Effective field theory approach to lepton number violation: dimension seven operators

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based on: Y. Liao and X.-D. Ma,

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Outline

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- SM very successful with all predicted particles discovered and their (gauge) interactions verified except that
  - neutrino mass and mixing?
  - dark matter?
- and that several fundamental questions remain to be answered:
  - gauge hierarchy problem or stability of Higgs mass
  - mechanism of spontaneous symmetry breaking
  - flavor puzzle
  - grand unification?
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Beyond SM – Focus on origin of Majorana m_ν

- Majorana neutrino?
  - Neutrino as a neutral fermion could be Majorana-type.
  - Underlying theory tends to favor Majorana neutrino mass – no special arrangement required.
  - Lepton number violation (LNV) – as a bonus baryon asymmetry in universe via leptogenesis?
- Question: Any other signals besides neutrino mass?
  - Experimentally:
    High-energy frontier: discover new particles/interactions at high energy colliders, with like-charge multi-leptons as typical signals for LNV
    High-intensity frontier: search for forbidden processes with large samples/extreme precision, e.g., nuclear $0\nu\beta\beta$ decay, $M_1^- \to M_2^+ \ell_\alpha^- \ell_\beta^-$, etc.

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    Study signals at colliders *model by model*; usually many free parameters, but definite answer once parameters fixed.
    Low energy processes are investigated by EFT; not specific for a particular model but possibly *for a class of models* with similar low energy effects; afflicted with hadronic/nuclear uncertainties.
- High and low energy studies are *complementary*.
  EFT offers a means to connect low and high energy processes: involving *a series of EFTs* from $\Lambda_{\rm NP}$ down to Λ_χ or nuclear phys scale
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Chart of EFT approach

Matching at scales	Series of EFTs	Interactions
	Underlying NP for LNV and m_ν	Fundamental theory/EFT
$\Lambda_{\rm NP} \gg \Lambda_{\rm EW}$		
	SMEFT	SM + high-dim operators
$\Lambda_{\rm EW} \sim 100$ GeV		
	LEFT	QCD + QED + high-dim operators
$\Lambda_\chi \sim$ GeV		
	Chiral effective theory	Interactions of mesons/nucleons
Nuclear~ MeV		
	Nuclear theory	Nuclear many body

This talk focuses on SMEFT.

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  - SM is a successful description of low energy physics.
  - $\Rightarrow$ SM better regarded as an EFT at Λ_{EW}.
- SM is *renormalizable* because it includes interactions from all possible operators up to dimension-4 (dim-4).
- If any underlying high-scale physics, its low-energy effects on SM particles amount to subleading higher-dim operators among SM fields – SMEFT.
- Summary of essential ingredients for SMEFT
  - Dynamical degrees of freedom – SM particles
  - Symmetries – guiding principle for interactions, SM gauge symmetry
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$$\mathscr{L}_{\rm SMEFT} = \mathscr{L}_{\rm SM} + \mathscr{L}_5 + \mathscr{L}_6 + \mathscr{L}_7 + \cdots .$$

- Higher-dim operators are suppressed by more powers of $1/\Lambda_{\rm NP}$.
- There is a finite number of independent operators at each dim.
- Two important things *at each dim*:
  - Find out a basis of indept, complete operators:

    necessary for S matrix and theory–experiment analysis;

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  - Renor. of operators $\mathscr{O}_i$ to incorporate RGE between $\Lambda_{\rm NP}$ and lower m:

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General literature on EFT

- H. Georgi, *On-shell effective field theory*, Nucl. Phys. B **361**, 339 (1991).
- H. Georgi, *Effective field theory*, Ann. Rev. Nucl. Part. Sci. **43**, 209 (1993).
- A. V. Manohar, *Effective field theories*, Lect. Notes Phys. **479**, 311 (1997) [hep-ph/9606222].
- D. B. Kaplan, *Five lectures on effective field theory*, nucl-th/0510023.
- C. P. Burgess, *Introduction to Effective Field Theory*, Ann. Rev. Nucl. Part. Sci. **57**, 329 (2007) [hep-th/0701053].
- W. Skiba, TASI lectures, *Effective Field Theory and Precision Electroweak Measurements*, arXiv:1006.2142 [hep-ph].

SMEFT: dim-5 operators

- Unique: $\Delta L = 2$, Majorana neutrino mass Weinberg 1979

$$\varepsilon_{ij}\varepsilon_{mn}(L_p^i C L_r^m) H^j H^n$$

L : LH lepton doublet
H : Higgs doublet
i, j, m, n : SU(2) indices
p, r, s, t : flavor indices

- RGE: in principle easy
  - 1 operator, involving only Yukawa and $\lambda(H^\dagger H)^2$, but
  - first result not quite correct: Babu-Leung-Pantaleone 1993
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SMEFT: dim-6 operators

- Long history on basis:
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Outline

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  20 independent and complete operators, including:

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- Our work: 1607.07309

  $18 = 12 + 6$ (count complete and indept. structures)
- Further extensions:

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Summary of basis of dim-7 operators

