A New Unified Theory of Electromagnetic and Gravitational Interactions

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Outline of the New Unified Theory

5-dim Einstein field eq.

$$\tilde{R}_{ab} - \frac{1}{2}\tilde{R}\tilde{g}_{ab} = \tilde{\kappa}\tilde{T}_{ab} ,$$

4+1 decomposition

$$R + K_{ab}K^{ab} - K^2 = -2\tilde{\kappa}\tilde{T}_{ab}n^a n^b ,$$

$$\nabla_a K^{ab} - \nabla^b K = \tilde{\kappa} g^{ab}\tilde{T}_{ac}n^c ,$$

$$G_{ab} = \tilde{\kappa} g_a{}^c g_b{}^d \tilde{T}_{cd} + g_{ab}{}^{cd}\tilde{\pounds}_n K_{cd} - (2K_a{}^c K_{cb} - KK_{ab}) + \frac{1}{2}\left(3K_{cd}K^{cd} - K^2\right) g_{ab} .$$

- The vector field eq. is equivalent to Einstein-Maxwell eq. with a curvature-coupled term

$$\nabla_a F^{ab} + 2R^b{}_a A^a = -4\pi J^b \ ,$$

- The tensor field eq. is just the 4-d Einstein field equation, with the r.h.s. being interpreted as matter fields.

- The scalar field eq. is a constraint eq.

Kaluza-Klein Theory

5-dim Einstein field eq.

$$\hat{G}_{AB} = 0$$

4+1 decomposition

$$\nabla^{\alpha} F_{\alpha\beta} = -3 \frac{\partial^{\alpha}\phi}{\phi} F_{\alpha\beta}$$

$$G_{\alpha\beta} = \frac{\kappa^2 \phi^2}{2} T^{\mathrm{EM}}_{\alpha\beta} - \frac{1}{\phi} [\nabla_{\alpha}(\partial_{\beta}\phi) - g_{\alpha\beta} \Box \phi]$$

$$\Box \phi = \frac{\kappa^2 \phi^2}{4} F_{\alpha\beta} F^{\alpha\beta}$$

Brane World Theory

5-dim Einstein field eq.

$$ {}^{(5)}G_{AB} = -\Lambda_5 \, {}^{(5)}g_{AB} + \kappa_5^2 \left[{}^{(5)}T_{AB} + T_{AB}^{\rm brane} \delta(y) \right] . $$

4+1 decomposition

$$ G_{\mu\nu} = -\frac{1}{2}\Lambda_5 g_{\mu\nu} + \frac{2}{3}\kappa_5^2 \left[{}^{(5)}T_{AB} g_\mu{}^A g_\nu{}^B + \left({}^{(5)}T_{AB} n^A n^B - \frac{1}{4} \, {}^{(5)}T \right) g_{\mu\nu} \right] $$
$$ + K K_{\mu\nu} - K_\mu{}^\alpha K_{\alpha\nu} + \frac{1}{2} \left[K^{\alpha\beta} K_{\alpha\beta} - K^2 \right] g_{\mu\nu} - \mathcal{E}_{\mu\nu} , $$

$$ \mathcal{E}_{\mu\nu} = {}^{(5)}C_{ACBD} \, n^C n^D g_\mu{}^A g_\nu{}^B , $$

The New Unified Theory

Consider a hypersurface embedded in a n+1 dimensional spacetime with coordinates {x^0,…, x^{n-1}, x^n=w}. The hypersurface is defined by w=0.

$$g_{ab} \equiv \tilde{g}_{ab} - n_a n_b,$$

$$w^a = N n^a + N^a,$$

$$n^a n_a = 1.$$

$$n^a N_a = 0.$$

$$K_{ab} \equiv g_a{}^c \tilde{\nabla}_c n_b = \frac{1}{2} \tilde{\pounds}_n g_{ab} = K_{ba},$$

$$a^b \equiv n^a \tilde{\nabla}_a n^b,$$

$$\tilde{R} = R - K_{ab} K^{ab} + K^2 - 2 \tilde{\nabla}_a v^a,$$

$$v^a \equiv n^a \tilde{\nabla}_c n^c - n^c \tilde{\nabla}_c n^a = K n^a - a^a.$$

