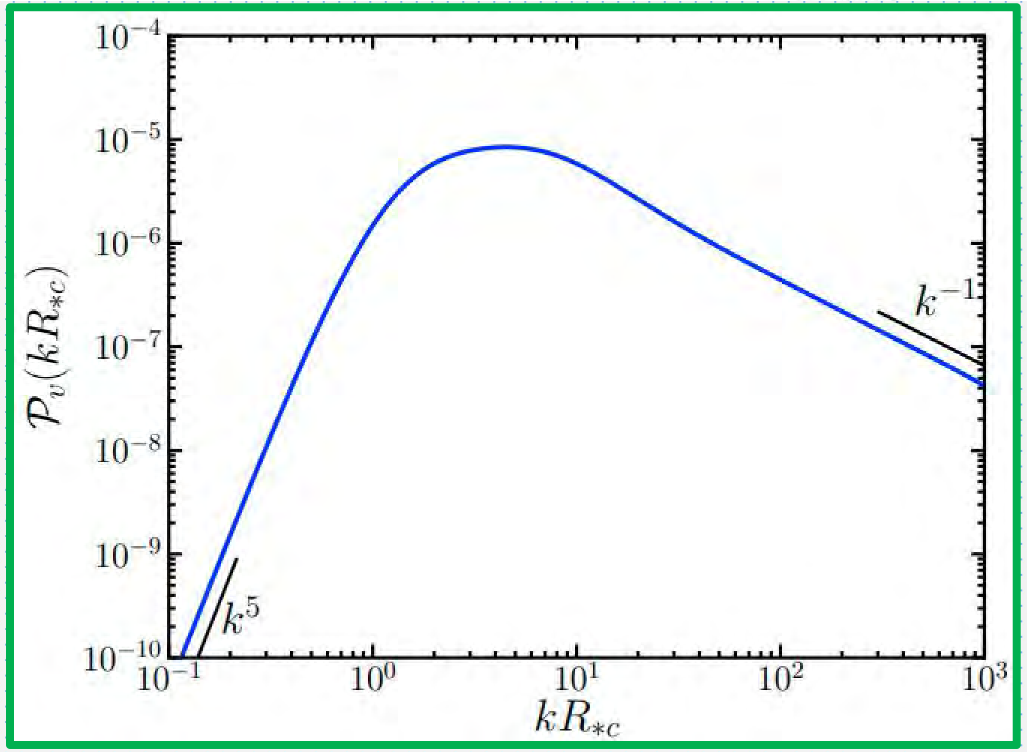


Gravitational Wave Power Spectrum

$$(\kappa_M y + 1 - \kappa_M)\frac{d^2 h_q}{dy^2} + \left[\frac{5}{2}\kappa_M + \frac{2(1-\kappa_M)}{y}\right]\frac{dh_q}{dy} + \tilde{\tilde{q}}^2 h_q = \frac{16\pi G a(y)^2 \pi_q^T(y)}{(a_s H_s)^2}$$

