

Leptonic Dirac CP Phase with Residual Symmetry & μ DAR

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- SFG**, Duane A. Dicus, Wayne W. Repko, *PLB* **702**, 220 (2011) [arXiv:1104.0602]
SFG, Duane A. Dicus, Wayne W. Repko, *PRL* **108**, 041801 (2012) [arXiv:1108.0964]
Andrew D. Hanlon, **SFG**, Wayne W. Repko, *PLB* **729**, 185-191 (2014) [arXiv:1308.6522]
SFG, [arXiv:1406.1985]
Jarah Evslin, **SFG**, Kaoru Hagiwara, *JHEP* **1602** (2016) 137 [arXiv:1506.05023]
SFG, Pedro Pasquini, M. Tortola, J. W. F. Valle, [arXiv:1605.01670]
SFG, Alexei Smirnov, [arXiv:1607.xxxxx]

ν Oscillation Data

(for NH)	-1σ	Best Value	$+1\sigma$
$\Delta m_s^2 \equiv \Delta m_{12}^2$ (10^{-5}eV^2)	7.42	7.60	7.79
$ \Delta m_a^2 \equiv \Delta m_{13}^2 $ (10^{-3}eV^2)	2.41	2.48	2.53
$\sin^2 \theta_s$ ($\theta_s \equiv \theta_{12}$)	0.307 (33.6°)	0.323 (34.6°)	0.339 (35.6°)
$\sin^2 \theta_a$ ($\theta_a \equiv \theta_{23}$)	0.439 (41.5°)	0.567 (48.9°)	0.599 (50.8°)
$\sin^2 \theta_r$ ($\theta_r \equiv \theta_{13}$)	0.0214 (8.4°)	0.0234 (8.8°)	0.0254 (9.2°)
δ_D, δ_{Mi}	?, ??	?, ??	?, ??

Forero, Tortola & Valle, arXiv:1405.7540

Evidence of μ - τ Symmetry

☞ Two small deviations (1σ level):

$$-3.5^\circ < \theta_a - 45^\circ < 5.8^\circ \quad 8.4^\circ < \theta_r < 9.2^\circ$$

with **Best Fit Value**: $\theta_a - 45^\circ = -3.9^\circ$ & $\theta_r = 8.8^\circ$.

☞ Zeroth Order Approximation:

$$\theta_a \approx 45^\circ, \quad \theta_r \approx 0^\circ.$$

⇒ **CP & μ - τ Symmetric** Mass Matrix:

$$M_\nu^{(0)} = \begin{pmatrix} A & \mathbf{B} & \mathbf{B} \\ & \mathbf{C} & \mathbf{D} \\ & & \mathbf{C} \end{pmatrix}$$

Mohapatra & Nussinov [hep-ph/9809415], Lam [hep-ph/0104116]

Horizontal Symmetry

[Lam, PRL101:121602(2008), PRD78:073015(2008)]

- ☞ Mass Matrix M_ν invariant under **Transformation**:

$$G_\nu^T M_\nu G_\nu = M_\nu$$

- ☞ **Diagonalization**:

$$V_\nu^T M_\nu V_\nu = D_\nu$$

- ☞ **Rephasing**:

$$D_\nu = d_\nu^T D_\nu d_\nu$$

with $d_\nu^2 = \mathbf{I}_3 \Rightarrow d_\nu = \text{diag}(\pm, \pm, \pm)$.

- ☞ Together

$$\begin{aligned} M_\nu &= G_\nu^T M_\nu G_\nu = \underline{G_\nu^T V_\nu^* D_\nu V_\nu^\dagger G_\nu} \\ &= \underline{V_\nu^* D_\nu V_\nu^\dagger} = \underline{V_\nu^* d_\nu^T D_\nu d_\nu V_\nu^\dagger} \end{aligned}$$

- ☞ **Consequence**: $V_\nu^\dagger G_\nu = d_\nu V_\nu^\dagger \Leftrightarrow \boxed{G_\nu = V_\nu d_\nu V_\nu^\dagger}$

- ☞ **For Leptons**: $\underline{F_\ell} = \underline{V_\ell d_\ell V_\ell^\dagger}$ with $d_\ell = \text{diag}(e^{i\beta_1}, e^{i\beta_2}, e^{i\beta_3})$.

Two Nontrivial Independent possibilities of \mathbf{d}_ν :

$$\mathbf{d}_\nu^{(1)} = \text{diag}(-1, 1, 1), \quad \mathbf{d}_\nu^{(2)} = \text{diag}(1, -1, 1), \quad \mathbf{d}_\nu^{(3)} = -\mathbf{d}_\nu^{(1)}\mathbf{d}_\nu^{(2)}.$$

θ_s parameterized in terms of \mathbf{k} : $\tan \theta_s = \sqrt{2}/k$

$$V_\nu(k) = \begin{pmatrix} \frac{k}{\sqrt{2+k^2}} & \frac{\sqrt{2}}{\sqrt{2+k^2}} & 0 \\ \frac{1}{\sqrt{2+k^2}} & \frac{k}{\sqrt{2(2+k^2)}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2+k^2}} & \frac{k}{\sqrt{2(2+k^2)}} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad \begin{array}{ll} \mathbf{k} = 2 & \theta_s = 35.3^\circ \text{ [TBM]} \\ \mathbf{k} = \frac{3}{\sqrt{2}} & \theta_s = 33.7^\circ \\ \mathbf{k} = \sqrt{6} & \theta_s = 30.0^\circ \end{array}$$

Two Independent Symmetry Transformations $\mathbf{G}_i = \mathbf{V}_\nu \mathbf{d}_\nu^{(i)} \mathbf{V}_\nu^\dagger$

$$\mathbf{G}_1 = \frac{1}{2+k^2} \begin{pmatrix} 2-k^2 & 2k & 2k \\ & k^2 & -2 \\ & & k^2 \end{pmatrix}, \quad \mathbf{G}_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$\mathbb{Z}_2^S (\times \mathbb{Z}_2^S) \times \mathbb{Z}_2^{\mu T} \equiv \mathcal{G} = \{\mathbf{E}, \mathbf{G}_1, \mathbf{G}_2 (\equiv \mathbf{G}_1 \mathbf{G}_3), \mathbf{G}_3\}$

Full Symmetries:

$\mathcal{H} \equiv \mathcal{G} \times \mathcal{F}$	\mathcal{G}	\mathcal{F}
S_4	$\mathbb{Z}_2^S \times \mathbb{Z}_2^{\mu T}$	$\mathbb{Z}_3 \equiv \{I, F, F^2\}$
$\{G_1, G_3, F\}$	$G_1(G_2), G_3$	$F \equiv \text{diag}(1, \omega, \omega^2)$

Bottom-Up \uparrow

\downarrow Top-Down

See also Smirnov et. al., 1204.0445, 1212.2149, 1510.00344

Residual Symmetries:

$$\nu_i: \quad \mathcal{G} \equiv \mathbb{Z}_2^S(\overline{\mathbb{Z}}_2) \times \mathbb{Z}_2^{\mu T} \quad \text{for} \quad d_\nu^i = \text{diag}(\pm 1, \pm 1, \pm 1)$$

$$\ell_i: \quad \mathcal{F} \in U(1) \times U(1) \quad \text{for} \quad d_\ell^i = \text{diag}(e^{i\beta_1}, e^{i\beta_2}, e^{i\beta_3})$$

Residual Symmetry as Effective Theory

☞ Full symmetry **HAS TO** be **Broken!**

☞ Fermion needs to acquire mass.