$$\begin{aligned} \tilde{S}_G &= \int \sqrt{-\tilde{g}}\, \tilde{R}\, \tilde{e} \\ &= \int \sqrt{-\tilde{g}}\, (R - K_{ab} K^{ab} + K^2)\, \tilde{e}, \end{aligned}$$

$$K_{ab} = \frac{1}{2} N^{-1} \left(\dot{g}_{ab} - M_{ab} \right),$$

$$\dot{g}_{ab} \equiv \frac{\partial}{\partial w} g_{ab} \equiv g_a{}^c g_b{}^d \tilde{\mathcal{L}}_w g_{cd}$$

$$M_{ab} \equiv \nabla_a N_b + \nabla_b N_a .$$

$$\tilde{S}_G = \int \tilde{\mathcal{L}}_G(N, N_a, g_{ab}) \tilde{e},$$

$$\begin{aligned} &\tilde{\mathcal{L}}_G(N, N_a, g_{ab}) \\ &= \sqrt{-g} N \left[R - \frac{1}{4} N^{-2} \left(M_{ab} M^{ab} - M^2 \right) \right] - \frac{1}{4} \sqrt{-g} N^{-1} \\ &\quad \times \left(g^{ac} g^{bd} - g^{ab} g^{cd} \right) \left(\dot{g}_{ab} \dot{g}_{cd} - 2 \dot{g}_{ab} M_{cd} \right). \end{aligned}$$

$$\tilde{\mathcal{L}}_G = \mathcal{L}_G + \mathcal{L}_{\mathrm{EM}} + \mathcal{L}_{\mathrm{Other}},$$

where

$$\mathcal{L}_G \equiv \sqrt{-g} N R,$$

$$\mathcal{L}_{\mathrm{EM}} \equiv -\frac{1}{4}\sqrt{-g} N^{-1}\left(M_{ab}M^{ab} - M^2\right),$$

and

$$\mathcal{L}_{\mathrm{Other}} \equiv -\frac{1}{4}\sqrt{-g} N^{-1}\left(g^{ac}g^{bd} - g^{ab}g^{cd}\right) \times \left(\dot{g}_{ab}\dot{g}_{cd} - 2\dot{g}_{ab}M_{cd}\right).$$

L_G: Lagrangian density of gravity

L_{EM}: Lagrangian density of electromagnetic fields

L_{Other}: Lagrangian density of other matter fields

Electromagnetic Field Equations

- Maxwell equation in a flat spacetime

$$\partial_a F^{ab} = -4\pi J^b$$

$$F_{ab} = \partial_a A_b - \partial_b A_a$$

- Minimal substitution rule: $\eta_{ab} \to g_{ab}, \ \partial_a \to \nabla_a$, hence we get the Einstein-Maxwell equation:

$$\nabla_a F^{ab} = -4\pi J^b$$

$$F_{ab} = \nabla_a A_b - \nabla_b A_a$$

Einstein was not satisfied with the EM field eq. copied from the Maxwell eq. in a flat spacetime. He spent his later half life to explore a unified theory for electromagnetic and gravitational interactions. Many other people also looked for such a theory, including Kaluza, Klein, Eddington, Weyl.

The minimal substitution rule usually does not lead to a unique result in a curved spacetime. For example, if one started from the EM wave eq. in a flat spacetime, by MSR one would get $\nabla^a \nabla_a A^b = 0$ in Lorents gauge. But the "correct" eq. derived from the Einstein-Maxwell eq. is $\nabla^a \nabla_a A^b - R^b{}_a A^a = 0$.

Maxwell equation can be expressed in terms of a symmetric tensor

$$H_{ab} \equiv \partial_a A_b + \partial_b A_a.$$

By the definitions of F_{ab} and H_{ab}, we have

$$F_{ab} = H_{ab} - 2\partial_b A_a.$$

Hence,

$$\partial_a F^{ab} = \partial_a H^{ab} - \partial^b H,$$

Hence, the Maxwell eq. becomes

$$\partial_a H^{ab} - \partial^b H = -4\pi J^b$$

Applying the minimal substitution rule to the Maxwell equation, we get

$$\nabla_a H^{ab} - \nabla^b H = -4\pi J^b$$

$$H_{ab} = \nabla_a A_b + \nabla_b A_a$$

By the identity

$$\nabla_a H^{ab} - \nabla^b H = \nabla_a F^{ab} + 2R^b{}_a A^a$$

we get

$$\nabla_a F^{ab} + 2R^b{}_a A^a = -4\pi J^b$$

Which differs from the Einstein-Maxwell eq. by a curvature-coupling term.

