universitätbonn | bctp | JÜLICH FORSCHUNGSZENTRUM

Roy-Steiner analysis of pion nucleon-scattering and a precision determination of the σ-term

Ulf-G. Meißner, Univ. Bonn & FZ Jülich

Supported by DFG, SFB/TR-16 and by DFG, SFB/TR-110 and by CAS, PIFI by HGF VIQCD VH-VI-417 and by VW Stiftung

CONTENTS

Introduction

TRACE ANOMALY

- Classical massles QCD is invariant under scale transformations

$$x \to \lambda x \; , \; q(x) \to \lambda^{3/2} q(\lambda x) \; , \; A_\mu(x) \to \lambda A_\mu(\lambda x)$$ (*dilatations*)

- Quantization/renormalization generates a scale $\Lambda_{\rm QCD}$ that breaks scale invariance: **dimensional transmutation**

$\Rightarrow$ **trace anomaly**

$$\theta_\mu^\mu = \frac{\beta_{\rm QCD}}{2g} G_{\mu\nu}^a G_a^{\mu\nu} + m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s + \ldots$$

$\ast$ trace anomaly = signal for the *generation of hadron masses*

$\ast$ the mass of any hadron made of light quarks mass is essentially field energy ("binding")

"Mass without mass" (Wheeler, 1962)

ANATOMY of the NUCLEON MASS

$$m_N = \langle N(p)|\theta^\mu_\mu|N(p)\rangle$$
$$= \langle N(p)|\underbrace{\frac{\beta_{\rm QCD}}{2g}G^a_{\mu\nu}G^{\mu\nu}_a}_{\text{trace anomaly}} + \underbrace{m_u\bar{u}u + m_d\bar{d}d + m_s\bar{s}s}_{\text{Higgs}}|N(p)\rangle$$

- Dissect the various contributions:

  $\star\ \langle N(p)|m_u\bar{u}u + m_d\bar{d}d|N(p)\rangle = 40\ldots70\ \text{MeV}\ \doteq \sigma_{\pi N}$

  $\star\ \langle N(p)|m_s\bar{s}s|N(p)\rangle = 0\ldots150\ \text{MeV}$

  from the analysis of the pion-nucleon sigma term & lattice QCD (before 2015)

  Gasser, Leutwyler, Sainio; Borasoy & M., Büttiker & M., Pavan et al., Alarcon et al. . . .

$\Rightarrow$ bulk of the nucleon mass is generated by the gluon fields / field energy

$\Rightarrow$ this is a central result of QCD

$\Rightarrow$ requires better Roy-Steiner analysis of πN and lattice data

$\hookrightarrow$ this talk

ROLE of the PION-NUCLEON σ-TERM

- Scalar couplings of the nucleon:

$$\langle N|m_q\bar{q}q|N\rangle = f_q^N\, m_N \quad (N = p, n)$$
$$(q = u, d, s)$$

$\hookrightarrow$ Dark Matter detection

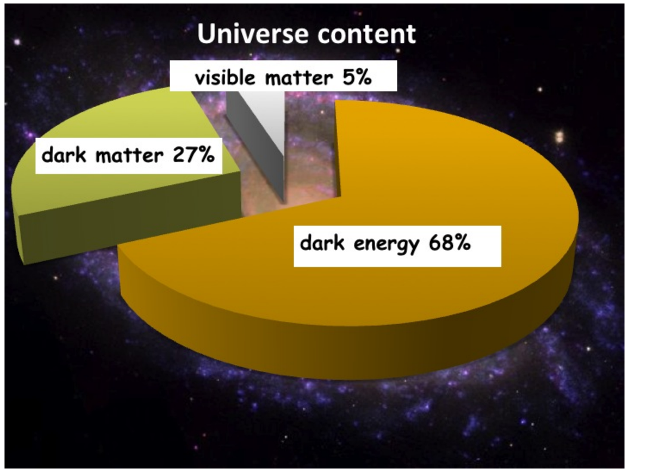
$\hookrightarrow$ $\mu \to e$ conversion in nuclei

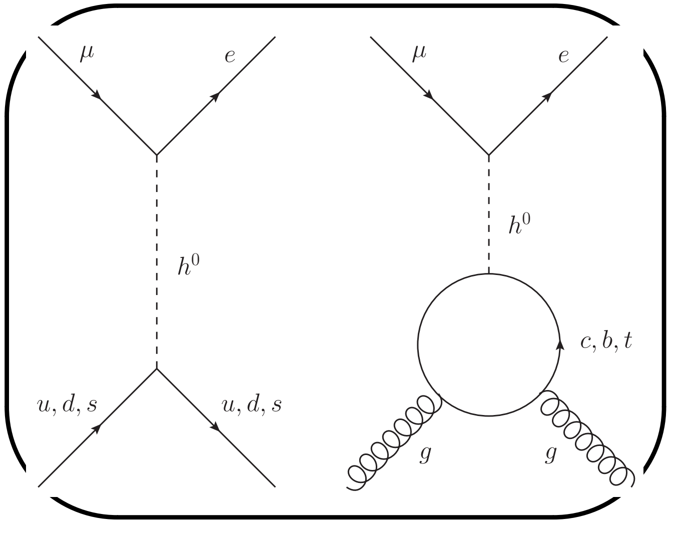
- Condensates in nuclear matter

$$\frac{\langle\bar{q}q\rangle(\rho)}{\langle 0|\bar{q}q|0\rangle} = 1 - \frac{\rho\,\sigma_{\pi N}}{F_\pi^2 M_\pi^2} + \ldots$$

- CP-violating πN couplings

$\hookrightarrow$ hadronic EDMs (nucleon, nuclei)