☞ Non-trivial mixing $V_{\text{PMNS}} = V_{\ell}^{\dagger} V_{\nu}$

☞ If mixing is **TRUELY determined by symmetry**, it has to be **residual symmetry**

☞ VEVs

☞ Yukawa couplings

☞ **Residual Symmetry as Custodial Symmetry**

☞ **Gauge symmetry has to be broken.** Otherwise, no mixing.

☞ **Weak mixing angle** is a function of gauge couplings, which **cannot be dictated by gauge symmetry** (and VEV).

☞ **Weak mixing angle is related to** the physical observables, the **gauge boson masses**, by **custodial symmetry**.

Lepton's Representation:

$$\begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix} \sim \mathbf{3}, \quad \begin{matrix} e_R \sim \mathbf{1} \\ \mu_R \sim \mathbf{1}' \\ \tau_R \sim \mathbf{1}'' \end{matrix}, \quad \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{pmatrix} \sim \mathbf{3}.$$

A_4 invariant Lagrangian:

$$\begin{aligned} \mathcal{L}_\ell &= \mathbf{y}_1 \bar{e}_R (\mathbf{1} \varphi_1^\dagger e_L + \mathbf{1} \varphi_2^\dagger \tau_L + \mathbf{1} \varphi_3^\dagger \tau_L) \\ &+ \mathbf{y}_2 \bar{\mu}_R (\omega \varphi_1^\dagger e_L + \mathbf{1} \varphi_2^\dagger \tau_L + \omega^2 \varphi_3^\dagger \tau_L) \\ &+ \mathbf{y}_3 \bar{\tau}_R (\omega^2 \varphi_1^\dagger e_L + \mathbf{1} \varphi_2^\dagger \tau_L + \omega \varphi_3^\dagger \tau_L). \end{aligned}$$

Mass term with $\langle \varphi_i \rangle = v_i$:

$$\mathcal{L}_\ell = \begin{pmatrix} \bar{e}_R & \bar{\mu}_R & \bar{\tau}_R \end{pmatrix} \begin{pmatrix} \mathbf{y}_1 & & \\ & \mathbf{y}_2 & \\ & & \mathbf{y}_3 \end{pmatrix} \begin{pmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \omega & \mathbf{1} & \omega^2 \\ \omega^2 & \mathbf{1} & \omega \end{pmatrix} \begin{pmatrix} \mathbf{v}_1 & & \\ & \mathbf{v}_2 & \\ & & \mathbf{v}_3 \end{pmatrix} \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix}.$$

$$\mathbf{v}_1 = \mathbf{v}_2 = \mathbf{v}_3 = \mathbf{v} \Rightarrow U_{\ell,R} = I, \quad \mathbf{U}_{\ell,L}(\omega), \quad m_{\ell,i} = \mathbf{y}_i \mathbf{v}.$$

$$\mathbf{y}_1 = \mathbf{y}_2 = \mathbf{y}_3 = \mathbf{y} \Rightarrow U_{\ell,L} = I, \quad \mathbf{U}_{\ell,R}(\omega), \quad m_{\ell,i} = \mathbf{y} \mathbf{v}_i.$$

Partial Residual Symmetry \mathbb{Z}_2^S or $\overline{\mathbb{Z}}_2^S$

- Although $\mathbb{Z}_2^{\mu T}$, represented by \mathbf{G}_3 , is Broken!
- No particular reason for \mathbb{Z}_2^S or $\overline{\mathbb{Z}}_2^S$ to be Broken!

$$\mathbb{Z}_2^S : \mathbf{G}_1(\mathbf{k}) = \frac{1}{2+k^2} \begin{pmatrix} 2-k^2 & 2k & 2k \\ & k^2 & -2 \\ & & k^2 \end{pmatrix},$$
$$\overline{\mathbb{Z}}_2^S : \mathbf{G}_2(\mathbf{k}) = \frac{1}{2+k^2} \begin{pmatrix} 2-k^2 & 2k & 2k \\ & -2 & k^2 \\ & & -2 \end{pmatrix}.$$

- \mathbb{Z}_2^S & $\overline{\mathbb{Z}}_2^S$ are **Dependent**

$$\mathbf{G}_1(\mathbf{k}) = \mathbf{G}_2(\mathbf{k})\mathbf{G}_3$$

- DIFFERENT Consequences!!!**

Correlation between Physical Observables

$$\mathbf{G}_\nu = \mathbf{V}_\nu \mathbf{d}_\nu \mathbf{V}_\nu^\dagger$$

$$\mathbb{Z}_2^S (G_1)$$

$$\overline{\mathbb{Z}}_2^S (G_2)$$

$$\cos \delta_D = \frac{(s_s^2 - c_s^2 s_r^2)(c_a^2 - s_a^2)}{4c_a s_a c_s s_s s_r}$$

$$\cos \delta_D = \frac{(s_s^2 s_r^2 - c_s^2)(c_a^2 - s_a^2)}{4c_a s_a c_s s_s s_r}$$

$$\sin \theta_r = \pm \left[\pm \sqrt{c_D^2 + \cot^2 2\theta_a - c_D} \right] \tan 2\theta_a (\tan \theta_s)^{\pm 1}$$

$$\frac{\delta_r}{\delta_a} = -\frac{\tan \theta_s}{\cos \delta_D}$$

$$\frac{\delta_r}{\delta_a} = +\frac{\cot \theta_s}{\cos \delta_D}$$

Model-Independence

$$\frac{\delta_r}{\delta_a} = -\frac{\tan \theta_s}{\cos \delta_D} \quad \cos \delta_D = \frac{(s_s^2 - c_s^2 s_r^2)(c_a^2 - s_a^2)}{4c_a s_a c_s s_s s_r},$$

☞ Minimal Seesaw [SFG, He, Yin, JCAP2010]

$$\frac{\delta_r}{\delta_a} = -\frac{\tan \theta_s}{\cos \delta_D}$$

Common origin of soft μ - τ & CP breaking

☞ Trimaximal Mixing ($k=2$) @ A_4/S_4 [King, Luhn, 1107.5332]