The effective current density

$$J^a_{\text{eff}} \equiv J^a + \frac{1}{2\pi} R^a{}_b A^b$$

which is conserved: $\nabla_a J^a_{eff} = 0.$

Define

$$\Theta_{ab} = \Theta_{ba} \equiv H_{ab} - H g_{ab}.$$

The new electromagnetic field eq. becomes

$$\nabla_a \Theta^{ab} = -4\pi J^b,$$

The new electromagnetic field eq. can be derived from the action

$$S_{\rm EM} = \int \mathcal{L}_{\rm EM}(g_{ab}, A_a) e$$

where

$$\begin{aligned} &\mathcal{L}_{\rm EM}(g_{ab}, A_a) \\ &= \sqrt{-g}\left(-\frac{1}{4}F_{ab}F^{ab} + R_{ab}A^a A^b + 4\pi A_a J^a\right) \\ &= \sqrt{-g}\left[-\frac{1}{4}\left(H_{ab}H^{ab} - H^2\right) + 4\pi A_a J^a\right] \\ &= \sqrt{-g}\left[-\frac{1}{4}\left(\Theta_{ab}\Theta^{ab} - \frac{1}{n-1}\Theta^2\right) + 4\pi A_a J^a\right] \end{aligned}$$

The Lagrangian density

$$\mathcal{L}_{\rm EM} \equiv -\frac{1}{4}\sqrt{-g}N^{-1}\left(M_{ab}M^{ab} - M^2\right),$$

is equivalent to the EM Lagrangian density for the new EM equation, if we assume $\nabla_a N = 0$ and make the substitution

$$N_a = 2N l_{\rm P}^{n/2-1} A_a,$$

Then

$$\mathcal{L}_{\rm EM} = -\sqrt{-g}N l_{\rm P}^{n-2}\left(H_{ab}H^{ab} - H^2\right).$$

$$\mathcal{L}_G + \mathcal{L}_{\rm EM} = \sqrt{-g}N l_{\rm P}^{n-2}\left[l_{\rm P}^{-n+2}R - \left(H_{ab}H^{ab} - H^2\right)\right].$$

Derivation of Field Equations

Recall that

$$\tilde{\mathcal{L}}_G = \mathcal{L}_G + \mathcal{L}_{\mathrm{EM}} + \mathcal{L}_{\mathrm{Other}},$$

where

$$\mathcal{L}_G \equiv \sqrt{-g} N R,$$

$$\mathcal{L}_{\mathrm{EM}} \equiv -\frac{1}{4}\sqrt{-g} N^{-1}\left(M_{ab}M^{ab} - M^2\right),$$

and

$$\mathcal{L}_{\mathrm{Other}} \equiv -\frac{1}{4}\sqrt{-g} N^{-1}\left(g^{ac}g^{bd} - g^{ab}g^{cd}\right) \times \left(\dot{g}_{ab}\dot{g}_{cd} - 2\dot{g}_{ab}M_{cd}\right).$$

Let us write

$$\tilde{\mathcal{L}}_G = \mathcal{L}_G + \mathcal{L}_{\rm EM} + \mathcal{L}_m + \mathcal{L}_{\rm int}.$$

$$\tilde{S}_G = S_G + S_{\rm EM} + S_m + S_{\rm int},$$

$$\mathcal{L}_G \equiv \sqrt{-g} N R,$$

$$\mathcal{L}_{\rm EM} = -\sqrt{-g} N \left(\Psi_{ab} \Psi^{ab} - \frac{1}{n-1} \Psi^2 \right),$$

$$\mathcal{L}_m = -\sqrt{-g} N \left(\Phi_{ab} \Phi^{ab} - \frac{1}{n-1} \Phi^2 \right),$$

and

$$\mathcal{L}_{\rm int} = -2\sqrt{-g} N \left(\Psi_{ab} \Phi^{ab} - \frac{1}{n-1} \Psi \Phi \right),$$

where

$$\Psi_{ab} \equiv \frac{1}{2} N^{-1} \left(M_{ab} - M g_{ab} \right),$$

$$\Phi_{ab} \equiv -\frac{1}{2} N^{-1} \left(g_a{}^c g_b{}^d - g_{ab} g^{cd} \right) \dot{g}_{cd},$$

The Lagrangian contains three independent variables: N, N_a, and g_{ab}.

