Existence of the critical endpoint in the vector meson extended linear sigma model

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27.04.2016

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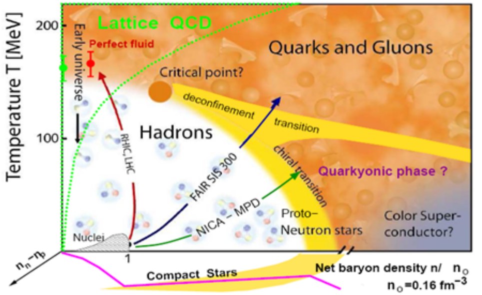
Overview

QCD phase diagram

Phase diagram in the $T - \mu_\mathrm{B} - \mu_\mathrm{I}$ space



- At $\mu_\mathrm{B} = 0$ $T_c = 151$ MeV Y. Aoki,*et al.*, PLB **643**, 46 (2006)
- Is there a CP? (T_{CP}=162 MeV, μ_{CP}=360 MeV, Fodor-Katz)
- At $T = 0$ in μ_B where is the phase boundary?

Details of the phase diagram are heavily studied theoretically (Lattice, EFT), and experimentally (RHIC, LHC, FAIR, NICA)

Addressed problems

- Which scalars are the chiral partners of the pseudoscalar nonet?
- Which parameterizations give phenomenologically good description of the phase transition?
- What is the thermodynamics of the model (compare with lattice)
- Which of them predict the existence of the CP?
- What is the order of phase transition on the T=0 line?
- How the order parameters behave at finite temperature/chemical potential?
- How the masses change in medium?

QCD Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\mathrm{QCD}} \equiv & -\frac{1}{4} G_a^{\mu\nu} G^a_{\mu\nu} + \sum_f \bar{q}_f \left(i\gamma^\mu D_\mu - m_f\right) q_f \\
= & -\frac{1}{4}(\partial^\mu G_a^\nu - \partial^\nu G_a^\mu)(\partial_\mu G^a_\nu - \partial_\nu G^a_\mu) \\
& + \sum_f \bar{q}_f^\alpha \left(i\gamma^\mu \partial_\mu - m_f\right) q_f^\alpha \\
& + g_s G_a^\mu \sum_f \bar{q}_f^\alpha \gamma_\mu \left(\frac{\lambda^a}{2}\right)_{\alpha\beta} q_f^\beta \\
& - \frac{g_s}{2} f^{abc} (\partial^\mu G_a^\nu - \partial^\nu G_a^\mu) G_\mu^b G_\nu^c \\
& - \frac{g_s^2}{4} f^{abc} f_{ade} G_b^\mu G_c^\nu G_\mu^d G_\nu^e
\end{aligned}
$$

1-2nd lines: kinetic terms, 3rd: interaction between quarks& gluons
4-5th lines: the cubic and quartic gluon self-interactions;

Efective models

Since QCD is very hard to solve $\longrightarrow$ low energy effective models were set up $\longrightarrow$ reflecting the global symmetries of QCD

- Nambu-Jona-Lasinio model (+Kobayashi-Maskawa-t'Hooft)
- Chiral perturbation theory
- Linear and nonlinear (it does not contain degrees of freedom relevant at high T) sigma model
- To study the phase diagram, we introduced the constituent quarks
- For mimicing confinement, we add the Polyakov loop variables.

extended Polyakov-Quark-Meson model

Chiral symmetry

If the quark masses are zero (chiral limit) $\Longrightarrow$ QCD invariant under the following global transformation (chiral symmetry):

$q_L = (1-\gamma_5)/2q, \quad q_R = (1+\gamma_5)/2q$ only the mass term mixes

$U(3)_V q = exp(-i\alpha t)q \quad U(3)_A q = exp(-i\beta\gamma_5 t)q$

$U(3)_L \times U(3)_R \simeq U(3)_V \times U(3)_A =$
$SU(3)_V \times SU(3)_A \times U(1)_V \times U(1)_A$

$U(1)_V$ term $\longrightarrow$ baryon number conservation

$U(1)_A$ term $\longrightarrow$ broken through axial anomaly

$SU(3)_A$ term $\longrightarrow$ broken down by any quark mass

$SU(3)_V$ term $\longrightarrow$ broken down to $SU(2)_V$ if $m_u = m_d \neq m_s$
$\longrightarrow$ totally broken if $m_u \neq m_d \neq m_s$ (in nature)

Meson fields - pseudoscalar and scalar meson nonets

$$\Phi_{PS} = \sum_{i=0}^{8} \pi_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\eta_N+\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N-\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix} \quad (\sim \bar{q}_i \gamma_5 q_j)$$

$$\Phi_{S} = \sum_{i=0}^{8} \sigma_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\sigma_N+a_0^0}{\sqrt{2}} & a_0^+ & K_S^+ \\ a_0^- & \frac{\sigma_N-a_0^0}{\sqrt{2}} & K_S^0 \\ K_S^- & \bar{K}_S^0 & \sigma_S \end{pmatrix} \quad (\sim \bar{q}_i q_j)$$

Particle content:

Pseudoscalars: $\pi(138), K(495), \eta(548), \eta'(958)$

Scalars: $a_0(980 \text{ or } 1450), K_0^\star(800 \text{ or } 1430)$,

(σ_N, σ_S) : 2 of $f_0(500, 980, 1370, 1500, 1710)$

Structure of scalar mesons

	Mass (MeV)	width (MeV)	decays
$A_0(980)$	980 ± 20	50 – 100	$\pi\pi$ dominant
$A_0(1450)$	1474 ± 19	265 ± 13	$\pi\eta$, $\pi\eta'$, $K\bar{K}$
$K_s(800) = \kappa$	682 ± 29	547 ± 24	$K\pi$
$K_s(1430)$	1425 ± 50	270 ± 80	$K\pi$ dominant
$f_0(500) = \sigma$	400–550	400 – 700	$\pi\pi$ dominant
$f_0(980)$	980 ± 20	40 – 100	$\pi\pi$ dominant
$f_0(1370)$	1200–1500	200 – 500	$\pi\pi \approx 250$, $K\bar{K} \approx 150$
$f_0(1500)$	1505 ± 6	109 ± 7	$\pi\pi \approx 38$, $K\bar{K} \approx 9.4$
$f_0(1710)$	1722 ± 6	135 ± 7	$\pi\pi \approx 30$, $K\bar{K} \approx 71$