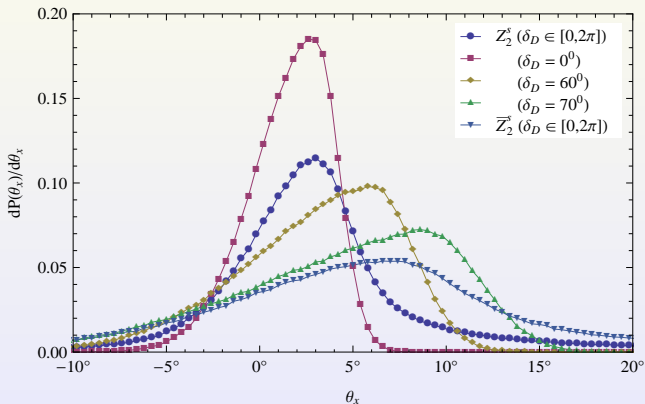
$$\sqrt{2}s_a - 1 = -\frac{1}{\sqrt{2}}s_r \cos \delta_D$$

☞ Unrealistic Bimaximal Mixing [Lam, 1105.5166]

$$\left\langle s_s, s_a, s_r e^{i\delta_D} \right\rangle = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}} e^{-\frac{i}{2}\pi} \right\rangle$$

Prediction of Large θ_r (θ_x)

$$\sin \theta_r = \pm \left[\pm \sqrt{\cos^2 \delta_D + \cot^2 2\theta_a} - \cos \delta_D \right] \tan 2\theta_a \tan^\pm \theta_s$$



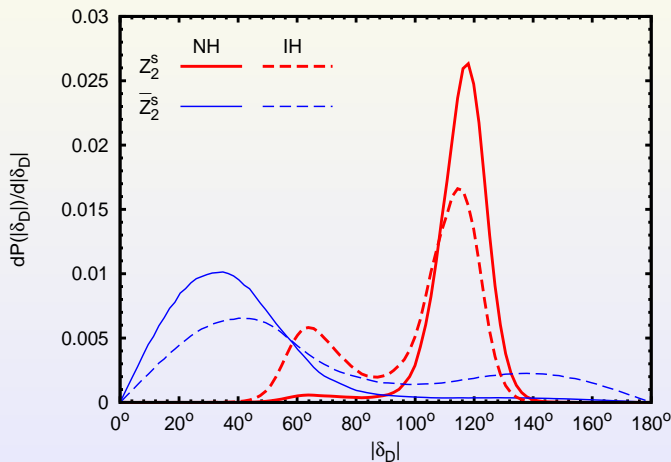
MINOS ($\theta_r \in [3^\circ, 9^\circ]/[5^\circ, 11^\circ]$)

T2K ($\theta_r \in [5^\circ, 16^\circ]/[6^\circ, 18^\circ]$) @ 90% C.L.

Prediction of Large δ_D

$$\cos \delta_D = \frac{(s_s^2 - c_s^2 s_r^2)(c_a^2 - s_a^2)}{4c_a s_a c_s s_s s_r}$$

$$\cos \delta_D = \frac{(s_s^2 s_r^2 - c_s^2)(c_a^2 - s_a^2)}{4c_a s_a c_s s_s s_r}$$



1σ Indication for $\delta_D = -74^\circ (-110^\circ)$ [Schwetz et.al. 1108.1376]

Precision Measurement of CP δ_D is Needed!

$$\mathbf{G}_\nu = \mathbf{V}_\nu \mathbf{d}_\nu \mathbf{V}_\nu^\dagger$$

$$\mathbb{Z}_2^S (G_1)$$

$$\overline{\mathbb{Z}}_2^S (G_2)$$

$$\cos \delta_D = \frac{(s_s^2 - c_s^2 s_r^2)(c_a^2 - s_a^2)}{4c_a s_a c_s s_s s_r}$$

$$\cos \delta_D = \frac{(s_s^2 s_r^2 - c_s^2)(c_a^2 - s_a^2)}{4c_a s_a c_s s_s s_r}$$

(for NH)	-1σ	Best Value	$+1\sigma$
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δ_D	?	?	?

Why neutrino mass & oscillation?

- ☞ Higgs boson for electroweak symmetry breaking & mass.
- ☞ Chiral symmetry breaking for mass.
- ☞ **The word seems not affected by the tiny neutrino mass!**
 - ☞ Neutrino mass \Rightarrow Mixing
 - ☞ 3 Neutrino \Rightarrow possible CP violation
 - ☞ CP violation \Rightarrow Leptogenesis
 - ☞ Leptogenesis \Rightarrow **Matter-Antimatter Asymmetry**
 - ☞ There is something left in the Universe.
 - ☞ Baryogenesis from quark mixing is not enough.

CP Measurement @ Accelerator Exp



The Dirac CP Phase δ_D @ Accelerator Exp

- To leading order in $\alpha = \frac{\delta M_{21}^2}{|\delta M_{31}^2|} \sim 3\%$, the oscillation probability relevant to measuring δ_D @ T2(H)K,

$$P_{\nu_\mu \rightarrow \nu_e} \approx 4s_a^2 c_r^2 s_r^2 \sin^2 \phi_{31} - 8c_a s_a c_r^2 s_r c_s s_s \sin \phi_{21} \sin \phi_{31} [\cos \delta_D \cos \phi_{31} \pm \sin \delta_D \sin \phi_{31}]$$

for ν & $\bar{\nu}$, respectively. $[\phi_{ij} \equiv \frac{\delta m_{ij}^2 L}{4E_\nu}]$

- $\nu_\mu \rightarrow \nu_\mu$ Exps measure $\sin^2(2\theta_a)$ precisely, but not $\sin^2 \theta_a$.

- Run both ν & $\bar{\nu}$ modes @ first peak $[\phi_{31} = \frac{\pi}{2}, \phi_{21} = \alpha \frac{\pi}{2}]$,

$$P_{\bar{\nu}_\mu \rightarrow \bar{\nu}_e} + P_{\nu_\mu \rightarrow \nu_e} = 2s_a^2 c_r^2 s_r^2,$$

$$P_{\bar{\nu}_\mu \rightarrow \bar{\nu}_e} - P_{\nu_\mu \rightarrow \nu_e} = \alpha \pi \sin(2\theta_s) \sin(2\theta_r) \sin(2\theta_a) \cos \theta_r \sin \delta_D.$$

The Dirac CP Phase δ_D @ Accelerator Exp

Accelerator experiment, such as **T2(H)K**, uses off-axis beam to compare ν_e & $\bar{\nu}_e$ appearance @ the oscillation maximum.

Disadvantages:

Efficiency:

- Proton accelerators produce ν more efficiently than $\bar{\nu}$ ($\sigma_\nu > \sigma_{\bar{\nu}}$).
- The $\bar{\nu}$ mode needs more beam time [**$T_{\bar{\nu}} : T_\nu = 2 : 1$**].
- Undercut statistics \Rightarrow Difficult to reduce the uncertainty.