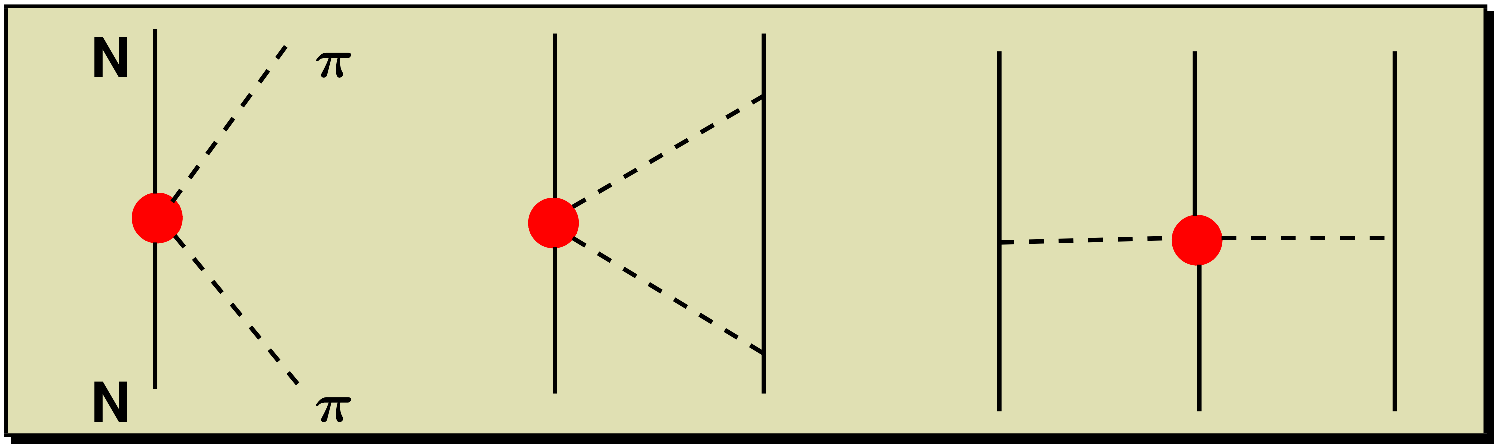


Crivellin, Hoferichter, Procura

PION–NUCLEON SCATTERING & NUCLEAR FORCES

- Low-energy constants (LECs) relate ***many*** processes

- e.g. the dimension-two LECs c_i in πN, NN, NNN, $\ldots$



● = operator from $\mathcal{L}^{(2)}_{\pi N} \propto c_i \;\; (i = 1, 2, 3, 4)$

Here: • determine the c_i from the purest process $\pi N \to \pi N$

$\to$ make parameter-free predictions for long-ranged nuclear forces

Pion-nucleon scattering: Fundamentals etc.

PION-NUCLEON SCATTERING

- s-channel: $\pi(q) + N(p) \;\to \pi(q') \;+\; N(p')$

  t-channel: $\pi(q) + \pi(-q') \to \bar{N}(-p) + N(p')$

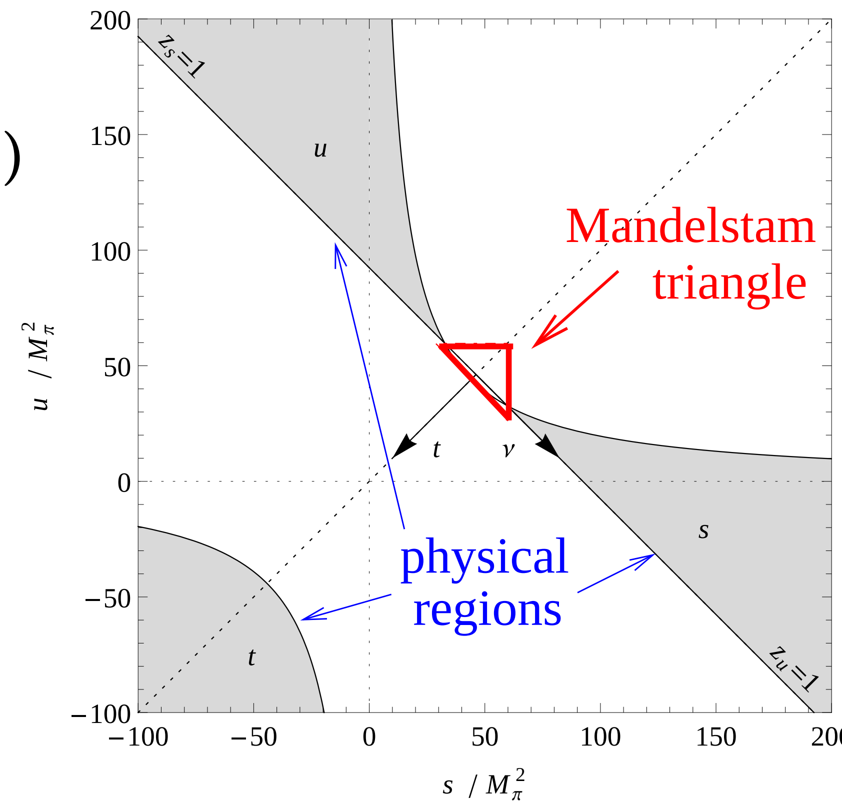
- Mandelstam variables:

$$s = (p+q)^2, t = (p-p')^2, u = (p-q')^2$$

$$s + t + u = 2m_N^2 + 2M_\pi^2, \;\; s = W^2$$

- Isospin structure:

$$T^{ba}(s,t) = \delta^{ba}\, T^+(s,t) + i\epsilon_{abc}\tau^c\, T^-(s,t)$$



- Lorentz structure:

$$8\pi\sqrt{s}T^I(s,t) = \bar{u}(p')\left\{A^I(s,t) + \tfrac{1}{2}(\not{q} + \not{q}')B^I(s,t)\right\}u(p)\;, \quad I = +,-$$

$$I = 1/2, 3/2$$

- Crossing:

$$A^\pm(\nu,t) = \pm A^\pm(-\nu,t)\;,\;\; B^\pm(\nu,t) = \mp B^\pm(-\nu,t)\;,\quad \nu = \frac{s-u}{4m_N}$$