Possible scalar states: $\bar{q}q$, $\bar{q}q\bar{q}q$, meson-meson molecules, glueballs

pseudoscalar nonet: π, K, η, η', scalar nonet: A_0, K_0, 2 f_0

multiquark states: $f_0(980)$,$A_0(980)$ $f_0(600)$, $K_0(800)$???

meson-meson bound state ($K\bar{K}$): $f_0(980)$???

glueballs: $f_0(1500)$ (weak coupling to $\gamma\gamma$), $f_0(1710)$???

Included fields - vector meson nonets

$$V^\mu = \sum_{i=0}^{8} \rho_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N+\rho^0}{\sqrt{2}} & \rho^+ & K^{\star+} \\ \rho^- & \frac{\omega_N-\rho^0}{\sqrt{2}} & K^{\star 0} \\ K^{\star-} & \bar{K}^{\star 0} & \omega_S \end{pmatrix}^\mu$$

$$A^\mu = \sum_{i=0}^{8} b_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{f_{1N}+a_1^0}{\sqrt{2}} & a_1^+ & K_1^+ \\ a_1^- & \frac{f_{1N}-a_1^0}{\sqrt{2}} & K_1^0 \\ K_1^- & \bar{K}_1^0 & f_{1S} \end{pmatrix}^\mu$$

Particle content:

Vector mesons: $\rho(770), K^\star(894), \omega_N = \omega(782), \omega_S = \phi(1020)$

Axial vectors: $a_1(1230), K_1(1270), f_{1N}(1280), f_{1S}(1426)$

Lagrangian (2/1)

$$
\begin{aligned}
\mathcal{L}_{\text{Tot}} = & \operatorname{Tr}[(D_\mu \Phi)^\dagger (D_\mu \Phi)] - m_0^2 \operatorname{Tr}(\Phi^\dagger \Phi) - \lambda_1 [\operatorname{Tr}(\Phi^\dagger \Phi)]^2 - \lambda_2 \operatorname{Tr}(\Phi^\dagger \Phi)^2 \\
& - \frac{1}{4}\operatorname{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) + \operatorname{Tr}\left[\left(\frac{m_1^2}{2} + \Delta\right)(L_\mu^2 + R_\mu^2)\right] + \operatorname{Tr}[H(\Phi + \Phi^\dagger)] \\
& + c_1(\det \Phi + \det \Phi^\dagger) + i\frac{g_2}{2}(\operatorname{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \operatorname{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) \\
& + \frac{h_1}{2}\operatorname{Tr}(\Phi^\dagger \Phi)\operatorname{Tr}(L_\mu^2 + R_\mu^2) + h_2 \operatorname{Tr}[(L_\mu \Phi)^2 + (\Phi R_\mu)^2] + 2h_3 \operatorname{Tr}(L_\mu \Phi R^\mu \Phi^\dagger). \\
& + g_3[\operatorname{Tr}(L_\mu L_\nu L^\mu L^\nu) + \operatorname{Tr}(R_\mu R_\nu R^\mu R^\nu)] + g_4[\operatorname{Tr}(L_\mu L^\mu L_\nu L^\nu) \\
& + \operatorname{Tr}(R_\mu R^\mu R_\nu R^\nu)] + g_5 \operatorname{Tr}(L_\mu L^\mu)\operatorname{Tr}(R_\nu R^\nu) + g_6[\operatorname{Tr}(L_\mu L^\mu)\operatorname{Tr}(L_\nu L^\nu) \\
& + \operatorname{Tr}(R_\mu R^\mu)\operatorname{Tr}(R_\nu R^\nu)] \\
& + \bar{\Psi} i \partial\!\!\!/ \Psi - g_F \bar{\Psi}(\Phi_S + i\gamma_5 \Phi_{PS})\Psi + g_V \bar{\Psi}\gamma^\mu \left(V_\mu + \frac{g_A}{g_V}\gamma_5 A_\mu\right)\Psi
\end{aligned}
$$

$+$ Polyakov loops

D. Parganlija, P. Kovacs, Gy. Wolf, F. Giacosa, D.H. Rischke, Phys. Rev. D87 (2013) 014011

Lagrangian (2/2)

where

$$D^\mu\Phi = \partial^\mu\Phi - ig_1(L^\mu\Phi - \Phi R^\mu) - ieA_e^\mu[T_3,\Phi]$$

$$\Phi = \sum_{i=0}^{8}(\sigma_i + i\pi_i)T_i, \quad H = \sum_{i=0}^{8} h_i T_i \qquad T_i : U(3) \text{ generators}$$

$$R^\mu = \sum_{i=0}^{8}(\rho_i^\mu - b_i^\mu)T_i, \quad L^\mu = \sum_{i=0}^{8}(\rho_i^\mu + b_i^\mu)T_i$$

$$L^{\mu\nu} = \partial^\mu L^\nu - ieA_e^\mu[T_3, L^\nu] - \{\partial^\nu L^\mu - ieA_e^\nu[T_3, L^\mu]\}$$

$$R^{\mu\nu} = \partial^\mu R^\nu - ieA_e^\mu[T_3, R^\nu] - \{\partial^\nu R^\mu - ieA_e^\nu[T_3, R^\mu]\}$$

$$\bar\Psi = (\bar u, \bar d, \bar s)$$

non strange – strange base:

$$\varphi_N = \sqrt{2/3}\varphi_0 + \sqrt{1/3}\varphi_8,$$

$$\varphi_S = \sqrt{1/3}\varphi_0 - \sqrt{2/3}\varphi_8, \qquad \varphi \in (\sigma_i, \pi_i, \rho_i^\mu, b_i^\mu, h_i)$$

broken symmetry: non-zero condensates $\langle\sigma_N\rangle, \langle\sigma_S\rangle \longleftrightarrow \phi_N, \phi_S$