Degeneracy:

- Only **$\sin \delta_D$** appears in $P_{\nu_\mu \rightarrow \nu_e}$ & $P_{\bar{\nu}_\mu \rightarrow \bar{\nu}_e}$.
- Cannot distinguish δ_D from $\pi - \delta_D$.

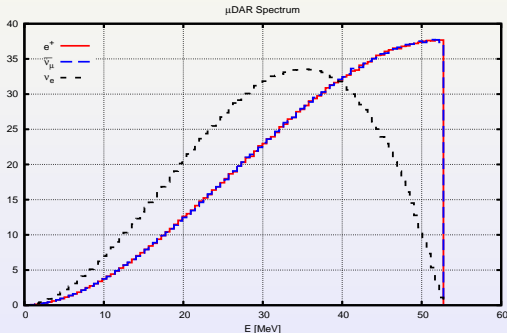
- CP Uncertainty** $\frac{\partial P_{\mu e}}{\partial \delta_D} \propto \cos \delta_D \Rightarrow \Delta(\delta_D) \propto \mathbf{1 / \cos \delta_D}$.

Solution:

Measure $\bar{\nu}$ mode with μ^+ decay @ rest (μ DAR)

μ DAR $\bar{\nu}$ Oscillation Experiments

- ☞ A cyclotron produces 800 MeV proton beam @ fixed target.
- ☞ Produce π^\pm which stops &
 - ☞ π^- is absorbed,
 - ☞ π^+ decays @ rest: $\pi^+ \rightarrow \mu^+ + \nu_\mu$.
- ☞ μ^+ stops & decays @ rest: $\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e$.



- ☞ $\bar{\nu}_\mu$ travel in all directions, oscillating as they go.
- ☞ A detector measures the $\bar{\nu}_e$ from $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillation.

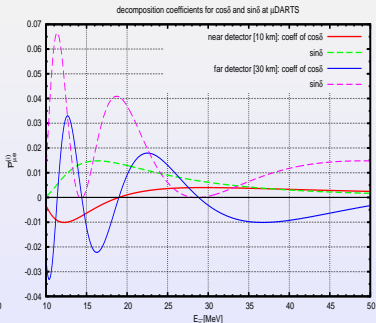
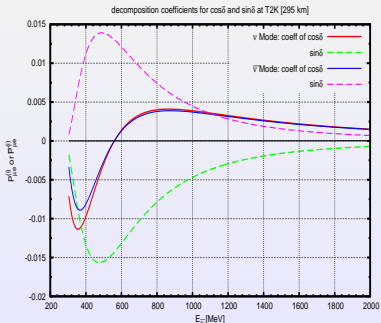
Accelerator + μ DAR Experiments

Combining $\nu_\mu \rightarrow \nu_e$ @ accelerator [narrow peak @ 550 MeV] & $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ @ μ DAR [wide peak ~ 45 MeV] solves the 2 problems:

Efficiency:

- $\bar{\nu}$ @ high intensity, μ DAR is plentiful enough.
- Accelerator Exps can devote all run time to the ν mode. With same run time, the statistical uncertainty drops by $\sqrt{3}$.

Degeneracy: (decomposition in propagation basis [1309.3176])



DAE δ ALUS Project

- ☞ It's the **FIRST** proposal along this line:
 - ☞ **3** μ DAR with **3** high-intensity cyclotron complexes.
 - ☞ **1** detector.
 - ☞ Different baselines: **1.5, 8 & 20** km to break degeneracies.
- ☞ **Disadvantages:**
 - ☞ The scattering lepton from IBD @ low energy is **isotropic**.
 - ☞ **Cannot** distinguish $\bar{\nu}_e$ from different sources
 - ☞ Baseline **cannot be measured**.
 - ☞ Cyclotrons **cannot** run simultaneously (20~25% duty factor).
 - ☞ **Large** statistical uncertainty.
 - ☞ **Higher intensity** is necessary.
 - ☞ **Expensive** & Technically **challenging**.

New Proposals

1 μ DAR source + 2 detectors

Advantages:

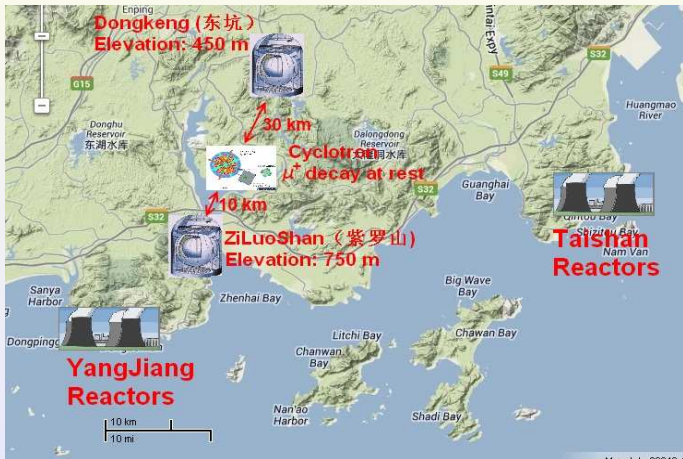
- ☞ Full (**100%**) duty factor!
- ☞ **Lower** intensity: $\sim 9\text{mA}$ [$\sim 4\times$ lower than DAE δ ALUS]
- ☞ Not far beyond the current state-of-art technology of cyclotron [**2.2mA** @ Paul Scherrer Institute]
- ☞ MUCH **cheaper** & technically **easier**.
 - ☞ Only one cyclotron.
 - ☞ Lower intensity.

Disadvantage?

- ☞ A second detector!
 - ☞ μ DAR with Two Scintillators (μ DARTS) [1401.3977]
 - ☞ Tokai 'N Toyama to(2) Kamioka (**TNT2K**) [1506.05023]

μ DARTS – JUNO & RENO50

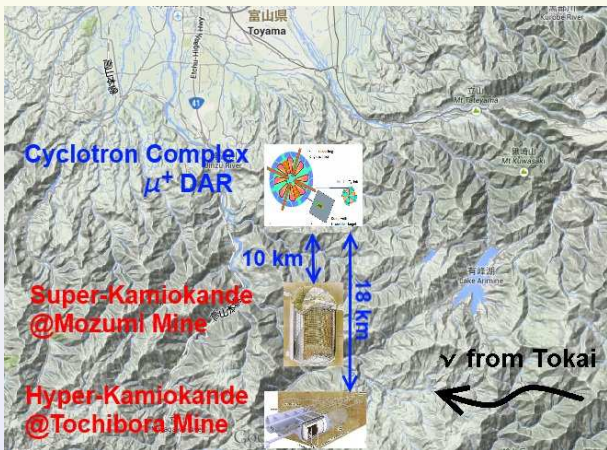
- Two detectors are suggested to overcome the **unknown** energy response. [Ciuffoli et al., PRD 2014; 1307.7419]