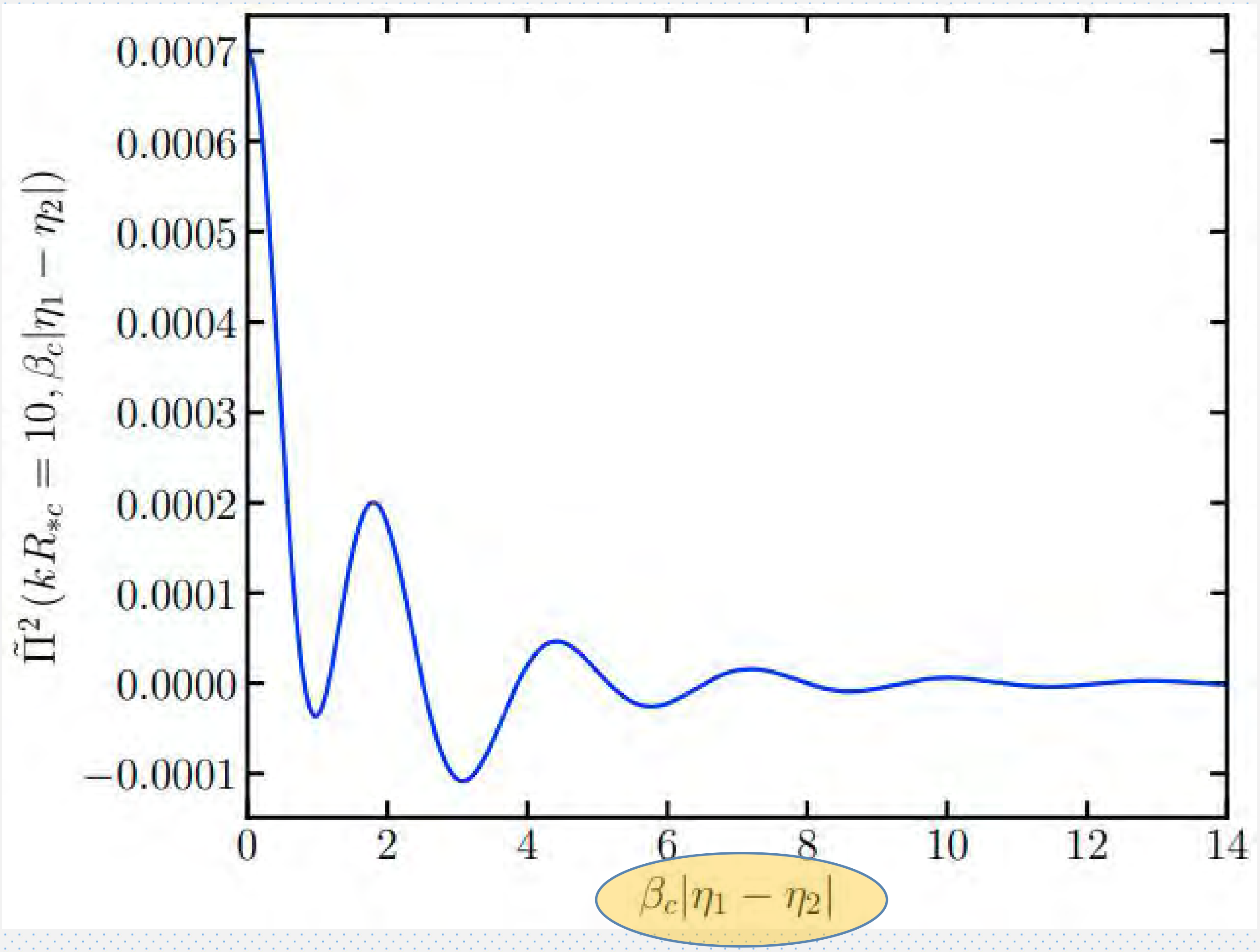
$y \equiv a/a_s$

$$h_{ij}(\tilde{y}, \mathbf{q}) = \int_{\tilde{y}_s}^{\tilde{y}} d\tilde{y}' G(\tilde{y}, \tilde{y}') \frac{16\pi G a(\tilde{y}')^2 \pi_{ij}^T(\tilde{y}', \mathbf{q})}{q^2}$$

$$P_{h'} = [16\pi G\,(\bar{\tilde{\epsilon}} + \bar{\tilde{p}})\,\bar{U}_f^2]^2 L_f^3 \int_{\tilde{y}_s}^{\tilde{y}} d\tilde{y}_1 \int_{\tilde{y}_s}^{\tilde{y}} d\tilde{y}_2 \left(\frac{\partial \tilde{y}}{\partial \tilde{\eta}}\right)^2 \frac{\partial G(\tilde{y}, \tilde{y}_1)}{\partial \tilde{y}} \frac{\partial G(\tilde{y}, \tilde{y}_2)}{\partial \tilde{y}} \times \frac{a_s^8}{a^2(\tilde{y}_1)a^2(\tilde{y}_2)} \frac{\tilde{\Pi}^2(kL_f, k\eta_1, k\eta_2)}{k^2}.$$

rescaled, dimensionless unequal time correlator

Gravitational Wave Power Spectrum

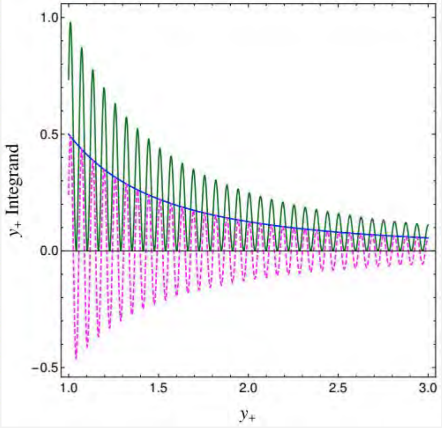


$y_- \sim \mathcal{O}(10^{-3})/v_w \times \beta_c(\eta_1 - \eta_2)$ source autocorrelation time very small