PION-NUCLEON SCATTERING continued

- Partial wave projection:

$$X_\ell^I(s) = \int\limits_{-1}^{+1} dz_s P_\ell(z_s) X^I(s,t)\Big|_{t=-2q^2(1-z_s)} , \quad X \in \{A, B\}$$

$\Rightarrow$ partial wave expansion (total isospin I, ang. mom. ℓ, $j = \ell \pm 1/2$):

$$f_{\ell\pm}^I(W) = \frac{1}{16\pi W}$$
$$\times\Big\{(E+m)\big[A_\ell^I(s) + (W-m)B_\ell^I(s)\big] + (E-m)\big[-A_{\ell\pm1}^I(s) + (W+m)B_{\ell\pm1}^I(s)\big]\Big\}$$

- MacDowell symmetry: $f_{\ell+}^I(W) = -f_{(\ell+1)-}^I(-W) \quad \forall\, l \geq 0$ MacDowell (1959)

- Low-energy region: only S- and P-waves are relevant $\boxed{f_{0+}^\pm,\, f_{1+}^\pm,\, f_{1-}^\pm}$

$\Rightarrow$ low-energy amplitude can eventually be matched to chiral perturbation theory

Büttiker, Fettes, UGM, Steiniger; Ellis, Tang; Becher, Leutwyler, . . .

SUBTHRESHOLD EXPANSION

- For the σ-term extraction, the πN amplitude $D = A + \nu B$ is most useful:

$$\bar{D}^+(\nu,t) = D^+(\nu,t) - \frac{g_{\pi N}^2}{m_N} - \nu g_{\pi N}^2 \left(\frac{1}{m_N^2 - s} - \frac{1}{m_N^2 - u} \right)$$

⋆ subtraction of pseudovector Born terms $\to$ $\bar{D}$

- Subthreshold expansion: expand around $\nu = t = 0$:

$$\boxed{X(\nu,t) = \sum_{m,n} x_{mn}\, \nu^{2m}\, t^n} \quad , \quad X \in \left\{ \bar{A}^+, \frac{\bar{A}^-}{\nu}, \frac{\bar{B}^+}{\nu}, \bar{B}^-, \bar{D}^+, \frac{\bar{D}^-}{\nu} \right\}$$

⋆ $\boxed{x_{mn}}$ are the **subthreshold parameters** $\to$ can be calculated via sum rules

⋆ inside the Mandelstam triangle, scattering amplitudes are real polynomials

σ-TERM BASICS

- Scalar form factor of the nucleon (isospin limit $\hat{m} = (m_u + m_d)/2$):

$$\sigma_{\pi N}(t) = \langle N(p')|\hat{m}(\bar{u}u + \bar{d}d)|N(p)\rangle \ , \ t = (p' - p)^2$$

- Cheng-Dashen Low-Energy Theorem (LET): Cheng, Dashen (1971)

$$\boxed{\bar{D}^+(0, 2M_\pi^2) = \sigma(2M_\pi^2) + \Delta_R}$$

$|\Delta_R| \lesssim 2$ MeV — Bernard, Kaiser, UGM (1996)

- Standard decomposition of the σ-term: $\sigma_{\pi N} = \sigma_{\pi N}(0)$

$$\sigma_{\pi N} = \Sigma_d + \Delta_D - \Delta_\sigma - \Delta_R$$

$$\Sigma_d = F_\pi^2(d_{00}^+ + 2M_\pi^2 d_{01}^+) \quad \rightarrow \text{full RS analysis}$$

$$\left.\begin{array}{l} \Delta_D = \bar{D}^+(0, 2M_\pi^2) - \Sigma_d \\ \Delta_\sigma = \sigma(2M_\pi^2) - \sigma_{\pi N} \end{array}\right\} \rightarrow \text{RS t-channel analysis}$$

- Strong $\pi\pi$ rescattering in Δ_D and Δ_σ, the difference is small!

Gasser, Leutwyler, Sainio (1991)

Hadronic Atoms

WHY HADRONIC ATOMS?

- Hadronic atoms: bound by the static Coulomb force (QED)
- Many species: $\pi^+\pi^-$, $\pi^\pm K^\mp$, $\pi^- p$, $\pi^- d$, $K^- p$, $K^- d$, $\ldots$
- Observable effects of QCD: strong interactions as **small** perturbations

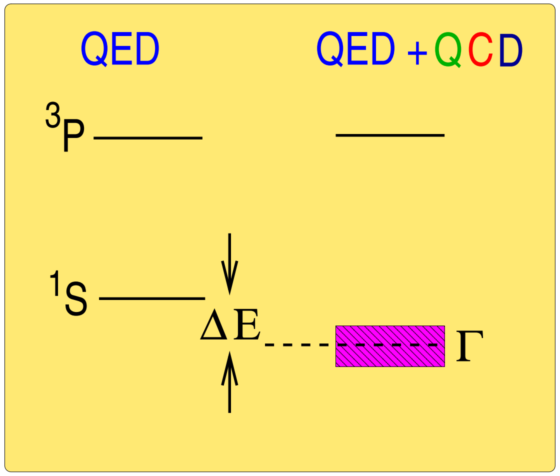
⋆ energy shift ΔE

⋆ deacy width Γ

$\Rightarrow$ access to scattering at zero energy!

= S-wave scattering lengths

$\Rightarrow$ best way to determine the scattering lengths!