ψ^2H^4		ψ^2H^3D	
$\mathcal{O}_{LH}$	$\epsilon_{ij}\epsilon_{mn}(L^iCL^m)H^jH^n(H^\dagger H)$	$\mathcal{O}_{LeHD}$	$\epsilon_{ij}\epsilon_{mn}(L^iC\gamma_\mu e)H^jH^miD^\mu H^n$
$\psi^2H^2D^2$		ψ^2H^2X	
$\mathcal{O}_{LHD1}$	$\epsilon_{ij}\epsilon_{mn}(L^iCD^\mu L^j)H^m(D_\mu H^n)$	$\mathcal{O}_{LHB}$	$\epsilon_{ij}\epsilon_{mn}(L^iCi\sigma_{\mu\nu}L^m)H^jH^nB^{\mu\nu}$
$\mathcal{O}_{LHD2}$	$\epsilon_{im}\epsilon_{jn}(L^iCD^\mu L^j)H^m(D_\mu H^n)$	$\mathcal{O}_{LHW}$	$\epsilon_{ij}(\tau^I\epsilon)_{mn}(L^iCi\sigma_{\mu\nu}L^m)H^jH^nW^{I\mu\nu}$
ψ^4D		ψ^4H	
$\mathcal{O}_{\bar{d}uLLD}$	$\epsilon_{ij}(\bar{d}\gamma_\mu u)(L^iCiD^\mu L^j)$	$\mathcal{O}_{\bar{e}LLLH}$	$\epsilon_{ij}\epsilon_{mn}(\bar{e}L^i)(L^jCL^m)H^n$
$\mathcal{O}_{\bar{L}QddD}$	$(\bar{L}\gamma_\mu Q)(dCiD^\mu d)$	$\mathcal{O}_{\bar{d}LQLH1}$	$\epsilon_{ij}\epsilon_{mn}(\bar{d}L^i)(Q^jCL^m)H^n$
$\mathcal{O}_{\bar{e}dddD}$	$(\bar{e}\gamma_\mu d)(dCiD^\mu d)$	$\mathcal{O}_{\bar{d}LQLH2}$	$\epsilon_{im}\epsilon_{jn}(\bar{d}L^i)(Q^jCL^m)H^n$
		$\mathcal{O}_{\bar{d}LueH}$	$\epsilon_{ij}(\bar{d}L^i)(uCe)H^j$
	all non-Herm., breaking L:	$\mathcal{O}_{\bar{Q}uLLH}$	$\epsilon_{ij}(\bar{Q}u)(LCL^i)H^j$
	$\Delta B = -\Delta L = 1$	$\mathcal{O}_{\bar{L}dud\tilde{H}}$	$(\bar{L}d)(uCd)\tilde{H}$
	$\Delta B = 0,\ \Delta L = 2$	$\mathcal{O}_{\bar{L}dddH}$	$(\bar{L}d)(dCd)H$
	flavor/color indices implied	$\mathcal{O}_{\bar{e}Qdd\tilde{H}}$	$\epsilon_{ij}(\bar{e}Q^i)(dCd)\tilde{H}^j$
	$(\psi C\chi) = \overline{\psi^C}\chi,\ (\psi^C)^C = \psi$	$\mathcal{O}_{\bar{L}dQQ\tilde{H}}$	$\epsilon_{ij}(\bar{L}d)(QCQ^i)\tilde{H}^j$

redundant operators

$\mathcal{O}^{(2)}_{\bar{d}uLLD}$	$\epsilon_{ij}(\bar{d}\gamma_\mu u)(L^iC\sigma^{\mu\nu}D_\nu L^j)$	$\mathcal{O}_{\bar{L}dQdD}$	$(\bar{L}iD^\mu d)(QC\gamma_\mu d)$

Removing redundancy

- Available: equations of motion (EoM), integration by parts (IBP), Fierz identities (Fierz), group identities.
- Notations:
  p,r,s,t,u,v,w for flavors;
  i,j,k,l for fundamental of $SU(2)_L$;
  α,β,σ,ρ for fundamental of $SU(3)_C$
- Examples of EoM from $\mathscr{L}_{\rm SM}$:

$$i\not{D}L = Y_e e H, \quad i\not{D}d = Y_d^\dagger H^\dagger Q$$

- Judicious application of generalized Fierz identities (non-contracted/involving C):
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Removing redundancy

subset $\Delta L = 2,\ \Delta B = 0$:

$$\begin{aligned}
\mathcal{O}_{\bar{d}uLLD}^{(2)prst} &= \epsilon_{ij}(\bar{d}_p\gamma_\mu u_r)(L_s^i C\sigma^{\mu\nu} D_\nu L_t^j) \qquad i\gamma^\mu\gamma^\nu - ig^{\mu\nu} \\
&= \epsilon_{ij}(\bar{d}_p\gamma_\mu u_r)(L_s^i C\gamma^\mu\gamma^\nu iD_\nu L_t^j) - \epsilon_{ij}(\bar{d}_p\gamma_\mu u_r)(L_s^i CiD^\mu L_t^j) \qquad \text{EoM} \\
&= (Y_e)_{tu}\epsilon_{ij}(\bar{d}_p\gamma_\mu u_r)(L_s^i C\gamma^\mu e_u)H^j - \mathcal{O}_{\bar{d}uLLD}^{prst} \\
&= (Y_e)_{tu} 2\mathcal{O}_{\bar{d}LueH}^{psru} - \mathcal{O}_{\bar{d}uLLD}^{prst} \qquad \text{Fierz}
\end{aligned}$$