Gravitational Wave Power Spectrum

Under reasonal assumptions, the power spectrum can be written down in a similar form as in Minkowski space time

It is also essential to neglect highly oscillatory part to obtain the following result



$$\mathcal{P}_{\mathrm{GW}}(y, kR_{*c}) = 3\Gamma^2\, \bar{U}_f^4 \frac{H_{R,s}^4}{H^2 H_s} (a_s R_{*c}) \frac{(kR_{*c})^3}{2\pi^2} \tilde{P}_{\mathrm{gw}}(kR_*) \times \frac{1}{y^4} \Upsilon(y)$$

spectrum generally very similar to Minkowski result
but might have additional features when the assumptions can not be made

dilution for modes deep inside the horizon

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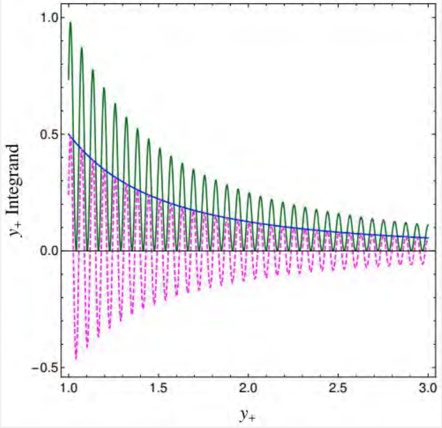


effect not captured before

$$\mathcal{P}_{\rm GW}(y,kR_{*c}) = 3\Gamma^2\,\bar{U}_f^4\,\frac{H_{R,s}^4}{H^2H_s}(a_sR_{*c})\frac{(kR_{*c})^3}{2\pi^2}\tilde{P}_{\rm gw}(kR_*)\times\frac{1}{y^4}\Upsilon(y)$$

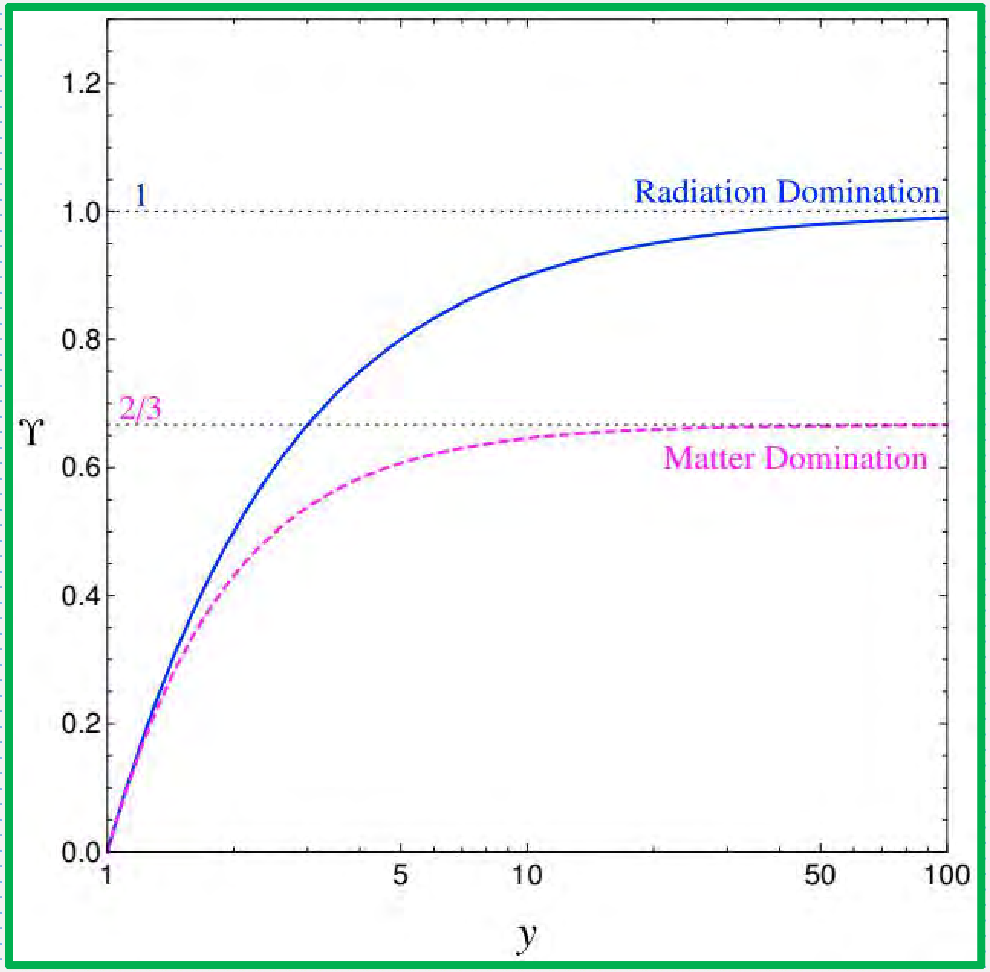
spectrum generally very similar to Minkowski result
but might have additional features when the assumptions can not be made