- can be analyzed in suitable NREFTs

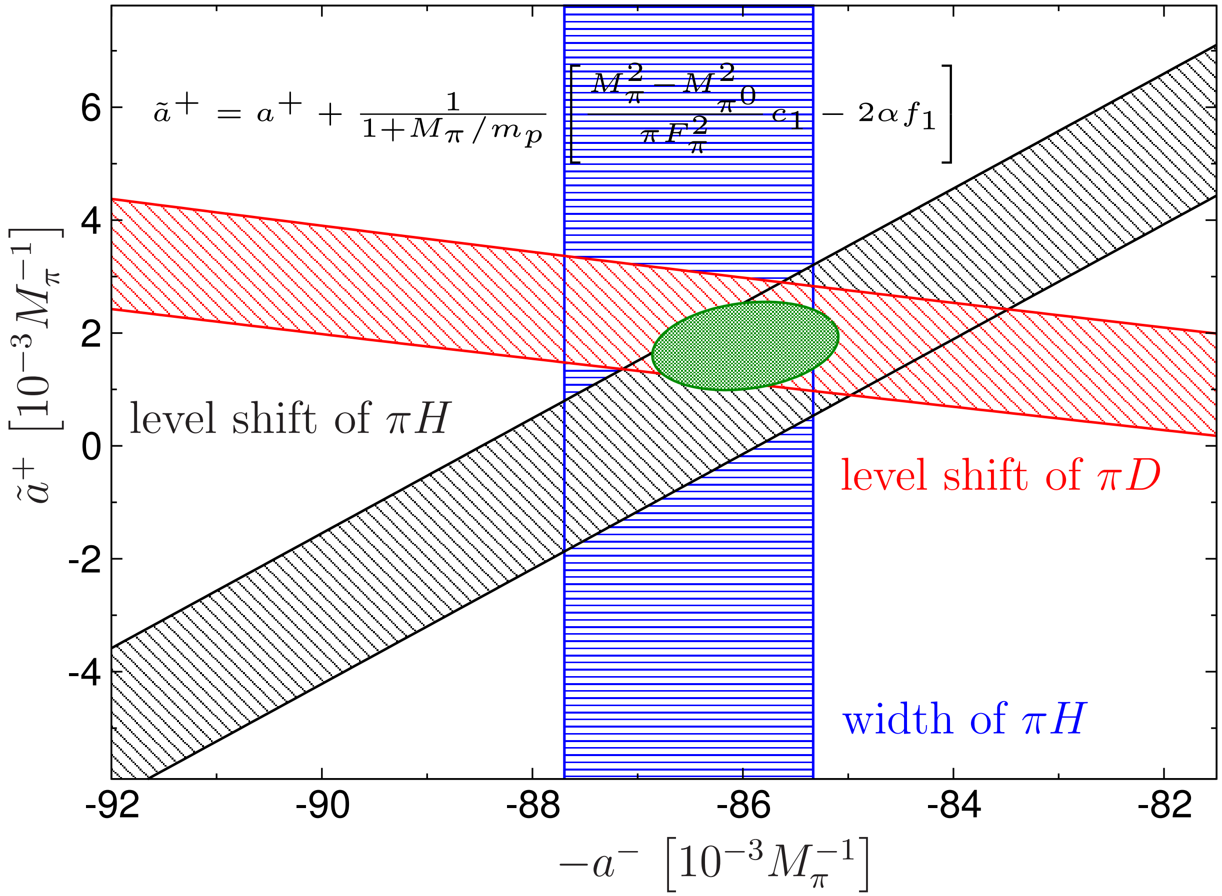
Pionic hydrogen (πH) — Gasser, Rusetsky, ... 2002

Pionic deuterium (πD) — Baru, Hoferichter, Kubis ... 2011

PION–NUCLEON SCATTERING LENGTHS

- superbe experiments performed at PSI

Gotta et al.



- πH level shift $\Rightarrow \pi^- p \to \pi^- p$
- πD level shift
  $\Rightarrow$ isoscalar $\pi^- N \to \pi^- N$
- πH width $\Rightarrow \pi^- p \to \pi^0 n$

$\downarrow$

$$a^+ = (7.6 \pm 3.1) \cdot 10^{-3}/M_\pi$$

$$a^- = (86.1 \pm 0.9) \cdot 10^{-3}/M_\pi$$

$\Rightarrow$ very precise value for a^- & first time definite sign for a^+

GMO SUM RULE

- Goldberger-Miyazawa-Oehme sum rule:

Goldberger, Miyazawa, Oehme 1955

$$\frac{g_{\pi N}^2}{4\pi} = \left[\left(\frac{m_p + m_n}{M_\pi}\right)^2 - 1\right] \left\{ \left(1 + \frac{M_\pi}{m_p}\right) \frac{M_\pi}{4} \underbrace{(a_{\pi^- p} - a_{\pi^+ p})}_{\text{just determined}} - \frac{M_\pi^2}{2} J^- \right\}$$

$$= 13.69 \pm 0.12 \pm 0.15$$

$$J^- = \frac{1}{4\pi^2} \int_0^\infty dk \, \frac{\sigma^{\rm tot}_{\pi^- p}(k) - \sigma^{\rm tot}_{\pi^+ p}(k)}{\sqrt{M_\pi^2 + k^2}}$$

- J^- is very well determined

Ericson et al. 2002, Abaev et al. 2007

- consistent with other determinations:

πN	13.75 ± 0.15	Arndt et al. 1994
NN	13.54 ± 0.05	de Swart et al. 1997

Roy-Steiner equations

Ditsche, Hoferichter, Kubis, UGM, JHEP **1206** (2012) 043

Hoferichter, Ditsche, Kubis, UGM, JHEP **1206** (2012) 063

HYPERBOLIC DISPERSION RELATIONS

- make use of hyperbolic dispersion relations (HDRs):

$$\boxed{(s-a)(u-a)=b}\,, \qquad a,b\in\mathbb{R} \qquad [b=b(s,t,a)]$$

Steiner (1968), Hite, Steiner (1973)

- why HDRs?