- China Atomic Energy Center has a proposal for cyclotron

TNT2K

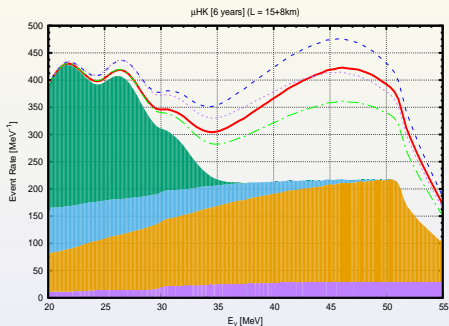
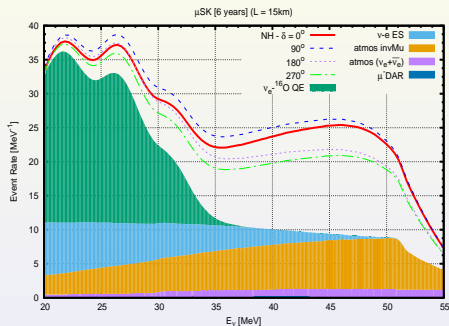
👁️ $T2(H)K + \mu SK + \mu HK$



👁️ μ DAR is also useful for **material**, **medicine** industries in Toyama

Event Shape @ TNT2K

Evslin, Ge & Hagiwara [1506.05023]

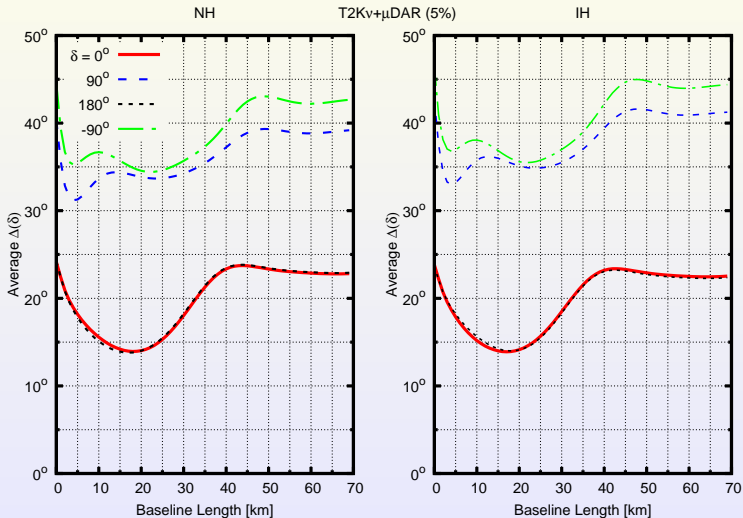


Expected μ DAR IBD signal from 6 yrs of running @ SK (15km) & HK (23km) with NH.

Simulated by [NuPro](http://nupro.hepforge.org/), <http://nupro.hepforge.org/>

δ_D Precision @ TNT2K

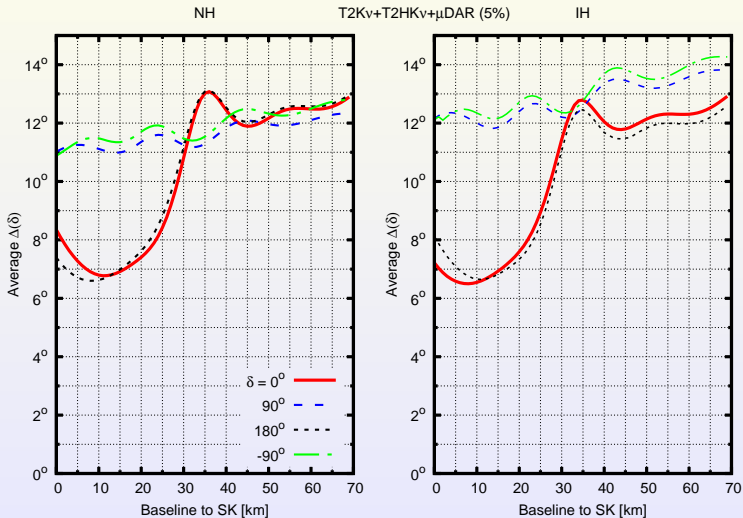
Evslin, Ge & Hagiwara [1506.05023]



Simulated by NuPro, <http://nupro.hepforge.org/>

δ_D Precision @ TNT2K

Evslin, Ge & Hagiwara [1506.05023]



Simulated by **NuPro**, <http://nupro.hepforge.org/>

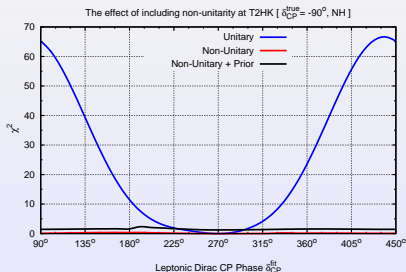
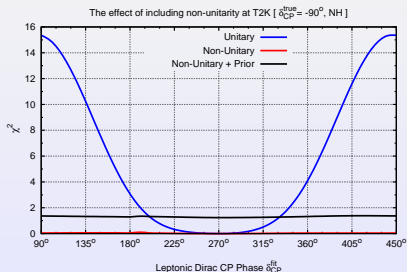
Non-Unitarity Mixing (NUM)

Ge, Pasquini, Tortola & Valle

[1605.01670]

$$N = N^{NP} U = \begin{pmatrix} \alpha_{11} & 0 & 0 \\ |\alpha_{21}| e^{i\phi} & \alpha_{22} & 0 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix} U.$$

$$P_{\mu e}^{NP} = \alpha_{11}^2 \left\{ \alpha_{22}^2 \left[c_a^2 |S'_{12}|^2 + s_a^2 |S'_{13}|^2 + 2c_a s_a (\cos \delta_D \mathbb{R} - \sin \delta_D \mathbb{I}) (S'_{12} S'_{13}^*) \right] + |\alpha_{21}|^2 P_{ee} \right. \\ \left. + 2\alpha_{22} |\alpha_{21}| \left[c_a (c_\phi \mathbb{R} - s_\phi \mathbb{I}) (S'_{11} S'_{12}^*) + s_a (c_{\phi+\delta_D} \mathbb{R} - s_{\phi+\delta_D} \mathbb{I}) (S'_{11} S'_{13}^*) \right] \right\}.$$