dilution for modes deep inside the horizon

Gravitational Wave Power Spectrum

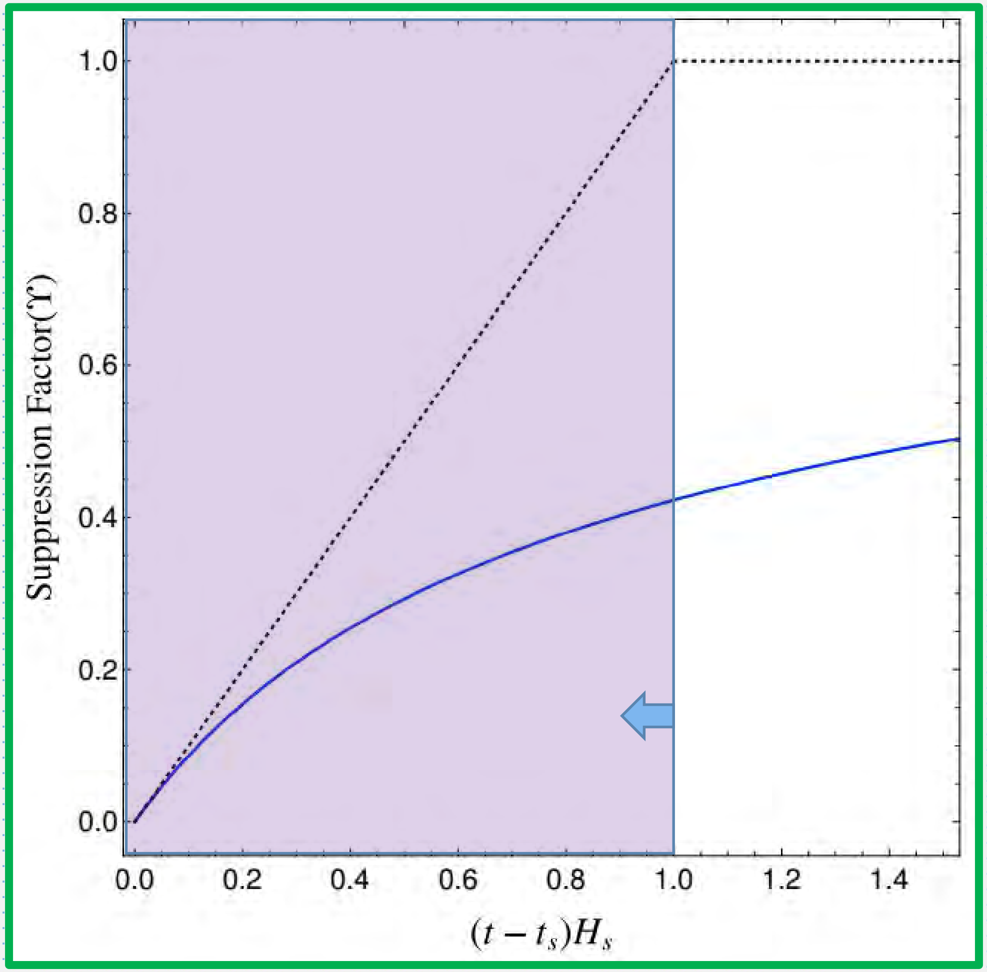
use spectrum from numerical simulation

$$h^2\Omega_{\rm GW}(f) = 8.5 \times 10^{-6} \left(\frac{100}{g_s(T_e)}\right)^{1/3} \Gamma^2 \bar{U}_f^4 \left[\frac{H_s}{\beta(v_w)}\right] v_w \mathcal{S}_{\rm SW}(f) \times \Upsilon(y)$$