$\hookrightarrow$ combine all *physical regions*
very important for a reliable continuation to the subthreshold region

Stahov (1999)

$\hookrightarrow$ especially powerful for the determination of the σ-term

Koch (1982)

$\hookrightarrow$ $s \leftrightarrow u$ crossing is explicit

$\hookrightarrow$ absorptive parts are only needed in regions where
the corresponding PW expansions converge

$\hookrightarrow$ judicious choice of a allows to increase the range of convergence

RS EQUATIONS: s-CHANNEL

- s-channel part of the full RS system:

$$f^I_{l+}(W) = N^I_{l+}(W) + n \text{ subtractions around } \nu = t = 0$$
$$+ \frac{1}{\pi}\int_{W_+}^{\infty} dW' \sum_{l'=0}^{\infty} \Big\{ K^I_{ll'}(W,W')\,\mathrm{Im} f^I_{l'+}(W') + K^I_{ll'}(W,-W')\,\mathrm{Im} f^I_{(l'+1)-}(W') \Big\}$$
$$+ \frac{1}{\pi}\int_{t_\pi}^{\infty} dt' \sum_J \Big\{ G_{lJ}(W,t')\,\mathrm{Im} f^J_{+}(t') + H_{lJ}(W,t')\,\mathrm{Im} f^J_{-}(t') \Big\}$$

$\hookrightarrow$ $N^I_{l+}(W)$ nucleon Born term contribution

$\hookrightarrow$ coupling to s-channel absorptive parts $\sim K^I_{ll'} = \dfrac{\delta_{ll'}}{W'-W} + \ldots$

$\hookrightarrow$ coupling to t-channel absorptive parts $\sim G_{lJ}, H_{lJ}$

$\hookrightarrow$ range of convergence: $a = -23.19\, M_\pi^2$

$$\Rightarrow \quad s \in \big[(m_N + M_\pi)^2, 97.30\, M_\pi^2\big] \quad \Leftrightarrow \quad W \in [1.08, 1.38] \text{ GeV}$$

RS EQUATIONS: t-CHANNEL

- t-channel part of the full RS system (only show $f_+^J(t)$):

$$f_+^J(t) = \tilde{N}_+^J(t) + \frac{1}{\pi}\int_{W_+}^{\infty} dW' \sum_{l=0}^{\infty} \Big\{ \tilde{G}_{Jl}(t,W')\,\mathrm{Im} f_{l+}^I(W') + \tilde{G}_{Jl}(t,-W')\,\mathrm{Im} f_{(l+1)-}^I(W') \Big\}$$

$$+ \frac{1}{\pi}\int_{t_\pi}^{\infty} dt' \sum_{J'} \Big\{ \tilde{K}^1_{JJ'}(t,t')\,\mathrm{Im} f_+^{J'}(t') + \tilde{K}^2_{JJ'}(t,t')\,\mathrm{Im} f_-^{J'}(t') \Big\} + n\,\mathbf{sub.} \quad \forall\, J \geq 0$$

$\hookrightarrow$ $\tilde{N}^I_{l+}(W)$ nucleon Born term contribution (no kinematical singularity)

$\hookrightarrow$ coupling to t-channel absorptive parts $\sim \tilde{K}^{1,2,3}_{JJ'} = \dfrac{\delta_{JJ'}}{t'-t} + \ldots$

$\hookrightarrow$ only higher t-channel PWs couple to lower ones!

$\hookrightarrow$ coupling to s-channel absorptive parts $\sim \tilde{G}_{Jl}, \tilde{H}_{Jl}$

$\hookrightarrow$ range of convergence: $\quad a = -2.71\, M_\pi^2$

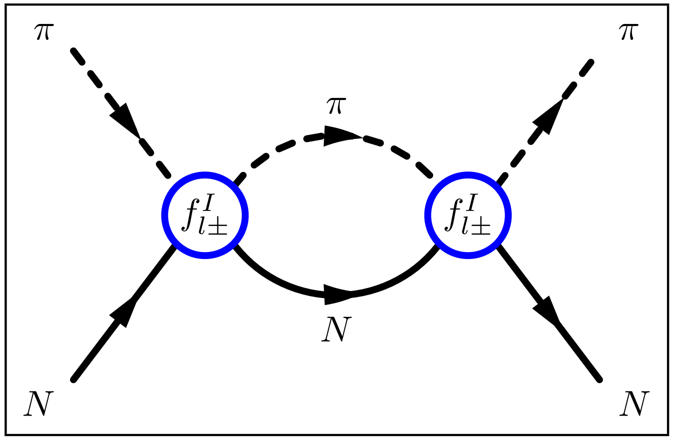
$$\Rightarrow \quad t \in \left[4M_\pi^2, 205.45\, M_\pi^2\right] \quad \Leftrightarrow \quad \sqrt{t} \in [0.28, 2.00]\ \text{GeV}$$

UNITARITY RELATIONS

- s-channel elastic unitarity ($I_s = 1/2, 3/2$):

$$\mathrm{Im}\, f_{l\pm}^{I_s}(W) =$$

$$\sqrt{\frac{\lambda(s,m_N^2,M_\pi^2)}{4s}} \left| f_{l\pm}^{I_s}(W) \right|^2 \theta(W - (m_N + M_\pi))$$

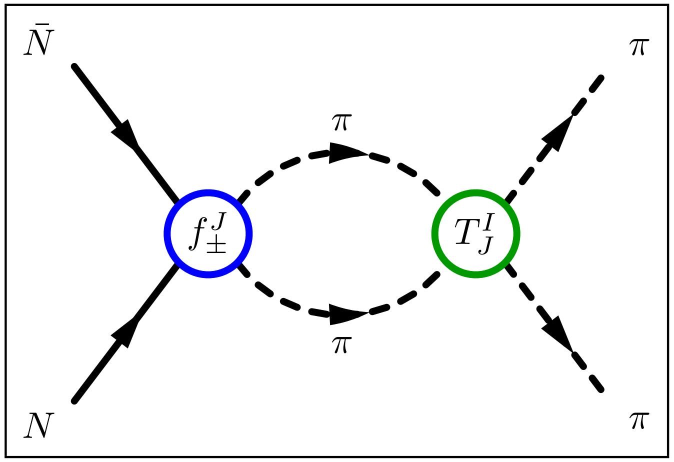


- t-channel extended unitarity:
  (two-body intermediate states)