subset $\Delta B = -\Delta L = 1$:

$$\begin{aligned}
\mathcal{O}_{\bar{L}QddD}^{prst} &= \epsilon_{\alpha\beta\sigma}\delta_{ij}(\bar{L}_{ip}\gamma_\mu Q_{j\alpha r})(d_{\beta s} CiD^\mu_{\sigma\rho} d_{\rho t}) \qquad \text{Fierz} \Downarrow \\
&= \epsilon_{\alpha\beta\sigma}\delta_{ij}\Big((\bar{L}_{ip} d_{\beta s})(Q_{j\alpha r} Ci\gamma_\mu D^\mu_{\sigma\rho} d_{\rho t}) + (\bar{L}_{ip} iD^\mu_{\sigma\rho} d_{\rho t})(Q_{j\alpha r} C\gamma_\mu d_{\beta s})\Big) \qquad \text{EoM} \\
&= (Y_d^\dagger)_{tu}\epsilon_{\alpha\beta\sigma}\delta_{ij}\delta_{kl}(\bar{L}_{ip} d_{\beta s})(Q_{j\alpha r} C Q_{k\sigma u})H_l^* + \mathcal{O}_{\bar{L}dQdD}^{ptrs} \\
&= (Y_d^\dagger)_{tu}\mathcal{O}_{\bar{L}dQQ\tilde{H}}^{psru} + \mathcal{O}_{\bar{L}dQdD}^{ptrs}
\end{aligned}$$

$$(\gamma_\mu P_\pm)_{\rho\sigma}(P_\mp)_{\alpha\beta} = (P_\mp)_{\rho\beta}(\gamma_\mu P_\pm)_{\alpha\sigma} + (P_\mp C^{-1})_{\rho\alpha}(C\gamma_\mu P_\mp)_{\sigma\beta}$$

Comparison of counting in different methods

(a) our work: *count operators without referring to flavors*
(b) Lehman, PRD **90**, 125023 (2014) [arXiv:1410.4193 [hep-ph]]: *same as above*
(c) Lehman-Martin, JHEP **1602**, 081 (2016) [arXiv:1510.00372 [hep-ph]]: *count operators wrt flavors in Hilbert series method*
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count operators wrt flavors using Hilbert series augmented by conformal algebra

- (a) has 1+1 less than (b) without referring to flavors, both in $\psi^4 D$.
- (c) counts each operator in each class (except for a few) while (d) only counts total # in each class. (d) found difference from (c) in class $\psi^2 H^2 D^2$.
- (a) also counts flavors by exhausting flavor sym of operators.
  (a) verified individual counts in (c) except for class $\psi^2 H^2 D^2$.
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- Conclusion
  (b) and (c) inconsistent, and inconsistent with (d) in 2 different classes;
  (a) consistent with (d) ✓.

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Story of basis continues: nontrivial flavor relations

- Previous count of $18 = 12 + 6$ operators not ready for *physical applications*.
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  For pheno analysis, we must be specific in flavors, *B* phys, *K* phys, etc.
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Summary of (non)trivial flavor relations at dim-7