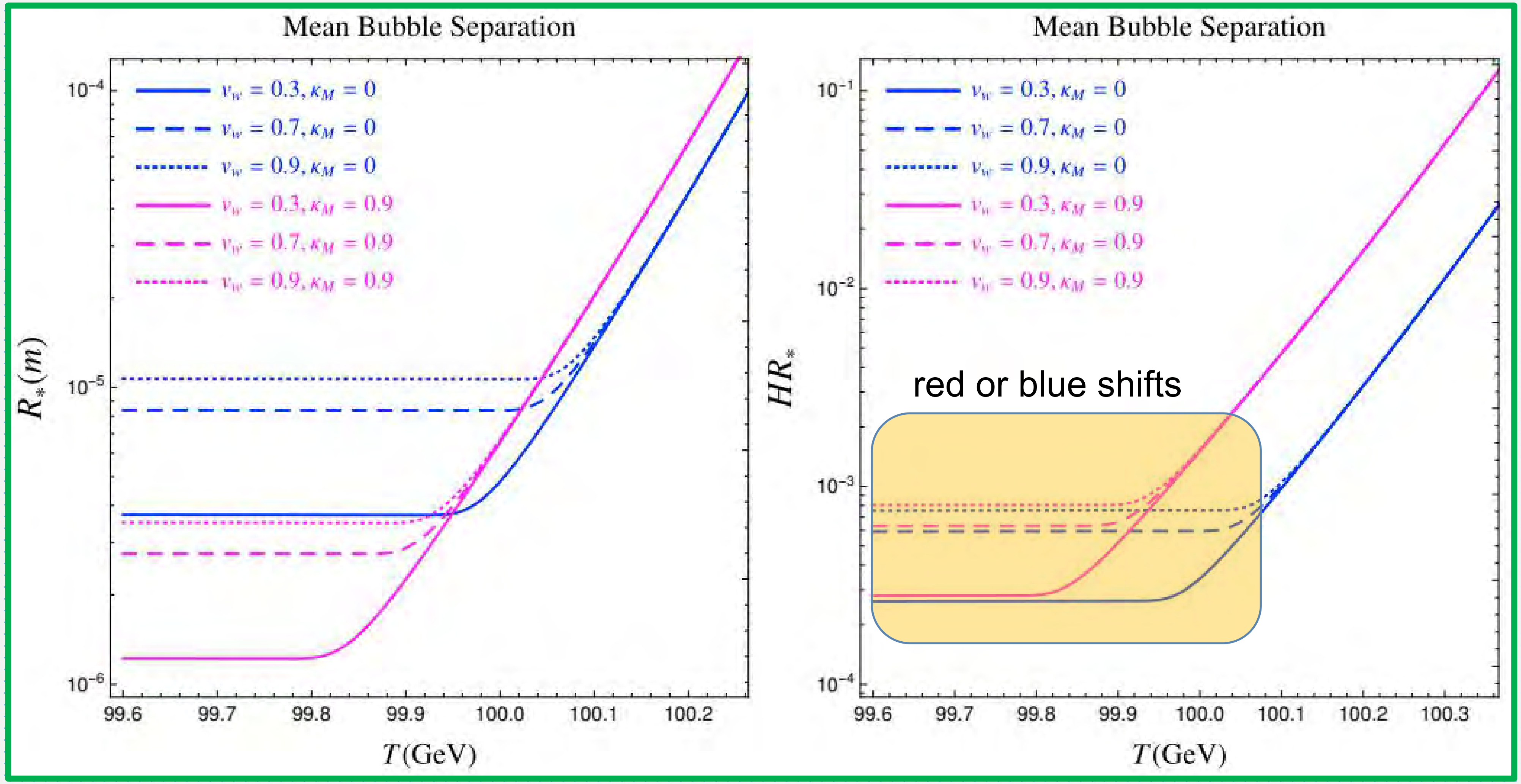
Lifetime of the Source

- Shocks, turbulence, dissipative processes all disrupt the source
- So lifetime is usually less than a Hubble time, meaning a suppression