$$\mathrm{Im}\, f_{l\pm}^{J}(t) =$$

$$T_J^I(t)^\star \; f_\pm^J \, \theta(t - 4M_\pi^2)$$

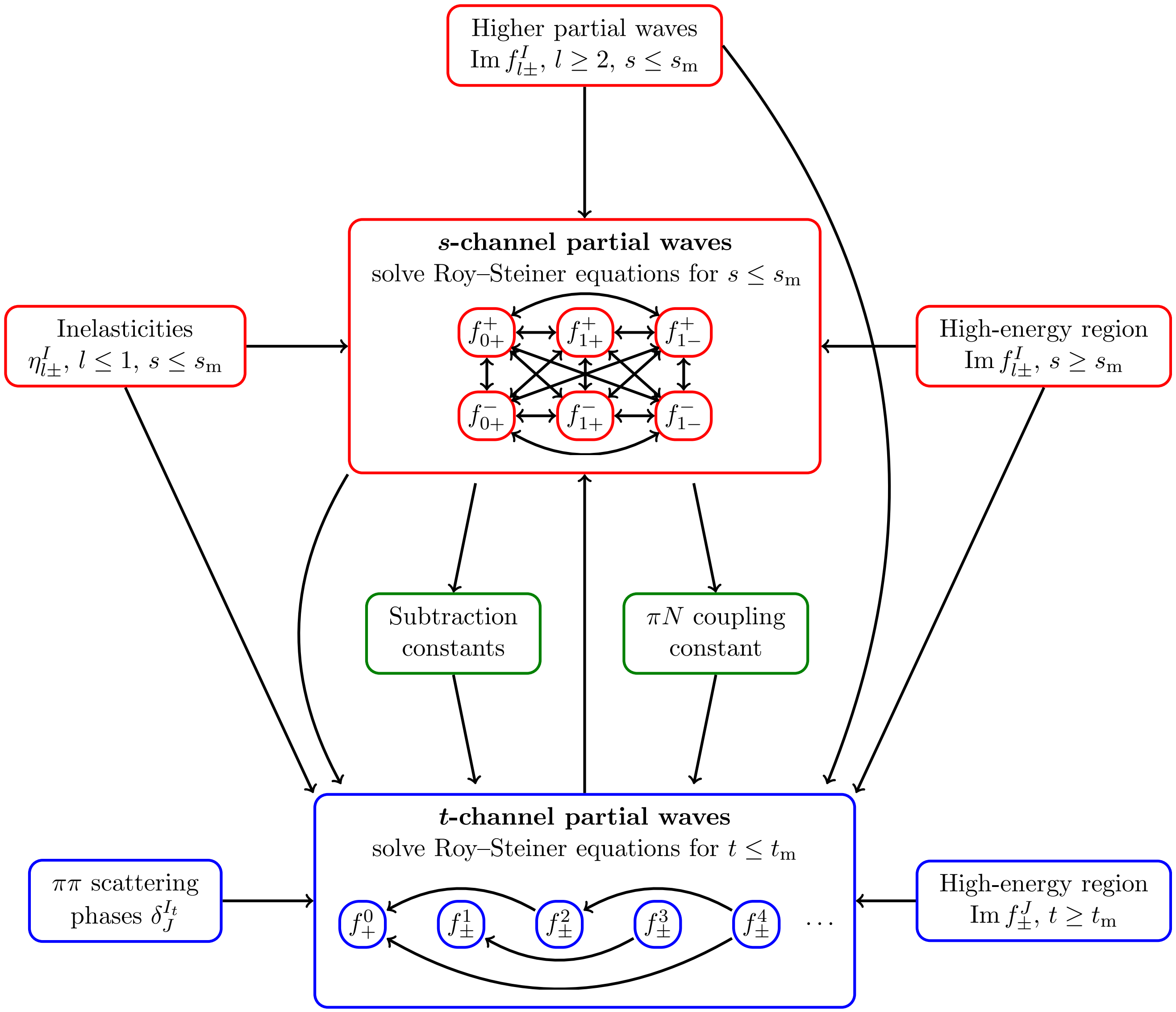
$$+\bar{K}K - \mathrm{loop} + \ldots$$



⇒ Muskhelishivili-Omnès (MO) problem !

⇒ precise $\pi\pi$ input (T_J^I) from Roy equations, Berne and Madrid/Cracow groups

SOLUTION STRATEGY



SOLUTION STRATEGY continued

- RS equations have a limited range of validity:

$$\sqrt{s} \leq \sqrt{s_m} = 1.38\,\text{GeV} \qquad \sqrt{t} \leq \sqrt{t_m} = 2.00\,\text{GeV}$$

- Input/Constraints

  S- and P-waves above the matching point ($s > s_m, t > t_m$)

  Inelasticities

  Higher waves (D-, F-, ...)

  Scattering lengths from hadronic atoms

- Output

  S- and P-wave phase shifts below the matching point ($s \leq s_m, t \leq t_m$)

  Subthreshold parameters

  → Pion-nucleon σ-term

  → LECs of pion-nucleon CHPT

  → N form factor spectral functions

- πN input from SAID/GWU, $\pi\pi$ input from Bern and Madrid/Cracow
- important check: recover KH80 phases with appropriate input

ERROR ANALYSIS

- Variation of the input:

  use KH80 input instead of GWU/SAID (higher PWs, inelasticities) $\to$ small effect

  very small effect from s-channel PWs with $\ell > 5$

  small effect from the S-wave extrapolation for $t > 1.3$ GeV

  negligible effect of the the ρ' and the ρ''

  very significant effect of the D-waves (esp. $f_2(1270)$)

  F-waves shown to be negligible

- Other sources of uncertainty:

  statistical errors (shallow fit minima)

  matching conditions (close to W_m) [no error on SAID, use smoothed KH80]

  scattering lengths errors (important for $\sigma_{\pi N}$)

$\Rightarrow$ First time this has been achieved in a dispersive analysis of πN scattering!