Symmetry properties of the model

Global U(3)$_L$xU(3)$_R$ transformation:

$$\Phi \rightarrow U_L \Phi U_R^\dagger$$
$$L^\mu \rightarrow U_L L^\mu U_L^\dagger$$
$$R^\mu \rightarrow U_R R^\mu U_R^\dagger$$

Consequences (using the unitarity of U's):

$$D^\mu \Phi \rightarrow U_L D^\mu \Phi U_R^\dagger$$
$$L^{\mu\nu} \rightarrow U_L L^{\mu\nu} U_L^\dagger$$
$$R^{\mu\nu} \rightarrow U_R R^{\mu\nu} U_R^\dagger$$

$$(\mathrm{Tr}(\Phi^\dagger\Phi))' = \mathrm{Tr}(U_R \Phi^\dagger U_L^\dagger U_L \Phi U_R^\dagger) = \mathrm{Tr}(U_R \Phi^\dagger \Phi U_R^\dagger) = \mathrm{Tr}(\Phi^\dagger \Phi U_R^\dagger U_R) = \mathrm{Tr}(\Phi^\dagger\Phi)$$

All terms are invariant except the determinant and the explicit symmetry breaking term.

Determinant term

$$U_L = e^{-i\omega_L^a T^a} \qquad U_R = e^{-i\omega_R^a T^a}$$

$$\omega_V^a = 0.5(\omega_L^a + \omega_R^a) \qquad \omega_A^a = 0.5(\omega_L^a - \omega_R^a)$$

By SU(3)$_L$xSU(3)$_R$ transformation (if $\omega_L^0 = \omega_R^0 = 0 = \omega_V^0 = \omega_A^0$)

$$(\det\Phi)' = \det(U_L \Phi U_R^\dagger) = \det U_L \det\Phi \det U_R^\dagger = \det\Phi$$

Similarly $\det\Phi^\dagger$ is also invariant.
If $\omega_V^0 \neq 0$ and all the other ω's are 0 ($[T^a, T^0] = 0$)

$$(\det\Phi)' = \det(e^{-i\omega_V^0 T^0}\Phi e^{i\omega_V^0 T^0}) = \det(e^{-i\omega_V^0 T^0} e^{i\omega_V^0 T^0}\Phi) = \det\Phi$$

On the other hand, if $\omega_A^0 \neq 0$ and all the other ω's are 0

$$(\det\Phi)' = \det(e^{-i\omega_A^0 T^0}\Phi e^{-i\omega_A^0 T^0}) = \det(e^{-i\omega_A^0 T^0} e^{-i\omega_A^0 T^0}\Phi) = e^{-i2\omega_A^0} \det\Phi \mathrm{Tr}\, T^0$$

So the determinant term is invariant under U(3)$_V$xSU(3)$_A$ transformation and breaks explicitely the U(1)$_A$ symmetry.

Explicit breaking term: $\mathrm{Tr}[\hat{\epsilon}(\Phi + \Phi^\dagger)]$

$$\hat{\epsilon} = \sum_{i=0}^{8} \epsilon_i T_i = \begin{pmatrix} \frac{\epsilon_N}{2} & 0 & 0 \\ 0 & \frac{\epsilon_N}{2} & 0 \\ 0 & 0 & \frac{\epsilon_S}{\sqrt{2}} \end{pmatrix} \qquad \text{only} \quad \epsilon^0, \epsilon^8 \neq 0$$

- axial transformation: if at least $\epsilon^0 \neq 0$ $U(3)_A$ is broken:

$$(\mathrm{Tr}[\hat{\epsilon}(\Phi)])' = \mathrm{Tr}(e^{-i2\omega_A^a T^a} \hat{\epsilon}\Phi)$$

- vector transformation

$$(\mathrm{Tr}[\hat{\epsilon}(\Phi)])' = \mathrm{Tr}(e^{-i\omega_V^a T^a} \hat{\epsilon} e^{i\omega_V^a T^a} \Phi)$$

Since $[\hat{\epsilon}, T^0] = 0$, $U(1)_V$ symmetry is preserved.
If all $\epsilon^a = 0$ except ϵ^0, $U(3)_V$ is preserved.
If ϵ^8 also non zero, then since $[T^K, T^8] = 0$ if $k = 1, 2, 3$, $U(1)_V$x$SU(2)_V$ survives (isospin symmetry)
(If $\epsilon^3 \neq 0$ too, then the isospin symmetry is broken, only $U(1)_V$.)

Spontaneous symmetry breaking

Interaction is approximately chiral symmetric, spectra not, so SSB:

$$\sigma_{N/S} \to \sigma_{N/S} + \phi_{N/S} \qquad \phi_{N/S} \equiv <\sigma_{N/S}>$$

For tree level masses we have to select all terms quadratic in the new fields. Some of the terms include mixings arising from terms like $\mathrm{Tr}[(D_\mu\Phi)^\dagger(D_\mu\Phi)]$:

$$\begin{aligned}
\pi_N - a_{1N}^{\mu} &: -g_1\phi_N a_{1N}^{\mu}\partial_\mu\pi_N, \\
\pi - a_1^{\mu} &: -g_1\phi_N(a_1^{\mu+}\partial_\mu\pi^- + a_1^{\mu0}\partial_\mu\pi^0) + \mathrm{h.c.}, \\
\pi_S - a_{1S}^{\mu} &: -\sqrt{2}g_1\phi_S a_{1S}^{\mu}\partial_\mu\pi_S, \\
K_S - K_\mu^\star &: \frac{ig_1}{2}(\sqrt{2}\phi_S - \phi_N)(\bar{K}_\mu^{\star0}\partial^\mu K_S^0 + K_\mu^{\star-}\partial^\mu K_S^+) + \mathrm{h.c.}, \\
K - K_1^{\mu} &: -\frac{g_1}{2}(\phi_N + \sqrt{2}\phi_S)(K_1^{\mu0}\partial_\mu\bar{K}^0 + K_1^{\mu+}\partial_\mu K^-) + \mathrm{h.c.}.
\end{aligned}$$