$\delta\tilde{S}_G/\delta N = 0$ leads to a scalar constraint equation

$$R + \Pi_{ab}\Pi^{ab} - \frac{1}{n-1}\Pi^2 = 0.$$

$$\Pi_{ab} \equiv \Psi_{ab} + \Phi_{ab},$$

$$\Pi_{ab} = -K_{ab} + K g_{ab}.$$

It is just

$$R + K_{ab}K^{ab} - K^2 = 0,$$

$\delta\tilde{S}_G/\delta N_a = 0$ leads to a vector field equation

$$\nabla_a \Pi^{ab} = 0,$$

or, equivalently,

$$\nabla_a \Psi^{ab} = -\nabla_a \Phi^{ab}.$$

Define

$$J^b \equiv \frac{1}{4\pi}\nabla_a \Phi^{ab},$$

we get

$$\nabla_a \Psi^{ab} = -4\pi J^b.$$

It is equivalent to

$$\nabla_a K^{ab} - \nabla^b K = 0,$$

$\delta\tilde{S}_G/\delta g^{ab} = 0$ leads to a tensor field equation--the gravitational field eq. on the hypersurface

$$G_{ab} = \kappa\left(T_{\mathrm{EM},ab} + T_{m,ab} + T_{\mathrm{int},ab}\right) + \frac{1}{N}\left(\nabla_a\nabla_b N - g_{ab}\nabla_c\nabla^c N\right).$$

$$\kappa T_{\mathrm{EM},ab} \equiv -\frac{1}{N\sqrt{-g}}\frac{\delta S_{\mathrm{EM}}}{\delta g^{ab}},$$

$$\kappa T_{m,ab} \equiv -\frac{1}{N\sqrt{-g}}\frac{\delta S_m}{\delta g^{ab}},$$

and

$$\kappa T_{\mathrm{int},ab} \equiv -\frac{1}{N\sqrt{-g}}\frac{\delta S_{\mathrm{int}}}{\delta g^{ab}}.$$

Stress-energy tensor of EM field

$$T_{\mathrm{EM},ab} = \frac{2}{\kappa}\left(\Psi_{ac}\Psi_b{}^c - \frac{1}{n-1}\Psi\Psi_{ab}\right) - \frac{1}{2\kappa}\left(\Psi_{cd}\Psi^{cd} - \frac{1}{n-1}\Psi^2\right)g_{ab} - \frac{1}{\kappa N}\nabla^c\left(2N_{(a}\Psi_{b)c} - N_c\Psi_{ab}\right),$$

$$T_{\mathrm{EM}} = \frac{2}{\kappa}\left(1-\frac{1}{4}n\right)\left(\Psi_{cd}\Psi^{cd} - \frac{1}{n-1}\Psi^2\right) - \frac{1}{\kappa N}\nabla^c\left(2N^a\Psi_{ac} - N_c\Psi\right).$$

Stress-energy tensor of matter fields

$$T_{m,ab} = -g_{ac}g_{bd}\frac{1}{\kappa N\sqrt{-g}}\frac{\partial}{\partial w}\left(\sqrt{-g}\Phi^{cd}\right)$$
$$+\frac{2}{\kappa}\left(\Phi_{ac}\Phi_b{}^c - \frac{1}{n-1}\Phi\Phi_{ab}\right)$$
$$-\frac{1}{2\kappa}\left(\Phi_{cd}\Phi^{cd} - \frac{1}{n-1}\Phi^2\right)g_{ab}.$$

$$T_m = -g_{cd}\frac{1}{\kappa N\sqrt{-g}}\frac{\partial}{\partial w}\left(\sqrt{-g}\Phi^{cd}\right)$$
$$+\frac{2}{\kappa}\left(1-\frac{1}{4}n\right)\left(\Phi_{cd}\Phi^{cd} - \frac{1}{n-1}\Phi^2\right).$$