Results

Hoferichter, Ruiz de Elvira, Kubis, UGM

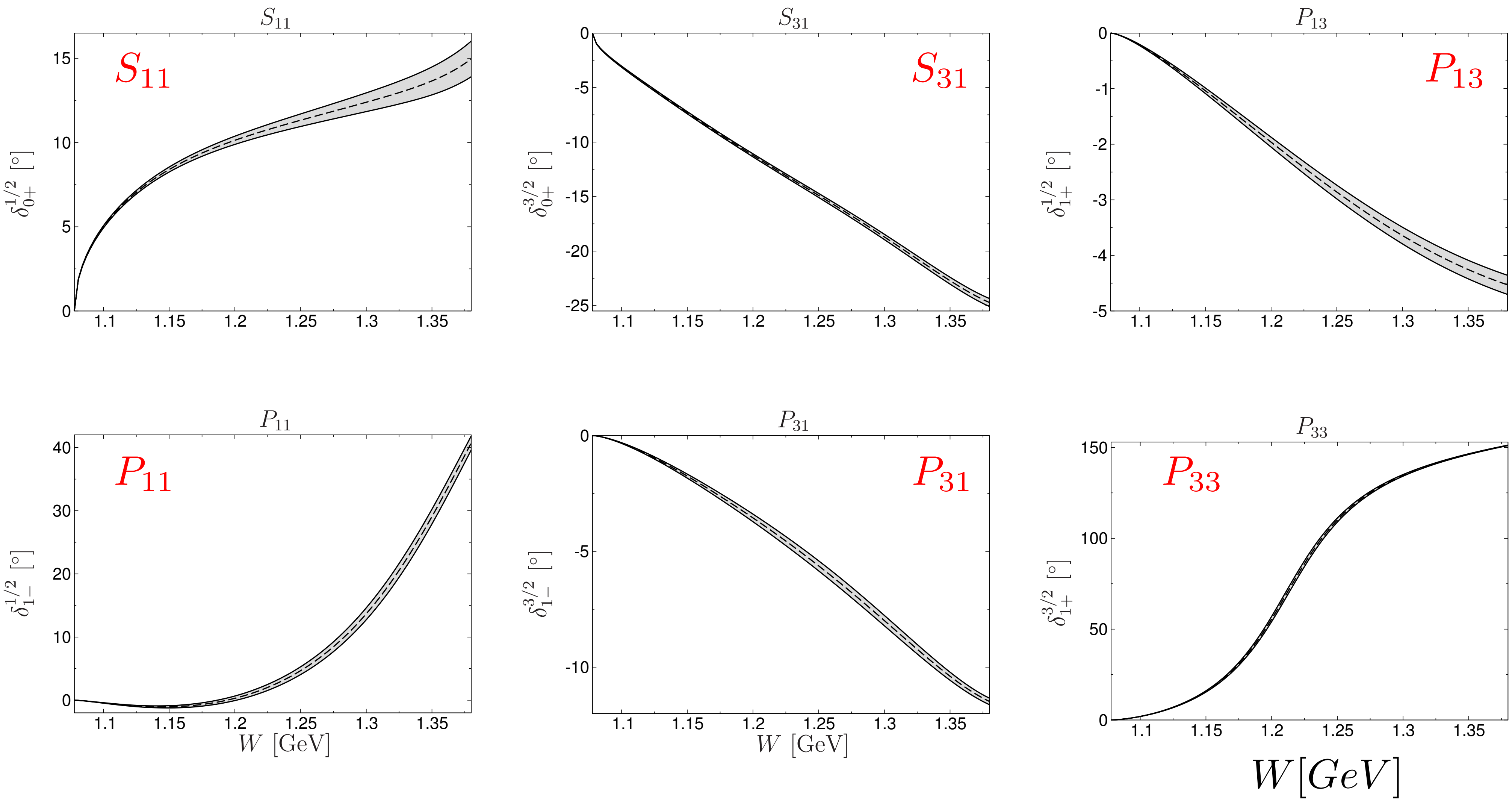
Phys. Rev. Lett. **115** (2015) 092301 [arXiv:1506.04142]

Phys. Rev. Lett. **115** (2015) 192301 [arXiv:1507.07552]

Phys. Rept. **625** (2016) 1 [arXiv:1507.07552]