NUM vs Seesaw Mechanism

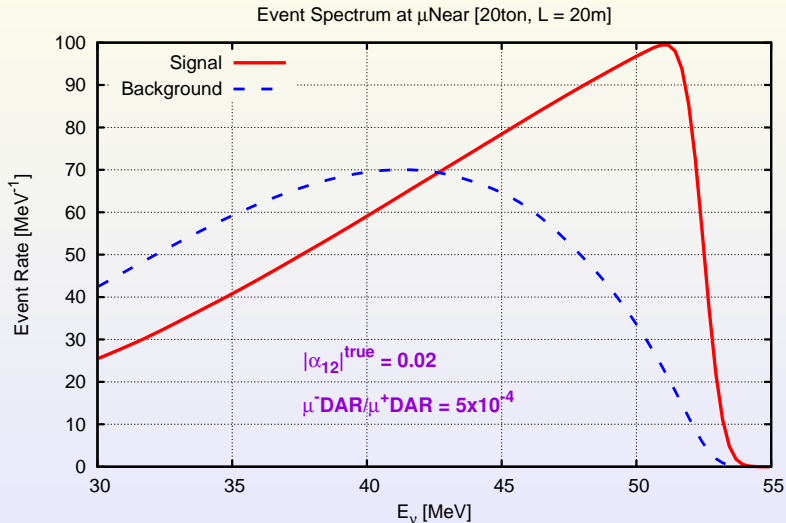
☞ Heavy neutrinos

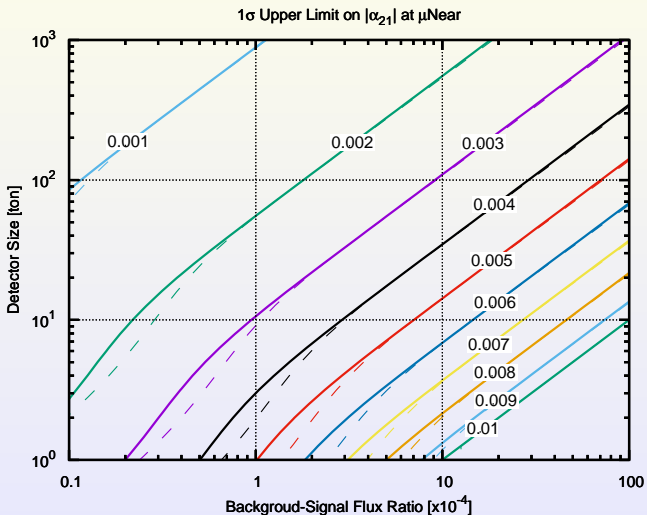
$$\bar{\nu} M_D \mathcal{N} + h.c. + \bar{\mathcal{N}} M_N \mathcal{N} = \begin{pmatrix} \bar{\nu} & \bar{\mathcal{N}} \end{pmatrix} \begin{pmatrix} 0 & M_D \\ M_D^T & M_N \end{pmatrix} \begin{pmatrix} \nu \\ \mathcal{N} \end{pmatrix}$$

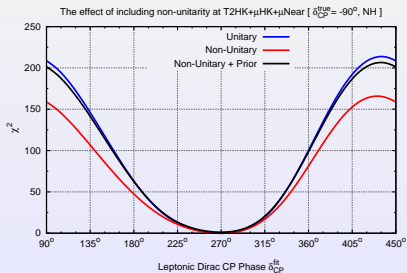
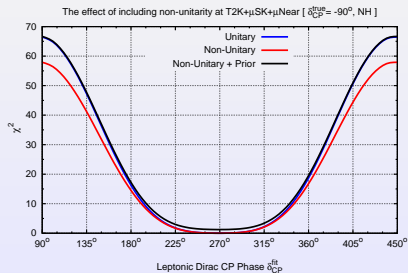
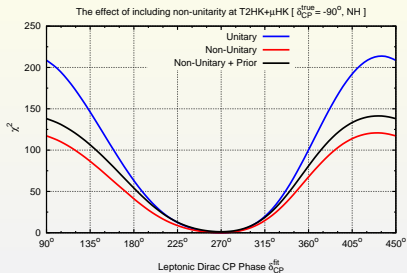
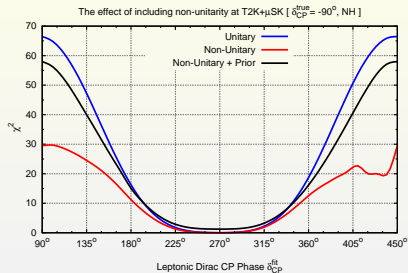
☞ Seesaw Mechanism

$$M_\nu = -M_D M_N^{-1} M_D^T, \quad \nu' = \nu + M_D M_N^{-1} \mathcal{N}$$







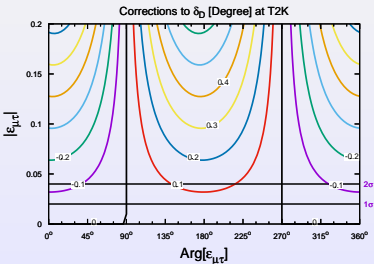
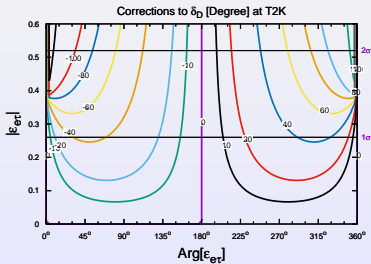
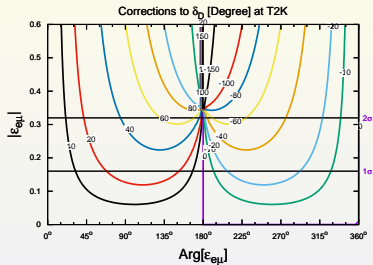
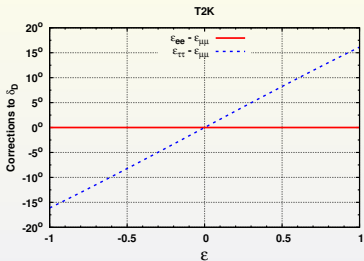


$$\mathcal{H} \equiv \frac{1}{2E_\nu} U \begin{pmatrix} 0 & & \\ & \Delta m_s^2 & \\ & & \Delta m_a^2 \end{pmatrix} U^\dagger + V_{cc} \begin{pmatrix} 1 + \epsilon_{ee} & \epsilon_{e\mu} & \epsilon_{e\tau} \\ \epsilon_{e\mu}^* & \epsilon_{\mu\mu} & \epsilon_{\mu\tau} \\ \epsilon_{e\tau}^* & \epsilon_{\mu\tau}^* & \epsilon_{\tau\tau} \end{pmatrix}$$

- Standard Interaction – V_{cc} (also V_{nc})
- Non-Standard Interaction – $\epsilon_{\alpha\beta}$
 - Diagonal $\epsilon_{\alpha\alpha}$ are real
 - Off-diagonal $\epsilon_{\alpha\neq\beta}$ are complex
 - Both can fake CP
- Z' in LMA-Dark model with $L_\mu - L_\tau$ gauged as $U(1)$
 - $M_{Z'} \sim \mathcal{O}(10)\text{MeV}$
 - $g_{Z'} \sim 10^{-5}$