Stress-energy tensor of interaction

$$T_{\mathrm{int},ab} = -g_{ac}g_{bd}\frac{1}{\kappa N\sqrt{-g}}\frac{\partial}{\partial w}\left(\sqrt{-g}\Psi^{cd}\right) + \frac{2}{\kappa}\left[2\Phi_{c(a}\Psi_{b)}{}^{c} - \frac{1}{n-1}\left(\Phi\Psi_{ab} + \Psi\Phi_{ab}\right)\right] - \frac{1}{\kappa}\left(\Phi_{cd}\Psi^{cd} - \frac{1}{n-1}\Phi\Psi\right)g_{ab} - \frac{1}{\kappa N}\nabla^{c}\left[2N_{(a}\Phi_{b)c} - N_{c}\Phi_{ab}\right].$$

$$T_{\mathrm{int}} = -g_{cd}\frac{1}{\kappa N\sqrt{-g}}\frac{\partial}{\partial w}\left(\sqrt{-g}\Psi^{cd}\right) + \frac{4}{\kappa}\left(1 - \frac{1}{4}n\right)\left(\Phi_{cd}\Psi^{cd} - \frac{1}{n-1}\Phi\Psi\right) - \frac{1}{\kappa N}\nabla^{c}\left(2N^{a}\Phi_{ac} - N_{c}\Phi\right).$$

Discussion on the New EM Equation

The new EM field eq. has the form

$$\nabla_a F^{ab} - \xi R^b{}_a A^a = -4\pi J^b$$

with ξ=-2. It has a few motivations:

- It is consistent with the minimal substitution rule
- It can be derived from 5-dim gravity
- It can describe a charged universe

EM Paradoxes in Cosmology

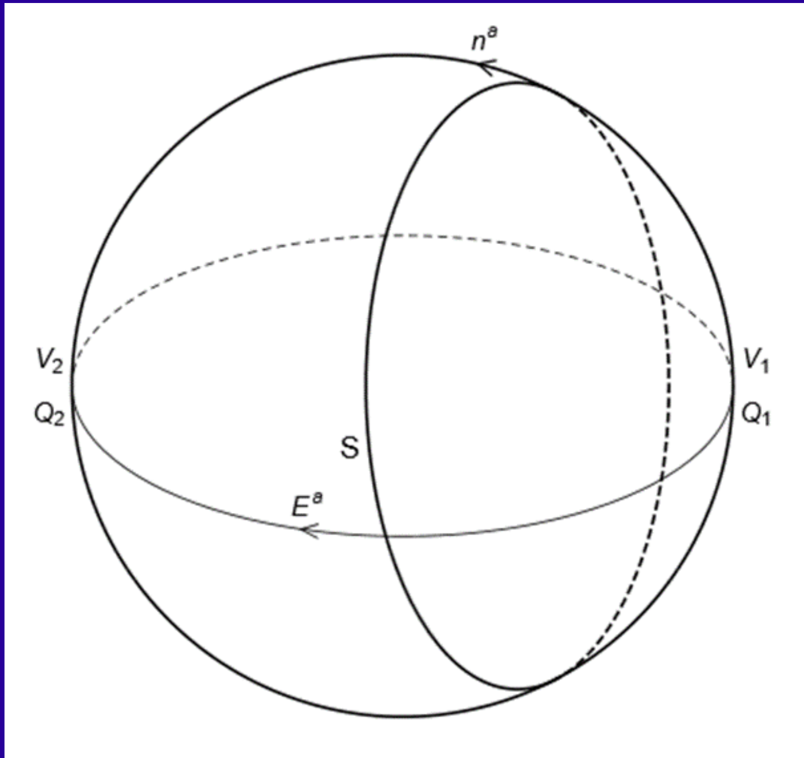
- Tyep I Parodox: The Maxwell equation fails when it is applied to a homogeneous and isotropic universe with a uniformly distributed net charge.

$$\nabla_a F^{ab} = -4\pi J^b$$

- Type II Paradox: if the spatial section of the universe is compact, the Maxwell equation always fails if the universe has a nonzero net charge.