Class	Operator	Flavor relations
$\psi^2 H^4$	$\mathcal{O}_{LH}$	$\mathcal{O}_{LH}^{pr} - p \leftrightarrow r = 0$
$\psi^2 H^2 D^2$	$\mathcal{O}_{LHD1}$	$(\mathcal{O}_{LDH1}^{pr} + \mathcal{K}^{pr}) - p \leftrightarrow r = 0$
	$\mathcal{O}_{LHD2}$	$\left[4\mathcal{O}_{LHD2}^{pr} + 2(Y_e)_{rv}\mathcal{O}_{LeHD}^{pv} - \mathcal{O}_{LHW}^{pr} + 2\mathcal{K}^{pr}\right] - p \leftrightarrow r = \mathcal{O}_{LHB}^{pr}$
$\psi^2 H^2 X$	$\mathcal{O}_{LHB}$	$\mathcal{O}_{LHB}^{pr} + p \leftrightarrow r = 0$
$\psi^4 H$	$\mathcal{O}_{\bar{e}LLLH}$	$(\mathcal{O}_{\bar{e}LLLH}^{prst} + r \leftrightarrow t) - r \leftrightarrow s = 0$
$\psi^4 D$	$\mathcal{O}_{\bar{d}uLLD}$	$\left[\mathcal{O}_{\bar{d}uLLD}^{prst} + (Y_d)_{vp}\mathcal{O}_{\bar{Q}uLLH}^{vrst} - (Y_u^\dagger)_{rv}\mathcal{O}_{\bar{d}LQLH2}^{psvt}\right] - s \leftrightarrow t = 0 \Rightarrow$ example
$\psi^4 H$	$\mathcal{O}_{\bar{L}dddH}$	$\mathcal{O}_{\bar{L}dddH}^{prst} + s \leftrightarrow t = 0, \quad \mathcal{O}_{\bar{L}dddH}^{prst} + \mathcal{O}_{\bar{L}dddH}^{pstr} + \mathcal{O}_{\bar{L}dddH}^{ptrs} = 0$
	$\mathcal{O}_{\bar{e}Qdd\tilde{H}}$	$\mathcal{O}_{\bar{e}Qdd\tilde{H}}^{prst} + s \leftrightarrow t = 0$
$\psi^4 D$	$\mathcal{O}_{\bar{L}QddD}$	$\left[\mathcal{O}_{\bar{L}QddD}^{prst} + (Y_u)_{rv}\mathcal{O}_{\bar{L}dud\tilde{H}}^{psvt}\right] - s \leftrightarrow t = -(Y_e^\dagger)_{vp}\mathcal{O}_{\bar{e}Qdd\tilde{H}}^{vrst} - (Y_d)_{rv}\mathcal{O}_{\bar{L}dddH}^{pvst}$
	$\mathcal{O}_{\bar{e}dddD}$	$\mathcal{O}_{\bar{e}dddD}^{prst} - r \leftrightarrow s = (Y_d^\dagger)_{tv}\mathcal{O}_{\bar{e}Qdd\tilde{H}}^{pvrs}$
		$(\mathcal{O}_{\bar{e}dddD}^{prst} + r \leftrightarrow t) - s \leftrightarrow t = (Y_e)_{vp}\mathcal{O}_{\bar{L}dddH}^{vrst}$

$$\mathcal{K}^{pr} = (Y_u)_{vw}\mathcal{O}_{\bar{Q}uLLH}^{vwpr} - (Y_d^\dagger)_{vw}\mathcal{O}_{\bar{d}LQLH2}^{vpwr} - (Y_e^\dagger)_{vw}\mathcal{O}_{\bar{e}LLLH}^{vwpr}.$$

An example of nontrivial flavor relations

$$
\begin{aligned}
\mathcal{O}_{\bar{d}uLLD}^{prst} - s \leftrightarrow t &= \epsilon_{ij}(\bar{d}_p \gamma_\mu u_r)(L_s^i C i D^\mu L_t^j) - s \leftrightarrow t \\
\text{(IBP)} &= -\epsilon_{ij}(\bar{d}_p i \overleftarrow{\not{D}} u_r)(L_s^i C L_t^j) - \epsilon_{ij}(\bar{d}_p i \not{D} u_r)(L_s^i C L_t^j) \\
\text{(EoM)} &= (Y_d)_{vp}\left[\epsilon_{ij}\delta_{mn}(\bar{Q}_v^m u_r)(L_s^i C L_t^j) H^n\right] - (Y_u^\dagger)_{rv}\left[\epsilon_{ij}\epsilon_{mn}(\bar{d}_p Q_v^m)(L_s^i C L_t^j) H^n\right] \\
\text{(Fierz)} &= \left[-(Y_d)_{vp}\mathcal{O}_{\bar{Q}uLLH}^{vrst} + (Y_u^\dagger)_{rv}\mathcal{O}_{\bar{d}LQLH2}^{psvt}\right] - s \leftrightarrow t
\end{aligned}
$$

True basis of dim-7 operators

- Still restricted to 18 structures, but counts flavors on imposing flavor relations; e.g.:
  1 family: 15 operators;
  3 families: 771 operators.
  Consistent with counts using Hilbert series.
- How to choose a basis?
  - In principle arbitrary, as long as operators in it are complete and indept.
  - In practice, must avoid singular inverse Yukawa when recasting loop-induced redundant operators in terms of chosen basis operators.
  - Inspection of relations suggests priority to reserve operators in basis: $\mathcal{O}$ without $D_\mu \Longrightarrow \mathcal{O}$ with one $D_\mu \Longrightarrow \mathcal{O}$ with two D_μ

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True basis of dim-7 operators: an example of choice

- For subset $\Delta L = 2,\ \Delta B = 0$

$$\frac{1}{2}(\mathcal{O}^{pr}_{LH}+\mathcal{O}^{rp}_{LH}),\quad \mathcal{O}^{pr}_{LeHD},\quad \frac{1}{2}(\mathcal{O}^{pr}_{LHD1}+\mathcal{O}^{rp}_{LHD1}),\quad \frac{1}{2}(\mathcal{O}^{pr}_{LHD2}+\mathcal{O}^{rp}_{LHD2}),\quad \frac{1}{2}(\mathcal{O}^{pr}_{LHB}-\mathcal{O}^{rp}_{LHB}),$$

$$\mathcal{O}^{pr}_{LHW},\quad \mathcal{O}^{prst}_{\bar{d}LQLH1},\quad \mathcal{O}^{prst}_{\bar{d}LQLH2},\quad \mathcal{O}^{prst}_{\bar{d}LueH},\quad \mathcal{O}^{prst}_{\bar{Q}uLLH},\quad \frac{1}{2}(\mathcal{O}^{prst}_{\bar{d}uLLD}+\mathcal{O}^{prst}_{\bar{d}uLLD}),$$