Diagonalization→ Wave function renormalization

Determination of the parameters of the Lagrangian

16 unknown parameters $(m_0, \lambda_1, \lambda_2, c_1, m_1, g_1, g_2, h_1, h_2, h_3, \delta_S, \Phi_N, \Phi_S, g_F, g_V, g_A) \longrightarrow$ Determined by the min. of χ^2:

$$\chi^2(x_1,\ldots,x_N) = \sum_{i=1}^{M}\left[\frac{Q_i(x_1,\ldots,x_N) - Q_i^{\text{exp}}}{\delta Q_i}\right]^2,$$

where $(x_1,\ldots,x_N) = (m_0, \lambda_1, \lambda_2, \ldots)$, $Q_i(x_1,\ldots,x_N)$ calculated from the model, while Q_i^{exp} taken from the PDG

multiparametric minimalization $\longrightarrow$ MINUIT

- PCAC $\rightarrow$ 2 physical quantities: f_π, f_K
- Tree-level masses $\rightarrow$ 15 physical quantities:
  $m_{u/d}, m_s, m_\pi, m_\eta, m_{\eta'}, m_K, m_\rho, m_\Phi, m_{K^\star}, m_{a_1}, m_{f_1^H}, m_{a_0}, m_{K_s}, m_{f_0^L}, m_{f_0^H}$
- Decay widths $\rightarrow$ 12 physical quantities:
  $\Gamma_{\rho\to\pi\pi}, \Gamma_{\Phi\to KK}, \Gamma_{K^\star\to K\pi}, \Gamma_{a_1\to\pi\gamma}, \Gamma_{a_1\to\rho\pi}, \Gamma_{f_1\to KK^\star}, \Gamma_{a_0}, \Gamma_{K_S\to K\pi},$
  $\Gamma_{f_0^L\to\pi\pi}, \Gamma_{f_0^L\to KK}, \Gamma_{f_0^H\to\pi\pi}, \Gamma_{f_0^H\to KK}$
- $T_c = 155$ MeV from lattice

Results

	Cal(GeV)	Mass		Cal(GeV)	Width
m_π	0.1405	0.1380	f_π	0.0955	0.0922
m_K	0.4995	0.4956	f_K	0.1094	0.1100
m_η	0.5421	0.5479	$\Gamma_{f_0 L\rightarrow KK}$	0.0	0.0
$m_{\eta'}$	0.9643	0.9578	$\Gamma_{f_0 H\rightarrow KK}$	0.0	0.0
m_ρ	0.8064	0.7755	Γ_ρ	0.1515	0.149
m_ϕ	0.9901	1.0195	Γ_ϕ	0.003534	0.003545
m_{K*}	0.9152	0.8938	Γ_{K*}	0.04777	0.048
$m_{f_1 H}$	1.4160	1.4264	$\Gamma_{f_1\rightarrow KK}$	0.04451	0.0445
m_{a_1}	1.0766	1.2300	$\Gamma_{A_1\rightarrow\rho\pi}$	0.1994	0.425
m_{K_1}	1.2999	1.2720	$\Gamma_{A_1\rightarrow\gamma\pi}$	0.0003670	0.000640
m_{a_0}	0.7208	0.980	Γ_{A_0}	0.06834	0.075
$m_{K_0^*}$	0.7529	0.682	$\Gamma_{K_0^*}$	0.006001	0.00547
$m_{f_0 L}$	0.2823	0.475	$\Gamma_{f_0 L\rightarrow\pi\pi}$	0.5542	0.550
$m_{f_0 H}$	0.7376	0.990	$\Gamma_{f_0 H\rightarrow\pi\pi}$	0.08166	0.07
m_{ud}	0.3224	0.308	m_s	0.4577	0.483

Parameters

Parameter	Value
ϕ_N [GeV]	0.1411
ϕ_S [GeV]	0.1416
m_0^2 [GeV^2]	$2.3925E-4$
m_1^2 [GeV^2]	$6.3298E-8$
λ_1	-1.6738
λ_2	23.5078
δ_S [GeV^2]	0.1133
c_1 [GeV]	1.3086
g_1	5.6156
g_2	3.0467
h_1	27.4617
h_2	4.2281
h_3	5.9839
g_F	4.5708
M_0 [GeV]	0.3511

- with this set $f_0^l = 0.2837$ GeV

Polyakov loops in Polyakov gauge

Polyakov loop variables: $\Phi(\vec{x}) = \frac{\mathrm{Tr}_c L(\vec{x})}{N_c}$ and $\bar{\Phi}(\vec{x}) = \frac{\mathrm{Tr}_c \bar{L}(\vec{x})}{N_c}$ with

$$L(x) = \mathcal{P} \exp\left[i \int_0^\beta d\tau A_4(\vec{x}, \tau) \right]$$

$\longrightarrow$ signals center symmetry ($\mathbb{Z}_3$) breaking at the deconfinement transition

low T: confined phase, $\langle \Phi(\vec{x}) \rangle, \langle \bar{\Phi}(\vec{x}) \rangle = 0$
high T: deconfined phase, $\langle \Phi(\vec{x}) \rangle, \langle \bar{\Phi}(\vec{x}) \rangle \neq 0$

Polyakov gauge: the temporal component of the gauge field is time independent and can be gauge rotated to a diagonal form in the color space

$$G_{4,d}(\vec{x}) = \phi_3(\vec{x})\lambda_3 + \phi_8(\vec{x})\lambda_8; \quad \lambda_3, \lambda_8 \text{ : Gell-Mann matrices.}$$