Mean Bubble Separation

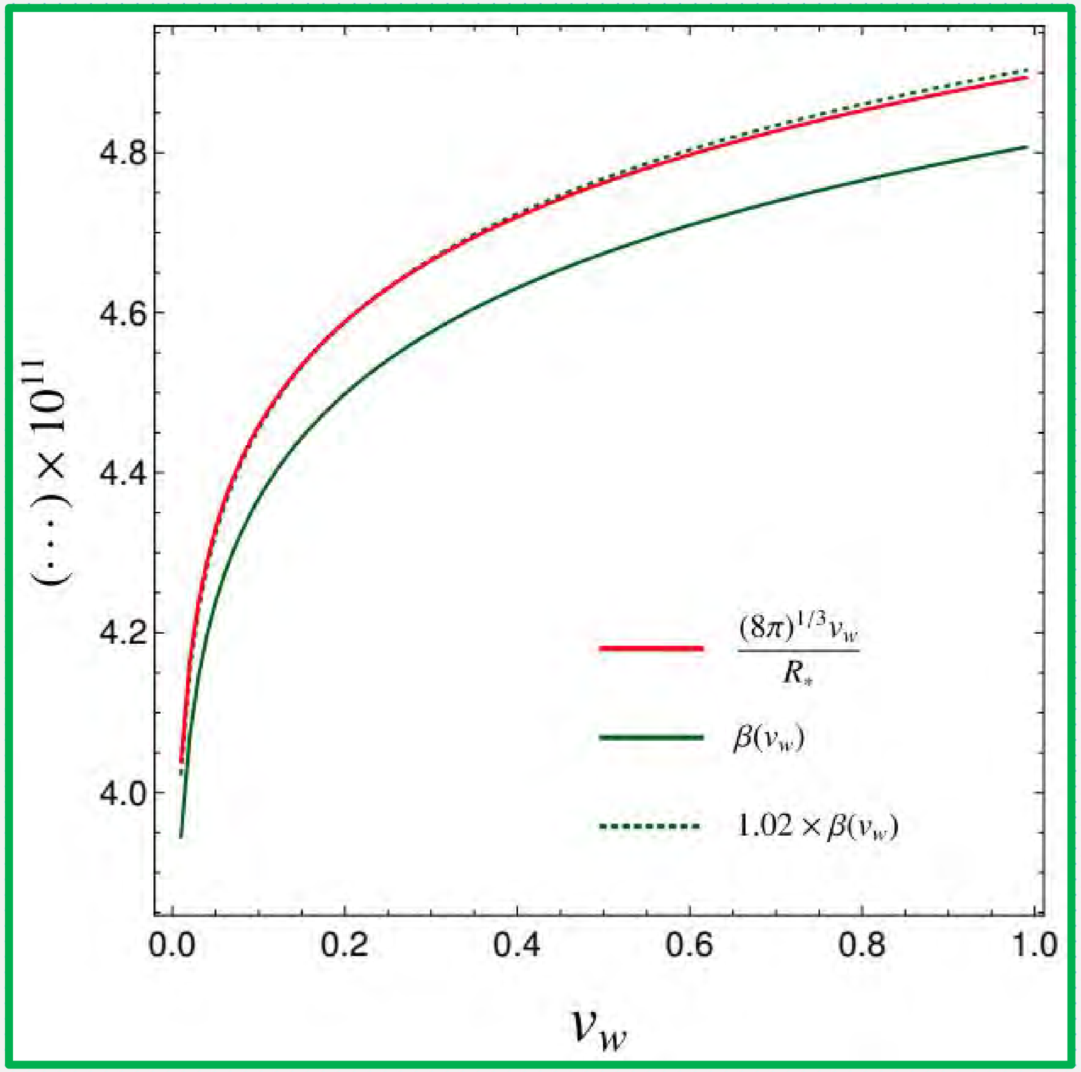
The (larger) scale that determines the spectral shape. Peak frequency at $kR_* = z_p$