$$\frac{1}{4}(\mathcal{O}^{prst}_{\bar{e}LLLH}+\mathcal{O}^{ptsr}_{\bar{e}LLLH}+\mathcal{O}^{psrt}_{\bar{e}LLLH}+\mathcal{O}^{ptrs}_{\bar{e}LLLH})\ \text{(with at least two of } r,s,t \text{ being equal)},$$

$$\mathcal{O}^{prst}_{\bar{e}LLLH},\ \mathcal{O}^{prts}_{\bar{e}LLLH},\ \mathcal{O}^{psrt}_{\bar{e}LLLH},\ \mathcal{O}^{pstr}_{\bar{e}LLLH}\ (\text{for } r<s<t).$$

- For subset $\Delta B = -\Delta L = 1$

$$\mathcal{O}^{prst}_{\bar{L}dud\tilde{H}},\quad \frac{1}{2}(\mathcal{O}^{prst}_{\bar{e}Qdd\tilde{H}}-\mathcal{O}^{prts}_{\bar{e}Qdd\tilde{H}}),\quad \mathcal{O}^{prst}_{\bar{L}dQQ\tilde{H}},\quad \frac{1}{2}(\mathcal{O}^{prst}_{\bar{L}QddD}+\mathcal{O}^{prts}_{\bar{L}QddD}),$$

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$$\mathcal{O}_{LHW}^{pr},\quad \mathcal{O}_{\bar{d}LQLH1}^{prst},\quad \mathcal{O}_{\bar{d}LQLH2}^{prst},\quad \mathcal{O}_{\bar{d}LueH}^{prst},\quad \mathcal{O}_{\bar{Q}uLLH}^{prst},\quad \frac{1}{2}(\mathcal{O}_{\bar{d}uLLD}^{prst} + \mathcal{O}_{\bar{d}uLLD}^{prst}),$$

$$\frac{1}{4}(\mathcal{O}_{\bar{e}LLLH}^{prst} + \mathcal{O}_{\bar{e}LLLH}^{ptsr} + \mathcal{O}_{\bar{e}LLLH}^{psrt} + \mathcal{O}_{\bar{e}LLLH}^{ptrs})\ \text{(with at least two of } r,s,t \text{ being equal)},$$

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Renormalization and anomalous dim of operators

- Two 'bases':
  Flavor-blind basis (FBB): 18 structures without referring to flavors.
  Flavor-specific basis (FSB): redundancy in FBB removed by flavor relations, genuine basis.
- FBB facilitates computing counterterms easily/*blindly*.
  But anomalous dimension is defined only for operators in FSB.

$$\begin{aligned}
\mathscr{L}_7 &= C_b\mathscr{O}_b + C_r\mathscr{O}_r \quad \text{written in 18 structures split into basis } \mathscr{O}_b \text{ and redundant } \mathscr{O}_r \\
\Rightarrow \text{c.t.} &= -\big(\langle C_b\mathscr{O}_b\rangle + \underbrace{\langle C_r\mathscr{O}_r\rangle}_{\text{dropped}}\big) \quad \text{one insertion of } C\mathscr{O} \text{ dressed by SM int.} \\
&\Downarrow \\
&= -(16\pi^2\varepsilon)^{-1} C_b^T (P\mathscr{O}_b + R\mathscr{O}_r) + \cdots \\
&= -(16\pi^2\varepsilon)^{-1} C_b^T (P + RM)\mathscr{O}_b + \cdots \quad \text{nonsingular flavor relations } \mathscr{O}_r = M\mathscr{O}_b \\
\Rightarrow &\quad 16\pi^2 \frac{dC_b}{d\ln\mu} = \gamma C_b, \quad \gamma = -\sum_\alpha \rho_\alpha g_\alpha \frac{\partial}{\partial g_\alpha}(P+RM) \\
&\quad g_\alpha = g_{1,2,3},\ Y_{e,d,u},\ \lambda;\ \ \rho_\alpha = 2 \text{ for } g_\alpha = \lambda, \text{ otherwise } \rho_\alpha = 1
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A few words on computation

- dim reg, MS, separate R_ξ gauges for $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$.
- All c.t. listed in 1901.10302, from which one reads out P, R once a basis is chosen. But with 3 families this should better be manipulated by codes.
- Our results on c.t. follow a nonren. theorem Cheung-Shen 1505.01844 and a power counting rule 1612.04527– another cross check.
- An example of c.t. computation:

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\begin{aligned}
& \langle (C\mathcal{O})_X \rangle \delta \quad (X = \bar{d}LueH,\ \delta = 16\pi^2\varepsilon) \\
= \quad & 3(Y_u^\dagger Y_d)_{vw} C_X^{wpvr} \mathcal{O}_{LeHD}^{pr} + (Y_u^\dagger)_{vs}(Y_e^\dagger)_{wt} C_X^{prvw} \mathcal{O}_{\bar{d}LQLH1}^{prst} - (Y_u^\dagger)_{vs}(Y_e^\dagger)_{wt} C_X^{prvw} \mathcal{O}_{\bar{d}LQLH2}^{prst} \\
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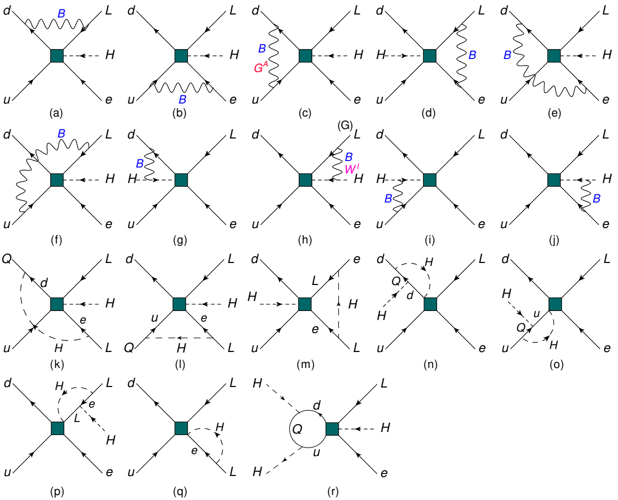
A few words on computation: diagrams for $\langle (C\mathscr{O})_{\bar{d}LueH} \rangle$



Outline

Low energy processes due to SMEFT dim-7 operators

- 2 classes of dim-7 operators:
  $\Delta B = -\Delta L = \pm 1$: unusual nucleon decay e.g., $p \to \nu\pi^+/K^+, e^-\pi^+\pi^+$; $n \to e^-\pi^+$,...
  $\Delta B = 0,\ \Delta L = \pm 2$: nuclear $0\nu\beta\beta$ decay, $K^- \to \pi^+\mu^-\mu^-$, ...
- Calculation of contribution of dim-7 operators to low energy processes involves a sequence of EFTs from LNV scale to hadronic/nuclear scale:
  - RGE within one EFT.
  - Matching between two neighboring EFTs –
    When nonpert. effects set in, symmetry acts as main guidance and new low-energy constants are introduced.
    $\Rightarrow$ uncertainties associated with hadronic/nuclear phys.
- Not attempt a complete analysis below but illustrate impact of RGE.

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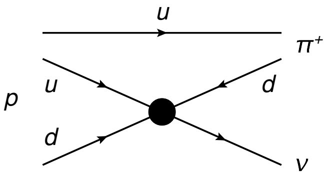
Example of $\Delta B = -\Delta L = \pm 1$: $p \to \nu \pi^+$

Low energy: $H \to v/\sqrt{2}$, $D_\mu \to 0$, only $\mathcal{O}^{p111}_{\bar{L}dud\tilde{H}}$, $\mathcal{O}^{p111}_{\bar{L}dQQ\tilde{H}}$ relevant.



Ignore quark mixing, drop all Yukawa couplings except for the top. RGEs are decoupled ($\alpha_i = g_i^2/(4\pi)$ $(i = 1,2,3)$, $\alpha_t = Y_t^2/(4\pi)$):

$$\mu \frac{d}{d\mu} C^{p111}_{\bar{L}dud\tilde{H}} = \frac{1}{4\pi}\left(-4\alpha_3 - \frac{9}{4}\alpha_2 - \frac{57}{12}\alpha_1 + 3\alpha_t\right) C^{p111}_{\bar{L}dud\tilde{H}}$$

$$\mu \frac{d}{d\mu} C^{p111}_{\bar{L}dQQ\tilde{H}} = \frac{1}{4\pi}\left(-4\alpha_3 - \frac{27}{4}\alpha_2 - \frac{19}{12}\alpha_1 + 3\alpha_t\right) C^{p111}_{\bar{L}dQQ\tilde{H}}$$

Example of $\Delta B = -\Delta L = \pm 1$: $p \to \nu \pi^+$

Rough estimate from $M \sim 10^{15}\ \mathrm{GeV}$ (GUT) to $\mu \sim m_p \sim 1\ \mathrm{GeV}$:

$$C^{p111}_{\bar{L}dud\tilde{H}}(m_p) = \left[\frac{\alpha_3(m_p)}{\alpha_3(M)}\right]^{2/\beta_3} \left[\frac{\alpha_2(M_Z)}{\alpha_2(M)}\right]^{9/(8\beta_2)} \left[\frac{\alpha_1(M_Z)}{\alpha_1(M)}\right]^{57/(24\beta_1)} (0.787) C^{p111}_{\bar{L}dud\tilde{H}}(M)$$
$$= (2.034)(1.158)(1.262)(0.787) C^{p111}_{\bar{L}dud\tilde{H}}(M)$$
$$= 2.34 C^{p111}_{\bar{L}dud\tilde{H}}(M)$$

$$C^{p111}_{\bar{L}dQQ\tilde{H}}(m_p) = \left[\frac{\alpha_3(m_p)}{\alpha_3(M)}\right]^{2/\beta_3} \left[\frac{\alpha_2(M_Z)}{\alpha_2(M)}\right]^{27/(8\beta_2)} \left[\frac{\alpha_1(M_Z)}{\alpha_1(M)}\right]^{19/(24\beta_1)} (0.787) C^{p111}_{\bar{L}dQQ\tilde{H}}(M)$$
$$= (2.034)(1.551)(1.081)(0.787) C^{p111}_{\bar{L}dQQ\tilde{H}}(M)$$
$$= 2.68 C^{p111}_{\bar{L}dQQ\tilde{H}}(M)$$