In this gauge the Polyakov loop operator is

$$L(\vec{x}) = \mathrm{diag}(e^{i\beta\phi_+(\vec{x})}, e^{i\beta\phi_-(\vec{x})}, e^{-i\beta(\phi_+(\vec{x})+\phi_-(\vec{x}))})$$

where $\phi_\pm(\vec{x}) = \pm\phi_3(\vec{x}) + \phi_8(\vec{x})/\sqrt{3}$

Form of the potential

I.) Simple polynomial potential invariant under $\mathbb{Z}_3$ and charge conjugation: R.D.Pisarski, PRD 62, 111501

$$\frac{\mathcal{U}_{\text{poly}}(\Phi,\bar{\Phi})}{T^4} = -\frac{b_2(T)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}\left(\Phi^3 + \bar{\Phi}^3\right) + \frac{b_4}{4}\left(\bar{\Phi}\Phi\right)^2$$

with $\quad b_2(T) = a_0 + a_1\frac{T_0}{T} + a_2\frac{T_0^2}{T^2} + a_3\frac{T_0^3}{T^3}$

II.) Logarithmic potential coming from the $SU(3)$ Haar measure of group integration K. Fukushima, Phys. Lett. **B591**, 277 (2004)

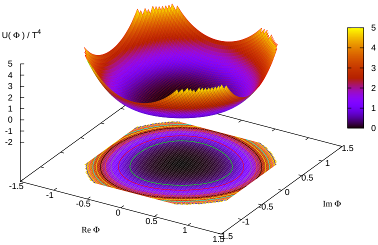
$$\frac{\mathcal{U}_{\log}(\Phi,\bar{\Phi})}{T^4} = -\frac{1}{2}a(T)\Phi\bar{\Phi} + b(T)\ln\left[1 - 6\Phi\bar{\Phi} + 4\left(\Phi^3 + \bar{\Phi}^3\right) - 3\left(\Phi\bar{\Phi}\right)^2\right]$$

with $\quad a(T) = a_0 + a_1\frac{T_0}{T} + a_2\frac{T_0^2}{T^2}, \qquad b(T) = b_3\frac{T_0^3}{T^3}$

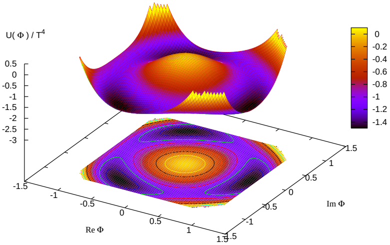
$\mathcal{U}\left(\Phi,\bar{\Phi}\right)$ models the free energy of a pure gauge theory
$\longrightarrow$ the parameters are fitted to the pure gauge lattice data

Polyakov loop potential

"Color confinement"
$\langle\Phi\rangle = 0 \longrightarrow$ no breaking of $\mathbb{Z}_3$
one minimum



"Color deconfinement"
$\langle\Phi\rangle \neq 0 \longrightarrow$ spontaneous breaking of $\mathbb{Z}_3$
minima at $0, 2\pi/3, -2\pi/3$
one of them spontaneously selected



from H. Hansen et al., PRD75, 065004 (2007)

Effects of quarks: Improved Polyakov loops potential

- N_f-dependence of T_0 estimated: $T_0(N_f = 2+1) = 182$ MeV for m_s=95 MeV
  L.M. Haas et al., PRD 87, 076004 see also B.-J. Schaefer et al., PRD 76, 074023
- Within FRG, the glue potential $U^{glue}(\Phi, \bar{\Phi})$ coming from the gauge dof propagating in the presence of dynamical quarks can be matched to the potential $U^{YM}(\Phi, \bar{\Phi})$ of the SU(3) YM theory by relating the reduced temperatures:

$$\frac{U^{glue}}{T^4}(\Phi, \bar{\Phi}, t_{glue}) = \frac{U^{YM}}{(T^{YM})^4}(\Phi, \bar{\Phi}, t_{YM}(t_{glue})), \quad t_{YM}(t_{glue}) \approx 0.57 t_{glue}$$

$$t_{glue} \equiv \frac{T - T_c^{glue}}{T_c^{glue}}, t_{YM} \equiv \frac{T^{YM} - T_0^{YM}}{T_0^{YM}}, \quad T_c^{glue} \in (180, 270) MeV$$

L.M.Haas et al., PRD 87, 076004 (2013)

Effects of Polyakov loops on FD statistics

Inclusion of the Polyakov loop modifies the Fermi-Dirac distribution function

$$f(E_p - \mu_q) \longrightarrow f_\Phi^+(E_p) = \frac{\left(\bar{\Phi} + 2\Phi e^{-\beta\left(E_p-\mu_q\right)}\right) e^{-\beta\left(E_p-\mu_q\right)} + e^{-3\beta\left(E_p-\mu_q\right)}}{1 + 3\left(\bar{\Phi} + \Phi e^{-\beta\left(E_p-\mu_q\right)}\right) e^{-\beta\left(E_p-\mu_q\right)} + e^{-3\beta\left(E_p-\mu_q\right)}}$$

$$f(E_p + \mu_q) \longrightarrow f_\Phi^-(E_p) = \frac{\left(\Phi + 2\bar{\Phi} e^{-\beta\left(E_p+\mu_q\right)}\right) e^{-\beta\left(E_p+\mu_q\right)} + e^{-3\beta\left(E_p+\mu_q\right)}}{1 + 3\left(\Phi + \bar{\Phi} e^{-\beta\left(E_p+\mu_q\right)}\right) e^{-\beta\left(E_p+\mu_q\right)} + e^{-3\beta\left(E_p+\mu_q\right)}}$$