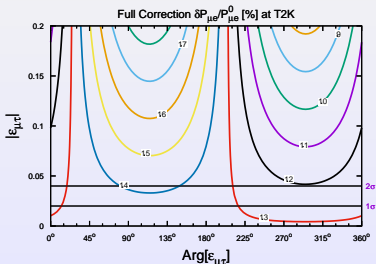
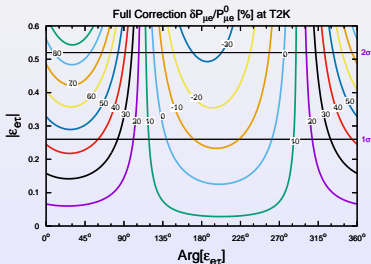
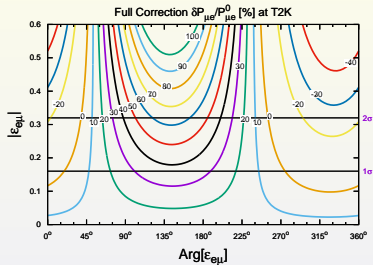
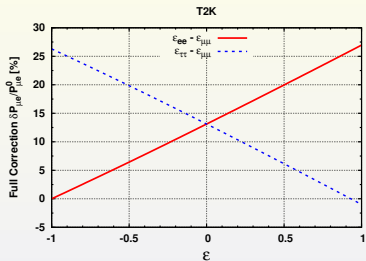
Faked CP with NSI

SFG & Smirnov [arXiv:1607.xxxxx]



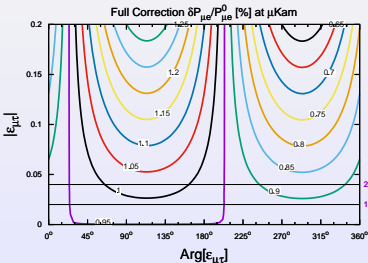
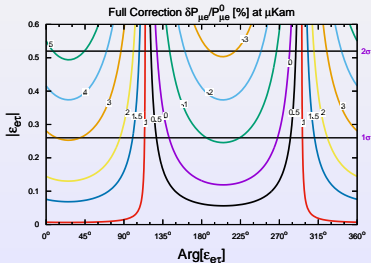
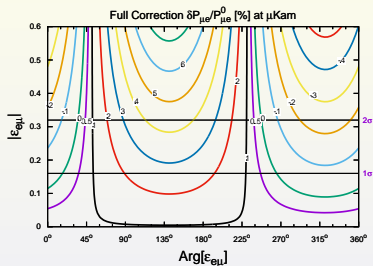
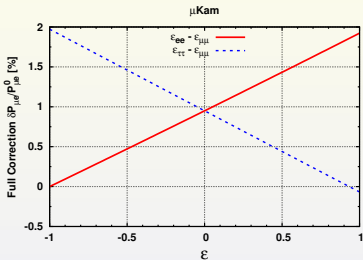
The effect of NSI @ T2K

SFG & Smirnov [arXiv:1607.xxxxx]

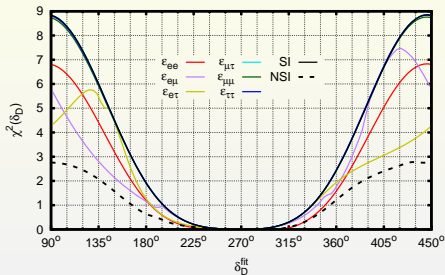


The effect of NSI @ μ Kam

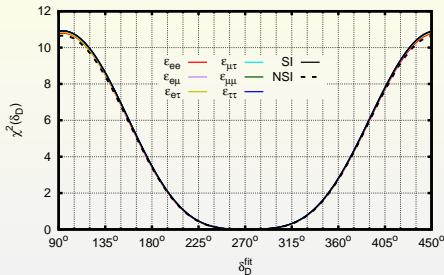
SFG & Smirnov [arXiv:1607.xxxxx]



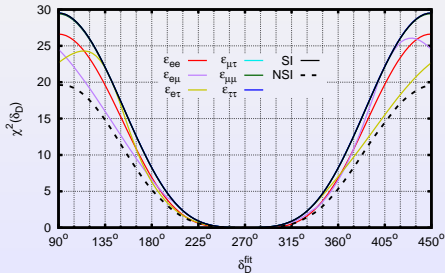
The effect of NSI on the CP sensitivity at T2K [$\delta_D^{\text{true}} = -90^\circ$]



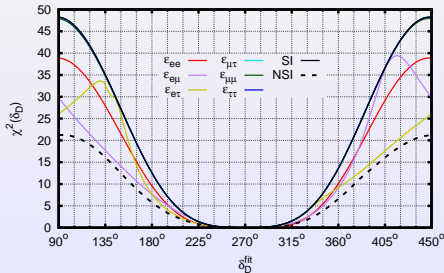
The effect of NSI on the CP sensitivity at μ SK [$\delta_D^{\text{true}} = -90^\circ$]



The effect of NSI on the CP sensitivity at T2K+ μ SK [$\delta_D^{\text{true}} = -90^\circ$]



The effect of NSI on the CP sensitivity at ν T2K+ μ SK [$\delta_D^{\text{true}} = -90^\circ$]



CP Sensitivities

SFG & Smirnov [arXiv:1607.xxxxx]

$\delta_D = -90^\circ$ vs 0° Event Numbers	T2K $57\nu + 28\bar{\nu}$		μ SK $212\bar{\nu}$		T2K+ μ SK $57\nu + 240\bar{\nu}$		ν T2K+ μ SK $171\nu + 212\bar{\nu}$	
χ^2 for SI & NSI	2.43	1.21	4.13	2.75	8.20	5.33	13.6	6.60
χ^2 ϵ_{ee} best fit	1.58	–	4.09	–	6.81	–	10.1	–
	0.60	0.55	0.47	0.47	0.69	0.65	0.90	0.69
χ^2 $\epsilon_{\mu\mu}$ best fit	2.42	–	4.13	–	8.20	–	13.6	–
	-0.02	-0.01	-0.02	-0.01	-0.02	-0.01	-0.02	0.09
χ^2 $\epsilon_{\tau\tau}$ best fit	2.43	–	4.13	–	8.20	–	13.6	–
	0.02	0.01	0.02	0.01	0.02	0.01	0.01	0.10
χ^2 $\epsilon_{e\mu}$ best fit	1.76	–	4.11	–	6.54	–	9.12	–
	0.10	0.04	0.02	0.01	0.15	0.11	0.19	0.10
χ^2 $\epsilon_{e\tau}$ best fit	1.94	–	4.13	–	7.02	–	9.17	–
	0.17	0.11	0.01	0.01	0.27	0.24	0.43	0.16
χ^2 $\epsilon_{\mu\tau}$ best fit	2.43	–	4.13	–	8.20	–	13.6	–
	0	0	0	0	0	0	0	0

Summary

☞ Horizontal Symmetry

- ☞ $\mathbb{Z}_2^{\mu\tau}$ [G_3 v.s. $d_\nu^{(3)}$]
- ☞ \mathbb{Z}_2^s [G_1 v.s. $d_\nu^{(1)}$] & $\overline{\mathbb{Z}}_2^s$ [G_2 v.s. $d_\nu^{(2)}$]

☞ Residual Symmetry as Custodial Symmetry

- ☞ Full symmetry has to be broken. Otherwise, no mixing.
- ☞ Mixing angles dictated by residual symmetry.
- ☞ Physical observables, mixing angles & CP phase, are correlated.