$$\frac{1}{4\pi}\int E_a n^a dS = Q$$

$$Q_1 + Q_2 = 0$$



The new electromagnetic field equation can solve the type I paradox

$$\nabla_a F^{ab} - \xi R^b{}_a A^a = -4\pi J^b$$

$$\xi R^b{}_a A^a = 4\pi J^b = 4\pi \rho_e(t) u^b$$

$$R_{ab} = \alpha(t) u_a u_b + \beta(t) h_{ab}$$

$$A^a = \Phi(t)\, u^a, \quad R^b{}_a A^a = -\alpha(t)\Phi(t)\, u^b$$

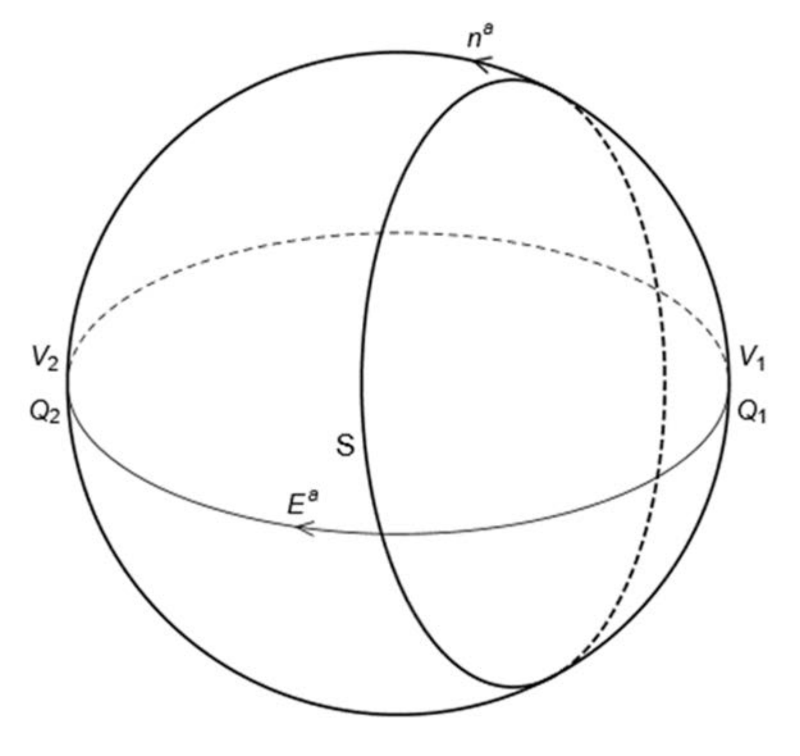
$$\Phi(t) = -\frac{4\pi}{\alpha(t)\xi}\rho_e(t), \quad A^a = -\frac{4\pi}{\alpha(t)\xi}\rho_e(t)\, u^a$$

The new electromagnetic field equation can solve the type II paradox

$$D_a E^a - \xi R_{ab} u^a A^b = 4\pi\rho_e$$

$$Q_1 + Q_2 = -\frac{\xi}{4\pi}\int_{V_1+V_2} R_{ab} u^a A^b dV$$

$Q_1 + Q_2 \neq 0$ in general.



The EM Stress-Energy Tensor

$$T_{\mathrm{EM},ab} = {}^{(0)}T_{\mathrm{EM},ab} + {}^{(1)}T_{\mathrm{EM},ab}$$

$${}^{(0)}T_{\mathrm{EM},ab} = \frac{1}{4\pi}\left(F_{ac}F_b{}^c - \frac{1}{4}g_{ab}F_{cd}F^{cd}\right)$$

$${}^{(1)}T_{\mathrm{EM},ab} = \frac{\xi}{8\pi}\{\nabla^c\nabla_c(A_aA_b) - 2\nabla^c\nabla_{(a}(A_{b)}A_c) + 4A^cR_{c(a}A_{b)}$$
$$+ g_{ab}\left[\nabla_c\nabla_d(A^cA^d) - R_{cd}A^cA^d\right]\}$$

$$\nabla^aT_{\mathrm{EM},ab} = -F_{ba}J^a + A_b\nabla_aJ^a$$

The last eq. is just the Lorentz force law when charge is conserved, i.e., when $\nabla_a J^a = 0$.

$$J^a_{\rm ps} = -\frac{\xi}{4\pi} R^a{}_b A^b \ .$$

$$\nabla^{a\,(0)}T_{{\rm EM},ab} = -F_{ba}\left(J^a + J^a_{\rm ps}\right) \ .$$

$$\nabla^{a\,(1)}T_{{\rm EM},ab} = F_{ba}J^a_{\rm ps} - A_b \nabla_a J^a_{\rm ps} \ .$$

${}^{(0)}T_{EM,ab}$ is related to interaction of EM fields with both true charges and pseudo charges.

${}^{(1)}T_{EM,ab}$ is related to interaction of EM fields with pseudo charges only.