$\Phi, \bar{\Phi} \to 0 \Longrightarrow f_\Phi^\pm(E_p) \to f(3(E_p \pm \mu_q))$ $\quad$ $\Phi, \bar{\Phi} \to 1 \Longrightarrow f_\Phi^\pm(E_p) \to f(E_p \pm \mu_q)$

three-particle state appears: mimics confinement of quarks within baryons

at $T = 0$ there is no difference between models with and without Polyakov loop

T/μ_B dependence of the Polyakov-loops

$$\frac{\partial\Omega}{\partial\Phi} = \frac{\partial\Omega}{\partial\bar{\Phi}}\bigg|_{\varphi_N=\phi_N,\varphi_S=\phi_S} = 0, \qquad \Omega : \text{grand canonical potential}$$

$$-\frac{d}{d\Phi}\left(\frac{U(\Phi,\bar{\Phi})}{T^4}\right) + \frac{2N_c}{T^3}\sum_{q=u,d,s}\int\frac{d^3\mathbf{p}}{(2\pi)^3}\left(\frac{e^{-\beta E_q^-(p)}}{g_q^-(p)} + \frac{e^{-2\beta E_q^+(p)}}{g_q^+(p)}\right) = 0$$

$$-\frac{d}{d\bar{\Phi}}\left(\frac{U(\Phi,\bar{\Phi})}{T^4}\right) + \frac{2N_c}{T^3}\sum_{q=u,d,s}\int\frac{d^3\mathbf{p}}{(2\pi)^3}\left(\frac{e^{-\beta E_q^+(p)}}{g_q^+(p)} + \frac{e^{-2\beta E_q^-(p)}}{g_q^-(p)}\right) = 0$$

$$g_q^+(p) = 1 + 3\left(\bar{\Phi} + \Phi e^{-\beta E_q^+(p)}\right)e^{-\beta E_q^+(p)} + e^{-3\beta E_q^+(p)}$$

$$g_q^-(p) = 1 + 3\left(\Phi + \bar{\Phi} e^{-\beta E_q^-(p)}\right)e^{-\beta E_q^-(p)} + e^{-3\beta E_q^-(p)}$$

$$E_q^\pm(p) = E_q(p) \mp \mu_B/3, \;\; E_{u/d}(p) = \sqrt{p^2 + m_{u/d}^2}, E_s(p) = \sqrt{p^2 + m_s^2}$$

T/μ_B dependence of the condensates ($\phi_{N/S}$)

$$\frac{\partial\Omega}{\partial\phi_N} = \frac{\partial\Omega}{\partial\phi_S}\bigg|_{\Phi,\bar{\Phi}} = 0, \qquad \text{(after the SSB)}$$

Hybrid approach: fermions at one-loop, mesons at tree-level (their effects are much smaller)

$$m_0^2\phi_N \quad + \quad \left(\lambda_1 + \frac{1}{2}\lambda_2\right)\phi_N^3 + \lambda_1\phi_N\phi_S^2 - h_N + \frac{g_F}{2}N_c\left(\langle u\bar{u}\rangle_T + \langle d\bar{d}\rangle_T\right) = 0$$

$$m_0^2\phi_S \quad + \quad (\lambda_1 + \lambda_2)\,\phi_S^3 + \lambda_1\phi_N^2\phi_S - h_S + \frac{g_F}{\sqrt{2}}N_c\langle s\bar{s}\rangle_T = 0$$

$$\langle q\bar{q}\rangle_T = -4m_q\int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{2E_q(p)}\left(1 - f_\Phi^-(E_q(p)) - f_\Phi^+(E_q(p))\right)$$

Masses

$$M^2_{i,ab} = \frac{\partial^2\Omega(T,\mu_f)}{\partial\varphi_{i,a}\partial\varphi_{i,b}}\Big|_{\min} = m^2_{i,ab} + \Delta_0 m^2_{i,ab} + \Delta_T m^2_{i,ab},$$

$m^2_{i,ab} \longrightarrow$ tree-level mass matrix,
$\Delta_{0/T} m^2_{i,ab} \longrightarrow$ fermion vacuum/thermal fluctuation,

$$\Delta_0 m^2_{i,ab} = \frac{\partial^2\Omega^{\mathrm{vac}}_{q\bar{q}}}{\partial\varphi_{i,a}\partial\varphi_{i,b}}\Big|_{\min} = -\frac{3}{8\pi^2}\sum_{f=u,d,s}\left[\left(\frac{3}{2}+\log\frac{m_f^2}{M^2}\right)m^{2(i)}_{f,a}m^{2(i)}_{f,b} + m_f^2\left(\frac{1}{2}+\log\frac{m_f^2}{M^2}\right)m^{2(i)}_{f,ab}\right],$$

$$\Delta_T m^2_{i,ab} = \frac{\partial^2\Omega^{\mathrm{th}}_{q\bar{q}}}{\partial\varphi_{i,a}\partial\varphi_{i,b}}\Big|_{\min} = 6\sum_{f=u,d,s}\int\frac{d^3p}{(2\pi)^3}\frac{1}{2E_f(p)}\Bigg[\left(f_f^+(p)+f_f^-(p)\right)\left(m^{2(i)}_{f,ab} - \frac{m^{2(i)}_{f,a}m^{2(i)}_{f,b}}{2E_f^2(p)}\right) + \left(B_f^+(p)+B_f^-(p)\right)\frac{m^{2(i)}_{f,a}m^{2(i)}_{f,b}}{2TE_f(p)}\Bigg],$$

where $m^{2(i)}_{f,a} \equiv \partial m_f^2/\partial\varphi_{i,a}$, $m^{2(i)}_{f,ab} \equiv \partial^2 m_f^2/\partial\varphi_{i,a}\partial\varphi_{i,b}$