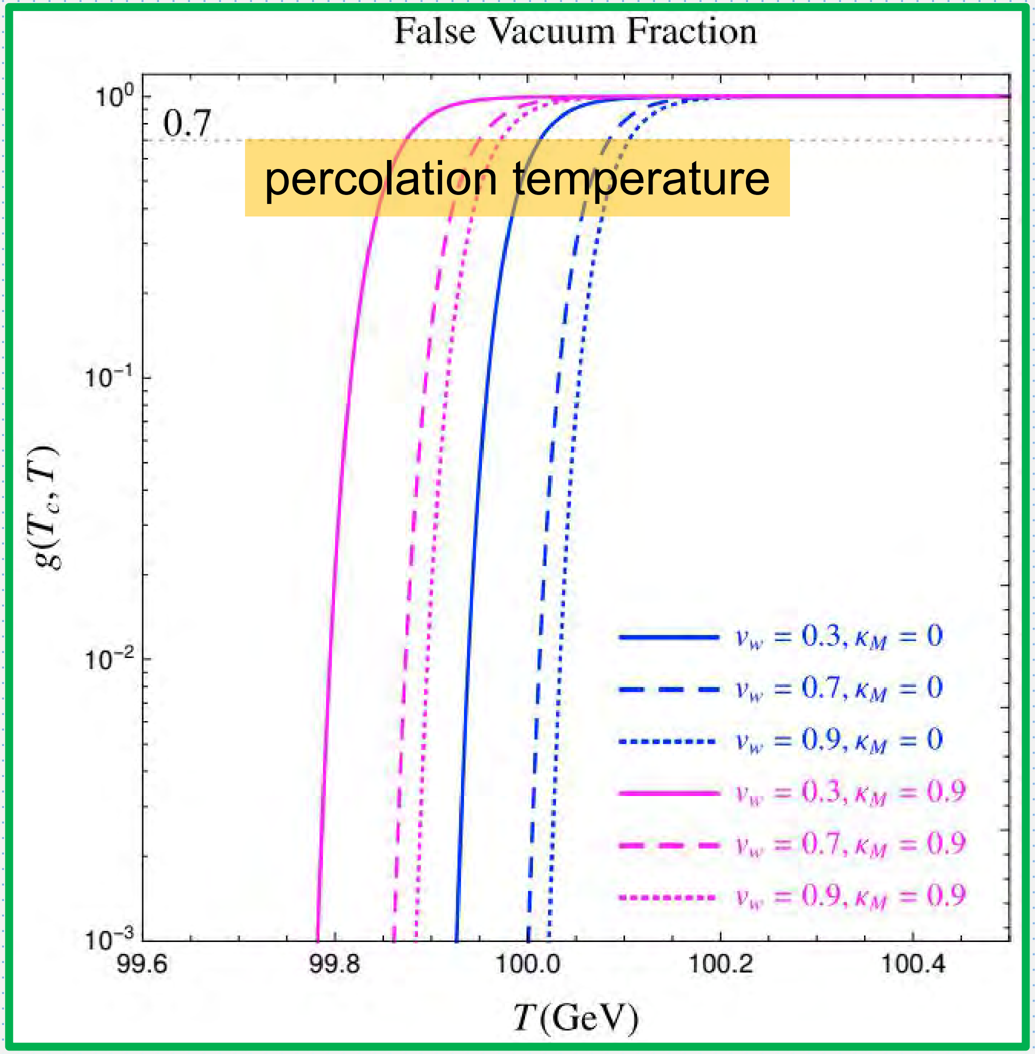
red or blue shifts

Relation between R* and Beta

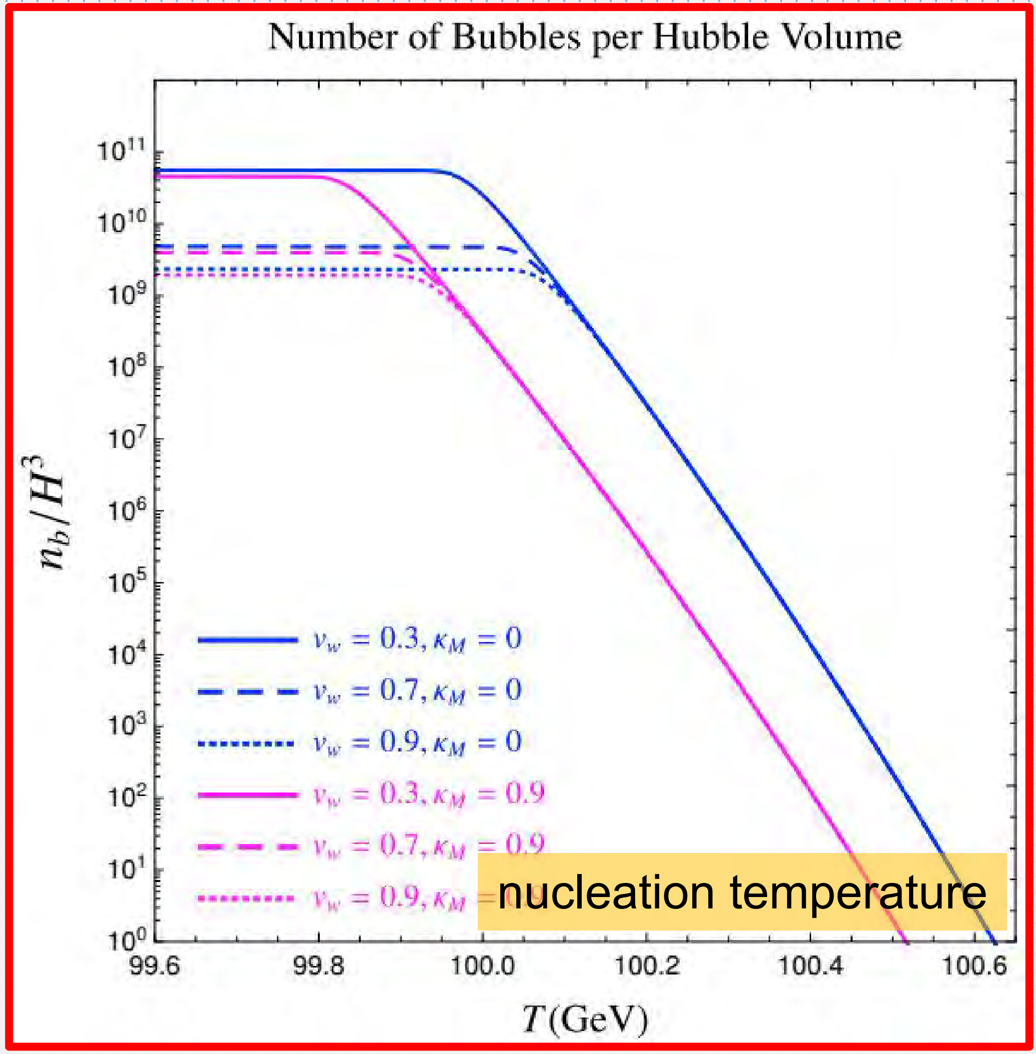
Relation unchanged for our redefined conformal quantities. For physical quantities: $R_*(\eta) = \frac{a(\eta)}{a(\eta_f)}(8\pi)^{1/3}\frac{v_w}{\beta(v_w)}$



Several Temperatuers


False Vacuum Fraction

percolation temperature


Number of Bubbles per Hubble Volume

nucleation temperature

Summary

- We have set up the framework for modelling the GW from sound waves, in an expanding unvierse

  generally no need for new simulations, rescaled quantities need to be used
- An additional multiplication factor is found and needs to be included to the generally used spectrum
- Details of the PT process is analyzed in standard and non-standard cosmic histories

Thanks!