On the Gauge Symmetry

Due to the curvature-coupling term, the new EM field equation is not invariance under the EM gauge transformation.

$$\nabla_a F^{ab} - \xi R^b{}_a A^a = -4\pi J^b$$

$$A_a \to A_a + \nabla_a \chi,$$

However, the 5-D Einstein field eq. is invariant under diffeomorphic transformations (general gauge symmetry). Violation of gauge symmetry in the new EM field eq. can be interpreted as symmetry breaking caused by 4+1 decomposition.

Relation to the KK Theory

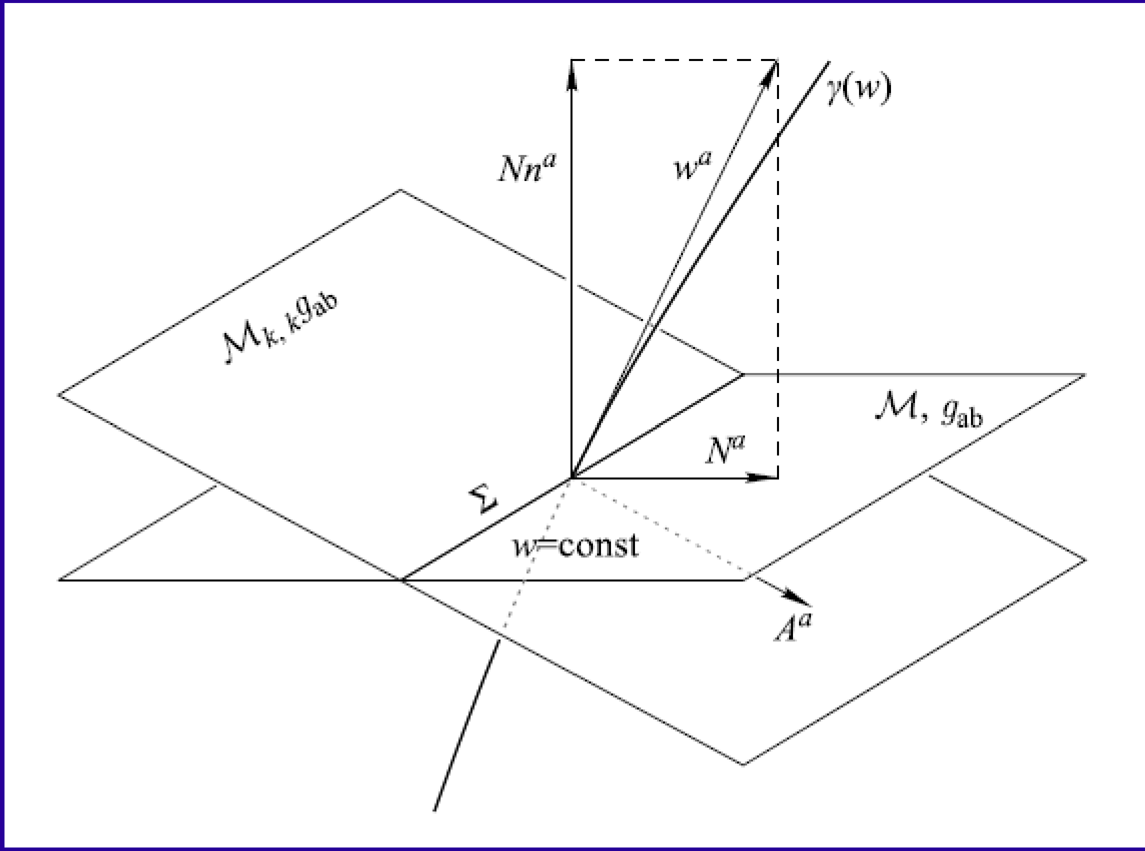
- Different decomposition of the same 5-D metric

$$\tilde{g}_{AB} = \begin{pmatrix} {}_k g_{\mu\nu} + \phi^2 A_\mu A_\nu & \phi^2 A_\mu \\ \phi^2 A_\nu & \phi^2 \end{pmatrix},$$

$$\tilde{g}_{AB} = \begin{pmatrix} g_{\mu\nu} & N_\mu \\ N_\nu & N^2 + N_\rho N^\rho \end{pmatrix},$$

- Defined on different hypersurfaces.

- Give rise to different EM field equations.