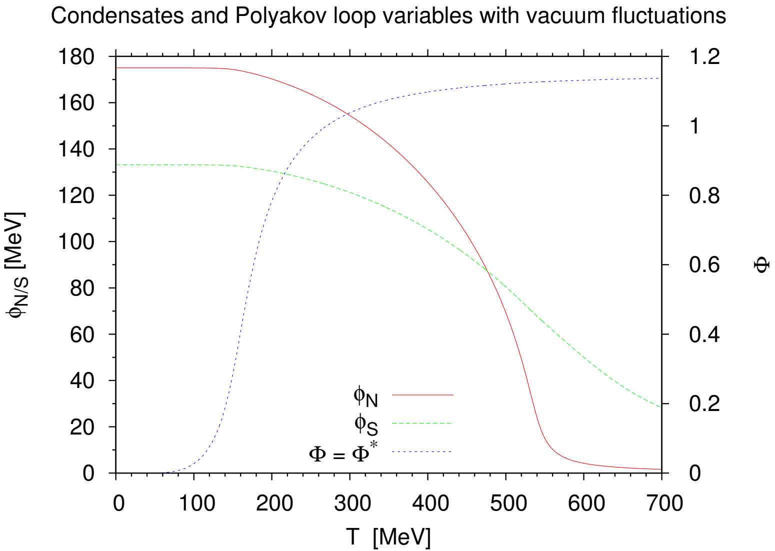
Features of our approach

- D.O.F's: scalar, pseudoscalar, vector, axial vector nonets,
- Polyakov loop variables, $\Phi, \bar{\Phi}$ with U^{YM} or U^{glue}
- u,d,s constituent quarks, $(m_u = m_d)$
- no mesonic fluctuations included in the grand canonical potential:

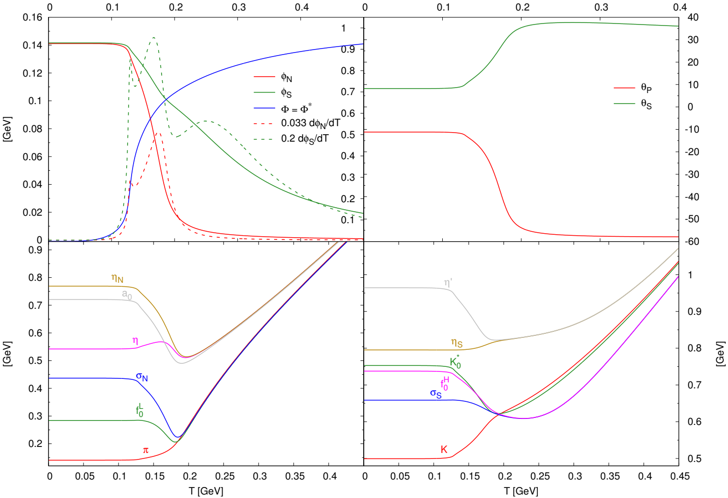
$$\Omega(T, \mu_q) = -\frac{1}{\beta V} ln(Z)$$

- Fermion vacuum and thermal fluctuations
- quarks do not couple to (axial) vector meson yet
- Four order parameters $(\phi_N, \phi_S, \Phi, \bar{\Phi}) \rightarrow$ four T/μ-dependent equations
- thermal contribution of π, K, f_0^L included in the pressure

$\phi_{N/S}(T), \Phi/\bar{\Phi}(T)$ *with* Polyakov loop $m_{f_0^L} = 1326$ MeV

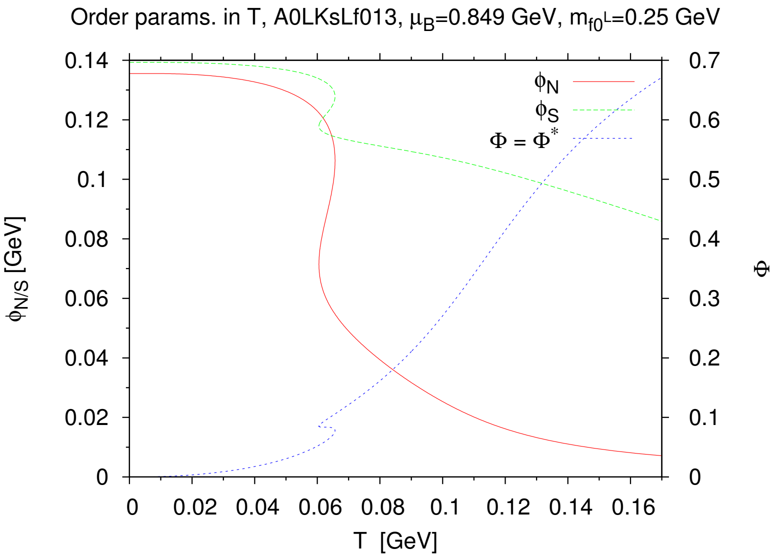


With low mass scalars, $m_{f_0^L} = 300$ MeV

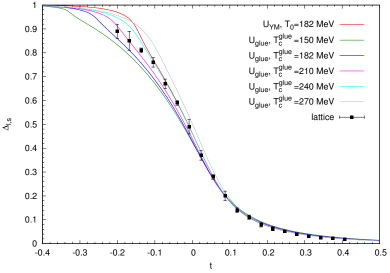


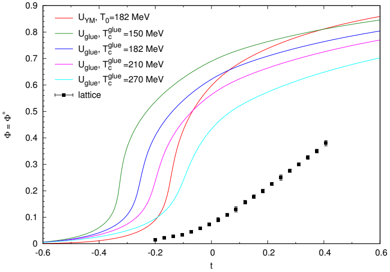
Our pattern: $m_\eta \leq m_{\eta_N} < m_{\eta_S} \leq m_{\eta'}$ in contrast to others
Schaefer, PRD79 014018, Tiwari PRD88, 074017

With low mass scalars 1st order phase transition



T-dependence of condensates compared to lattice results





subtracted chiral condensate

$$\Delta = \frac{(\Phi_N - h_N/h_S\Phi_S)_T}{(\Phi_N - h_N/h_S\Phi_S)_{T=0}}$$

U^{glue} with $T_c^{glue} \in (210-240)$MeV gives good agreement with the lattice result of Borsanyi et al.,JHEP 1009, 073 (2010)

- lattice shows smooth transition
- our result is completely off
- renormalization of the Polyakov loop may explain part of the discrepancy Andersen et al., PRD92, 114504