Input:

$$\beta_3 = 7, \quad \beta_2 = \frac{19}{6}, \quad \beta_1 = -\frac{41}{10}$$
$$\alpha_1(M_Z) = 0.0169225 \pm 0.0000039, \quad \alpha_2(M_Z) = 0.033735 \pm 0.000020,$$
$$\alpha_3(M_Z) = 0.1173 \pm 0.00069, \quad \alpha_t(M_Z) = 0.07514$$

Example of $\Delta B = 0,\ \Delta L = \pm 2:\ 0\nu\beta\beta$ decay

- $0\nu\beta\beta$ decay most extensively studied both experimentally and theoretically.
  - KamLAND-Zen, Gerda, EXO-200, SNO+, Majorana, CUORE, CUPID-0, ...
    Best limit on half lifetime on $^{136}\mathrm{Xe}$: $1.07\times 10^{26}\ \mathrm{yr}$ by KamLAND-Zen
    $\Leftrightarrow m_{\beta\beta} < 0.1\ \mathrm{eV}$ assuming light neutrino mass dominance
    Expected to be improved by 1 or 2 orders of mag. in future
  - Theoretical status:
    comprehensive analysis Cirigliano et al 1806.02780, Horoi-Neacsu 1706.05391
    review Rodejohann 1106.1334
    hadronic and nuclear uncertainties of order one
- Sequence of EFTs:
  underlying theory/EFT – SMEFT – LEFT – chiral eff. theory – nuclear phys
  Here focus on impact of complete 1-loop RGE in SMEFT;
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$0\nu\beta\beta$ decay: RGE in SMEFT

Between scales $\Lambda_{\rm NP}$ and $\Lambda_{\rm EW}$ relevant RGEs are

$$
\begin{aligned}
\frac{d}{d\ln\mu}C^{11\dagger}_{LHD1} &= \frac{1}{4\pi}\left(-\frac{9}{10}\alpha_1+\frac{11}{2}\alpha_2+6\alpha_t\right)C^{11\dagger}_{LHD1}+\frac{1}{4\pi}\left(-\frac{33}{20}\alpha_1-\frac{19}{4}\alpha_2-2\alpha_\lambda\right)C^{11\dagger}_{LHD2},\\
\frac{d}{d\ln\mu}C^{1111\dagger}_{\bar{d}uLLD} &= \frac{1}{4\pi}\left(\frac{1}{10}\alpha_1-\frac{1}{2}\alpha_2\right)C^{1111\dagger}_{\bar{d}uLLD},\\
\frac{d}{d\ln\mu}C^{11\dagger}_{LeHD} &= \frac{1}{4\pi}\left(-\frac{9}{10}\alpha_1+6\alpha_\lambda+9\alpha_t\right)C^{11\dagger}_{LeHD},\\
\frac{d}{d\ln\mu}C^{1111\dagger}_{\bar{d}LueH} &= \frac{1}{4\pi}\left(-\frac{69}{20}\alpha_1-\frac{9}{4}\alpha_2+3\alpha_t\right)C^{1111\dagger}_{\bar{d}LueH},\\
\frac{d}{d\ln\mu}C^{1111\dagger}_{\bar{Q}uLLH} &= \frac{1}{4\pi}\left(\frac{1}{20}\alpha_1-\frac{3}{4}\alpha_2-8\alpha_3+3\alpha_t\right)C^{1111\dagger}_{\bar{Q}uLLH},\\
\frac{d}{d\ln\mu}C^{1111\dagger}_{\bar{d}LQLH1} &= \frac{1}{4\pi}\left(\frac{13}{20}\alpha_1+\frac{9}{4}\alpha_2-8\alpha_3+3\alpha_t\right)C^{1111\dagger}_{\bar{d}LQLH1}+\frac{1}{4\pi}\left(6\alpha_2\right)C^{1111\dagger}_{\bar{d}LQLH2},\\
\frac{d}{d\ln\mu}C^{1111\dagger}_{\bar{d}LQLH2} &= \frac{1}{4\pi}\left(-\frac{121}{60}\alpha_1-\frac{15}{4}\alpha_2+\frac{8}{3}\alpha_3+3\alpha_t\right)C^{1111\dagger}_{\bar{d}LQLH2}+\frac{1}{4\pi}\left(-\frac{4}{3}\alpha_1+\frac{16}{3}\alpha_3\right)C^{1111\dagger}_{\bar{d}LQLH1},\\
\frac{d}{d\ln\mu}C^{11\dagger}_{LHD2} &= \frac{1}{4\pi}\left(\frac{12}{5}\alpha_1+3\alpha_2+4\alpha_\lambda+6\alpha_t\right)C^{11\dagger}_{LHD2}+\frac{1}{4\pi}\left(-8\alpha_2\right)C^{11\dagger}_{LHD1},\\
\frac{d}{d\ln\mu}C^{11\dagger}_{LHW} &= \frac{1}{4\pi}\left(-\frac{6}{5}\alpha_1+\frac{13}{2}\alpha_2+4\alpha_\lambda+6\alpha_t\right)C^{11\dagger}_{LHW}+\frac{1}{4\pi}\left(\frac{5}{8}\alpha_2\right)C^{11\dagger}_{LHD1}+\frac{1}{4\pi}\left(-\frac{9}{80}\alpha_1+\frac{11}{16}\alpha_2\right)C^{11\dagger}_{LHD2}.
\end{aligned}
$$