☞ Phenomenological consequences

- ☞ Nonzero θ_r
- ☞ Large δ_D
- ☞ Non-maximal θ_a
- ☞ Distinguishing \mathbb{Z}_2^s & $\overline{\mathbb{Z}}_2^s$ with $\text{NO}\nu\text{A}$, T2K, μDAR

☞ Better CP measurement than T2K

- ☞ Much larger event numbers
- ☞ Much better CP sensitivity around maximal CP
- ☞ Solve degeneracy between δ_D & $\pi - \delta_D$
- ☞ Guarantee CP sensitivity against NUM
- ☞ Guarantee CP sensitivity against NSI

☞ Better configuration than DAE δ LUS

- ☞ Only one cyclotron
- ☞ 100% duty factor
- ☞ Much lower flux intensity
- ☞ Much easier
- ☞ Much cheaper
- ☞ Single near detector

Thank You!

Mixing Dictated by \mathbb{Z}_2^5

☞ Mixing matrix,

$$\mathbf{V}_\nu \equiv \mathcal{P}_\nu \mathcal{U}_\nu \mathcal{Q}_\nu$$

☞ $\mathbf{G}_\nu = \mathbf{V}_\nu \mathbf{d}_\nu \mathbf{V}_\nu^\dagger \quad \Rightarrow \quad \tilde{\mathbf{G}} \equiv \mathcal{P}_\nu^\dagger \mathbf{G}_1 \mathcal{P}_\nu = \mathcal{U}_\nu \mathbf{d}_\nu^{(1)} \mathcal{U}_\nu^\dagger$

$$\tilde{\mathbf{G}}_1 = \frac{1}{2 + k^2} \begin{pmatrix} 2 - k^2 & 2ke^{-i(\phi_1 - \phi_2)} & 2ke^{-i(\phi_1 - \phi_3)} \\ 2ke^{i(\phi_1 - \phi_2)} & k^2 & -2e^{-i(\phi_2 - \phi_3)} \\ 2ke^{i(\phi_1 - \phi_3)} & -2e^{i(\phi_2 - \phi_3)} & k^2 \end{pmatrix}$$

$$\tilde{\mathbf{G}}_{11} = s_r^2 - c_r^2(c_s^2 - s_s^2),$$

$$\tilde{\mathbf{G}}_{12} = \tilde{\mathbf{G}}_{21}^* = 2c_r c_s (c_s s_a s_r e^{i\delta_D} - c_a s_s),$$

$$\tilde{\mathbf{G}}_{13} = \tilde{\mathbf{G}}_{31}^* = -2c_r c_s (c_s c_a s_r e^{i\delta_D} + s_a s_s),$$

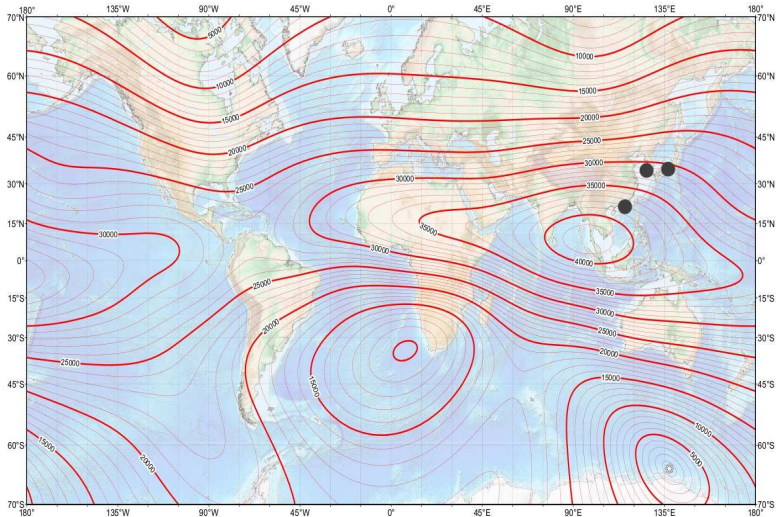
$$\tilde{\mathbf{G}}_{22} = (c_s^2 - s_s^2)c_a^2 + 4c_a s_a c_s s_s s_r \cos \delta_D + [c_r^2 - (c_s^2 - s_s^2)s_r^2] s_a^2,$$

$$\tilde{\mathbf{G}}_{23} = \tilde{\mathbf{G}}_{32}^* = [3c_s^2 - 2 - c_s^2(c_r^2 - s_r^2)] c_a s_a - 2[(c_a^2 - s_a^2) \cos \delta_D + i \sin \delta_D] c_s s_s s_r,$$

$$\tilde{\mathbf{G}}_{33} = (c_s^2 - s_s^2)s_a^2 - 4c_a s_a c_s s_s s_r \cos \delta_D + [c_r^2 - (c_s^2 - s_s^2)s_r^2] c_a^2.$$

Lowest Atmospheric Neutrino Background

US/UK World Magnetic Model -- Epoch 2010.0
Main Field Horizontal Intensity (H)



Backgrounds to IBD ($\bar{\nu}_e + p \rightarrow e^+ + n$)

- ☞ Reactor $\bar{\nu}_e$: $E_\nu < 10$ MeV
- ☞ Accelerator ν_e : $E_\nu > 100$ MeV
- ☞ Spallation: $E_\nu \lesssim 20$ MeV
- ☞ Supernova Relic Neutrino: $E_\nu \lesssim 20$ MeV

Cut with $30 \text{ MeV} < E_\nu < 55 \text{ MeV}$

- ☞ Accelerator $\nu_\mu \rightarrow$ **Invisible muon**
- ☞ Atmospheric Neutrino Background
 - ☞ **Invisible muon** (below Cherenkov limit)
 - ☞ $E_\mu \lesssim 1.5 \times m_\mu$, $\mu^\pm \rightarrow e^\pm$
 - ☞ $E_\pi \lesssim 1.5 \times m_\pi$, $\pi^+ \rightarrow \mu^+ \rightarrow e^+$
 - ☞ 1 neutron
 - ☞ No prompt photon
 - ☞ Irreducible $\bar{\nu}_e$: $30 \text{ MeV} \lesssim E_\nu \lesssim 55 \text{ MeV}$
 - ☞ Reducible ν_e : $60 \text{ MeV} \lesssim E_\nu \lesssim 100 \text{ MeV}$
 - ☞ 1 neutron
 - ☞ No prompt photon
 - ☞ **Lowest** at μ DARTS & TNT2K sites