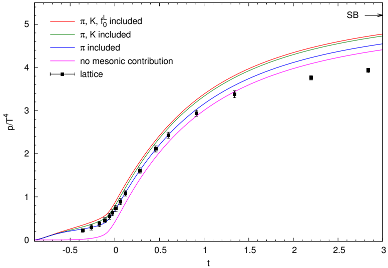
Thermodynamical Observables

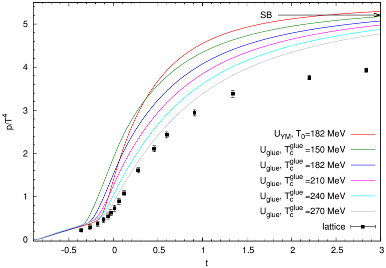
We include mesonic thermal contribution to p for (π, K, f_0^l)

$$\Delta p(T) = -nT \int \frac{d^3q}{(2\pi)^3} ln(1 - e^{-\beta E(q)}), \quad E(q) = \sqrt{q^2 + m^2}$$

- pressure: $p(T, \mu_q) = \Omega_H(T = 0, \mu_q) - \Omega_H(T, \mu_q)$
- entropy density: $s = \frac{\partial p}{\partial T}$
- quark number density: $\rho_q = \frac{\partial p}{\partial \mu_q}$
- energy density: $\epsilon = -p + Ts + \mu_q \rho_q$
- scaled interaction measure: $\frac{\Delta}{T^4} = \frac{\epsilon - 3p}{T^4}$
- speed of sound at $\mu_q = 0$: $c_s^2 = \frac{\partial p}{\partial \epsilon}$

Normalized pressure



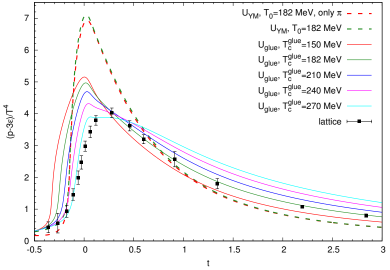


we use U^{glue} with $T_c^{glue} = 270$ MeV
pion dominates at low T
at high T pressure overshoots the lattice data

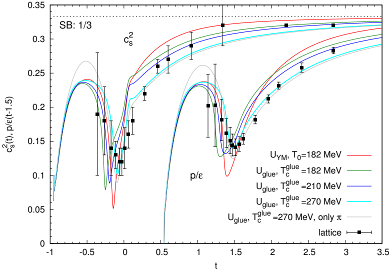
overshooting increases with decreasing T_c^{glue}
lattice: Borsányi et al., JHEP 1011, 077 (2010)

Observables

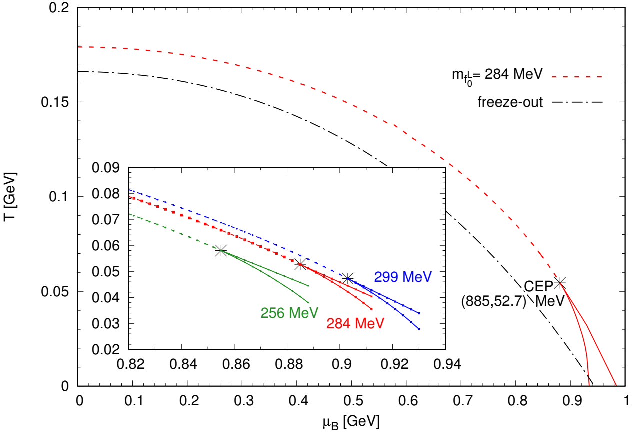
interaction measure



speed of sound

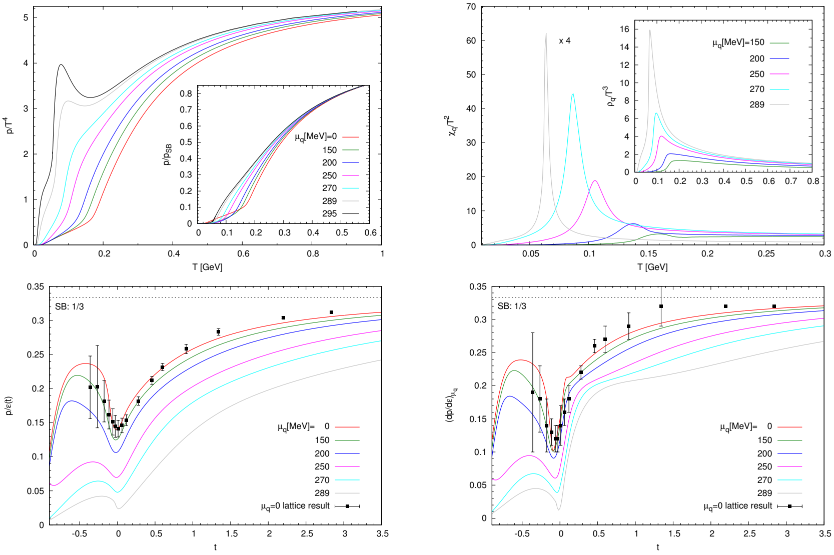


$T - \mu_B$ phase diagram



- we use U^{glue} with $T_c^{glue} = 210$ MeV
- freeze-out curve from Cleymans et al., J.Phys.G32, S165

Observables at $\mu_B \neq 0$



Summary and Conclusions

- The thermodynamics of the ePQM was studied after parametrizing of the model with a modification of the method used in Parganlija et al., PRD87, 014011
- 40 possible assignments of the scalars to the nonet states were investigated. Lowest χ^2 for $a_0^{\bar{q}q} \to a_0(980)$, $K_0^{*,\bar{q}q} \to K_0^*(980)$, $f_0^{l,\bar{q}q} \to f_0(500)$, $f_0^{h,\bar{q}q} \to f_0(980)$
- The phase transition temperature requires low mass (≤ 400 MeV) f_0
- For the best set of parameters CEP was found in the $T - \mu_B$ plane
- The T-dependence of various thermodynamical observables measured on the lattice is reasonable well reproduced with an improved Polyakov loop potential. L.M. Hass et al., PRD87, 076004
- The model can be used to give predictions at finite μ_B

→ To do . . .

→ Improve the vacuum phenomenology by tetraquarks (and glueballs)

→ coupling the quarks to the (axial)vectors

→ including mesonic fluctuations

Thank you for your attention!