$0\nu\beta\beta$ decay: Matching SMEFT and LEFT

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  - LEFT: basis of dim-5 and dim-6 operators, 1-loop QCD+QED RGE;
  - Matching at tree level between SMEFT up to dim-6 and LEFT.
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  - Integrate out heavy particles in SMEFT to yield operators in LEFT
  - Diagrams in SMEFT and LEFT (next slide) classified into 3 types:
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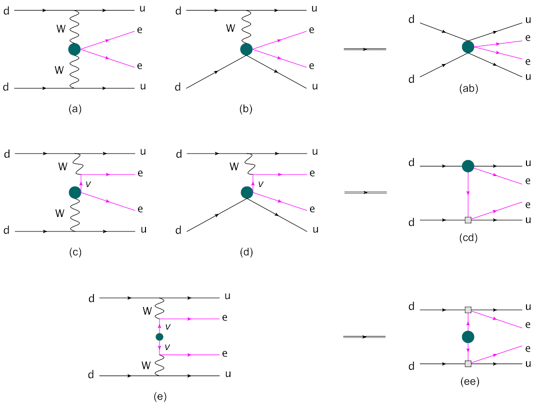
$0\nu\beta\beta$ decay: Matching SMEFT and LEFT

SMEFT $\Downarrow$ LEFT $\Downarrow$ chiral and nuclear $\Downarrow$

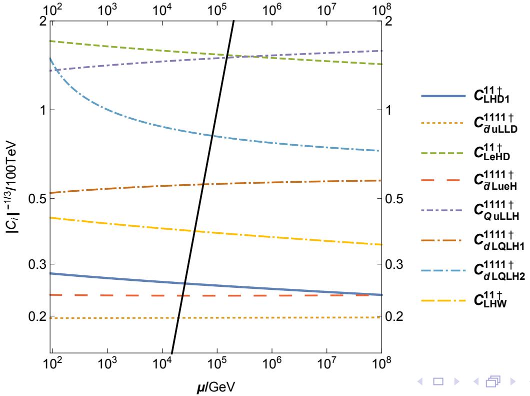


$0\nu\beta\beta$ decay: Upper bounds on couplings in SMEFT

At $\mu = m_W$, $^{136}\mathrm{Xe}$ data implies upper bounds [in $(100\ \mathrm{TeV})^{-3}$]:

$\lvert C_{LHD1}^{11\dagger}\rvert$	$\lvert C_{\bar{d}uLLD}^{1111\dagger}\rvert$	$\lvert C_{LeHD}^{11\dagger}\rvert$	$\lvert C_{\bar{d}LueH}^{1111\dagger}\rvert$	$\lvert C_{\bar{Q}uLLH}^{1111\dagger}\rvert$	$\lvert C_{\bar{d}LQLH1}^{1111\dagger}\rvert$	$\lvert C_{\bar{d}LQLH2}^{1111\dagger}\rvert$	$\lvert C_{LHW}^{11\dagger}\rvert$
46	131	0.2	76	0.4	0.7	0.3	12

RGE of couplings to scale $\mu > m_W$:



$0\nu\beta\beta$ decay: Summary of numerical results

- Except for operators $\mathcal{O}_{\bar{d}uLLD}$, $\mathcal{O}_{\bar{d}LueH}$, RGE effects are significant for others.
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Outline

Conclusions

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- Flavors must be taken into account for a *genuine basis* of operators. We found *nontrivial flavor relations* first appear at dim-7 in SMEFT, which makes choice of a basis complicated. We suggested a convenient way to choose a *correct basis*.
- We renormalized at one loop all dim-7 operators in SMEFT, and demonstrated how to extract *anomalous dimension matrix* once an appropriate basis is chosen.
- We illustrated *RGE effects* with two low-energy processes:
  $\Delta B=-\Delta L=1:\ p\to\nu\pi^+$ – RGE from m_p to typical GUT scale amounts to an enhancement factor of ~ 2.
  $\Delta B=0,\ \Delta=2:\ 0\nu\beta\beta$ – RGE in SMEFT is significant, improving previous pure QCD analysis and constraining NP scale to be above $\sim 10^2\ \mathrm{GeV}$.

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