Relation to the KK Theory

$$g_{ab} = {}_k g_{ab} + \frac{1}{1+\phi^2 A_c A^c} \times \left(A_c A^c w_a w_b + A_a w_b + A_b w_a - \phi^2 A_a A_b\right)$$

and

$$\begin{cases} N^a = \dfrac{\phi^2}{1+\phi^2 A_c A^c}\left(A^a + A_c A^c w^a\right), \\ N = \dfrac{\phi}{\sqrt{1+\phi^2 A_c A^c}}. \end{cases}$$

- Two theories are not related by diffeomorphic transformations. Hence they are different in physics.

Testability of the Model

- When $R_{ab} = 0$, the new EM field eq. becomes the Maxwell eq. Gauge symmetry is also restored.

- The curvature term is equivalent to a pseudo-charge current vector

$$J^a_{\text{ps}} = -\frac{\xi}{4\pi} R^a{}_b A^b$$

$$Q_{\text{tot}} = Q + \frac{\xi}{4\pi} \int_V R_{ab} u^a A^b dV$$

$$j^a_{\text{tot}} = j^a - \frac{\xi}{4\pi} h^{ab} R_{bc} A^c$$

Testability of the Model

The effect of the curvature term is important only if the probed scale of EM fields is larger than the scale of curvature radius. This condition is satisfied in neutron stars and the early universe.

TABLE I: Density ρ, radius (height) r, and the curvature radius r_c of some objects. The r_c is estimated by equation (230), except for the universe where both r and r_c are taken to be the Hubble distance d_H.[a]

Object	ρ (g/cm^3)	r (cm)	r_c (cm)
Atmosphere[b]	0.001225	$\sim 10^6$	6.6×10^{14}
Earth[c]	5.5	6.4×10^8	1.0×10^{13}
Jupiter[c]	1.3	7.0×10^9	2.0×10^{13}
Sun[c]	1.4	7.0×10^{10}	2.0×10^{13}
White Dwarf[d]	10^6	7×10^8	2×10^{10}
Neutron Star[e]	5×10^{14}	10^6	1.0×10^6
Universe[f]	$2 \times 10^{-29} h^2$	$9 \times 10^{27} h^{-1}$	$9 \times 10^{27} h^{-1}$

[a]In the case of the universe, the r_c estimated by equation (230) is of the same order as d_H.
[b]The density is measured at sea level and 15 ℃. The r refers to the approximate height above sea level.
[c]The density ρ and radius r are averaged values.
[d]The ρ is the averaged density, and r is the radius of a typical white dwarf.
[e]The ρ is the core density, and r is the radius of a typical neutron star.
[f]The ρ is the critical density of the universe, where h is the Hubble constant in units of 100 km s^{-1} Mpc^{-1}.

Dark Electromagnetic Energy

- The extra term in the stress-energy tensor of electromagnetic fields does not interact with electric charges and current. So its effect cannot be measured through EM experiments.

- However, it affects the spacetime geometry through the Einstein field equation.

- It is usually not small. It can be comparable to the normal stress-energy term so may affect the early evolution of the universe.

The Cosmological Costant

$$\begin{cases} g_{ab}(w) = g_{ab}(1+\lambda w), \\ g^{ab}(w) = g^{ab}(1-\lambda w), \end{cases}$$

$$\dot{g}_{ab} = \lambda g_{ab}, \quad \dot{g}^{ab} = -\lambda g^{ab},$$

and

$$\dot{g}_a{}^b = \ddot{g}_{ab} = \ddot{g}^{ab} = 0.$$

$$T_{m,ab} = \frac{3}{2\kappa}\lambda N^{-3}\dot{N} g_{ab},$$

which corresponds to a cosmological constant

$$\Lambda = -\frac{3}{2}\lambda N^{-3}\dot{N}$$

Hence, in appropriate conditions, the matter represented by $\dot{g}_{ab}$ behaves like a cosmological constant in a 4-dim spacetime.

Summary

- A new unified theory for electromagnetic and gravitational interactions is proposed.
- A new electromagnetic field equation is derived, which contains a term coupled to the spacetime curvature.
- The new EM field equation is well motivated and is testable.
- The new unified theory is testable.

References

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2. L.-X. Li, Electrodynamics on cosmological scales, Gen. Relativ. Gravit. 48, 28 (2016), arXiv:1508.06910.