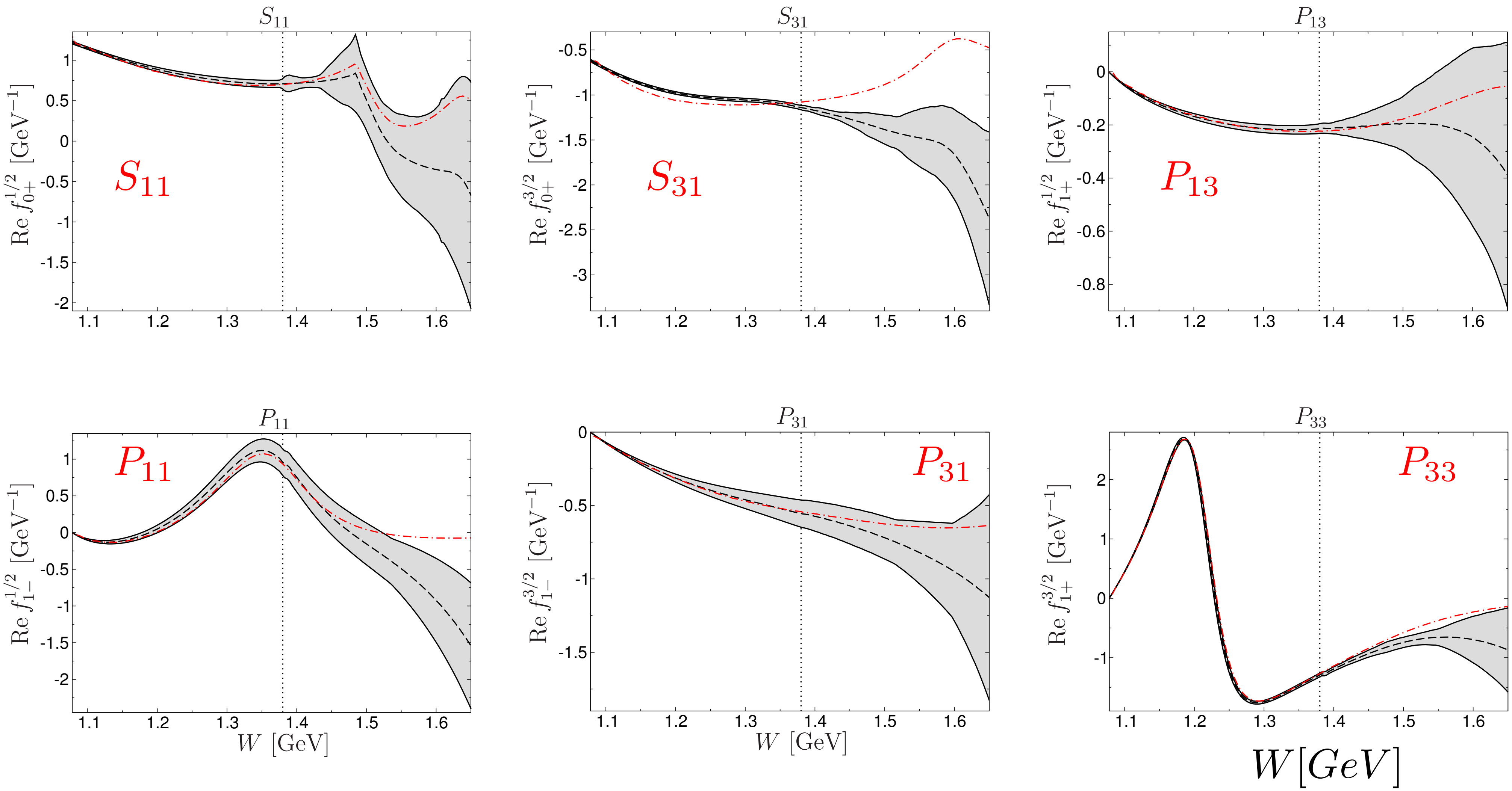
PHASE SHIFTS

- S- and P-waves up to the matching point [Notation: L_{2I_s2J}]



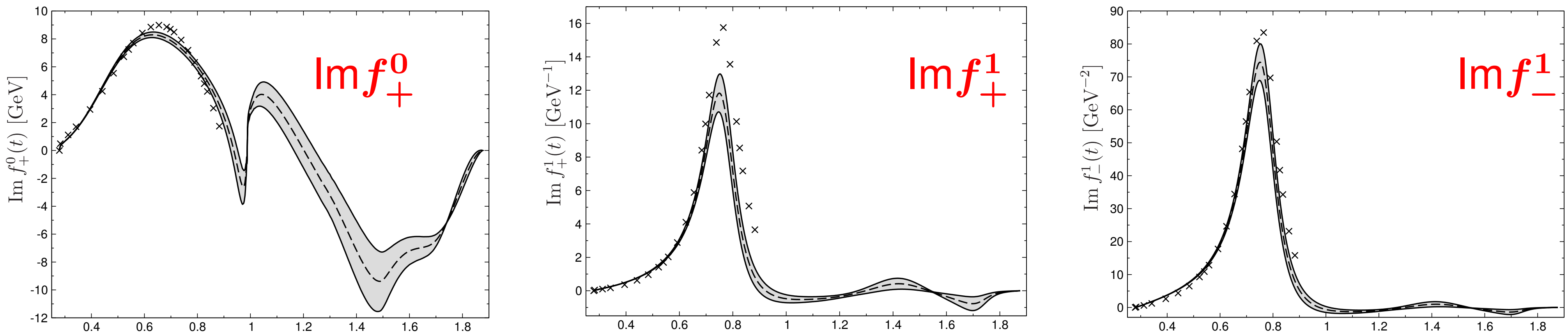
PHASE SHIFTS II

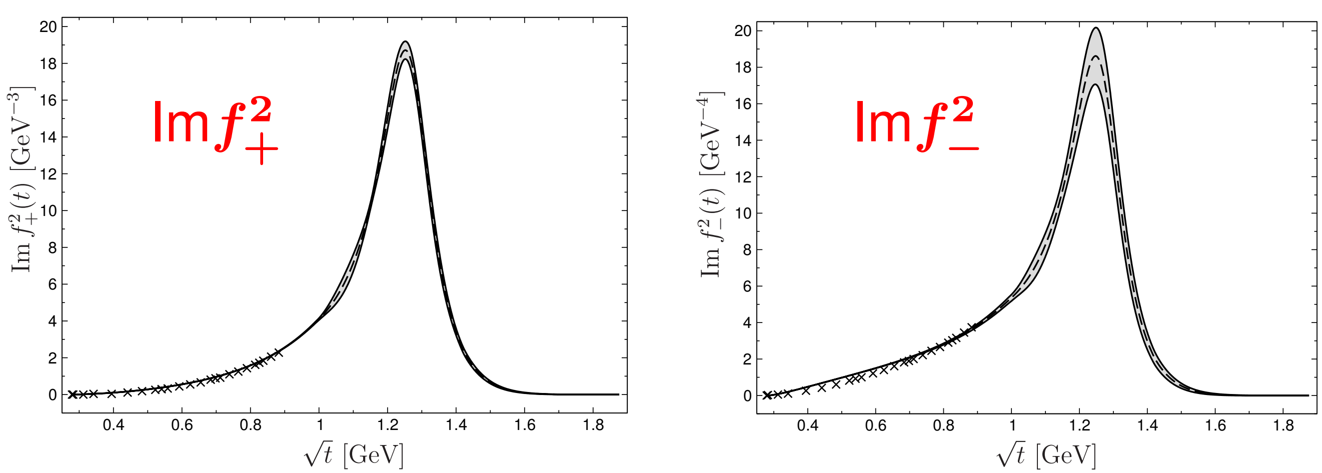
- Above the matching point (cf SAID/GWU)



t-CHANNEL PARTIAL WAVES

- Imaginary parts of the t-channel partial waves (cf KH80)





xxx = KH80

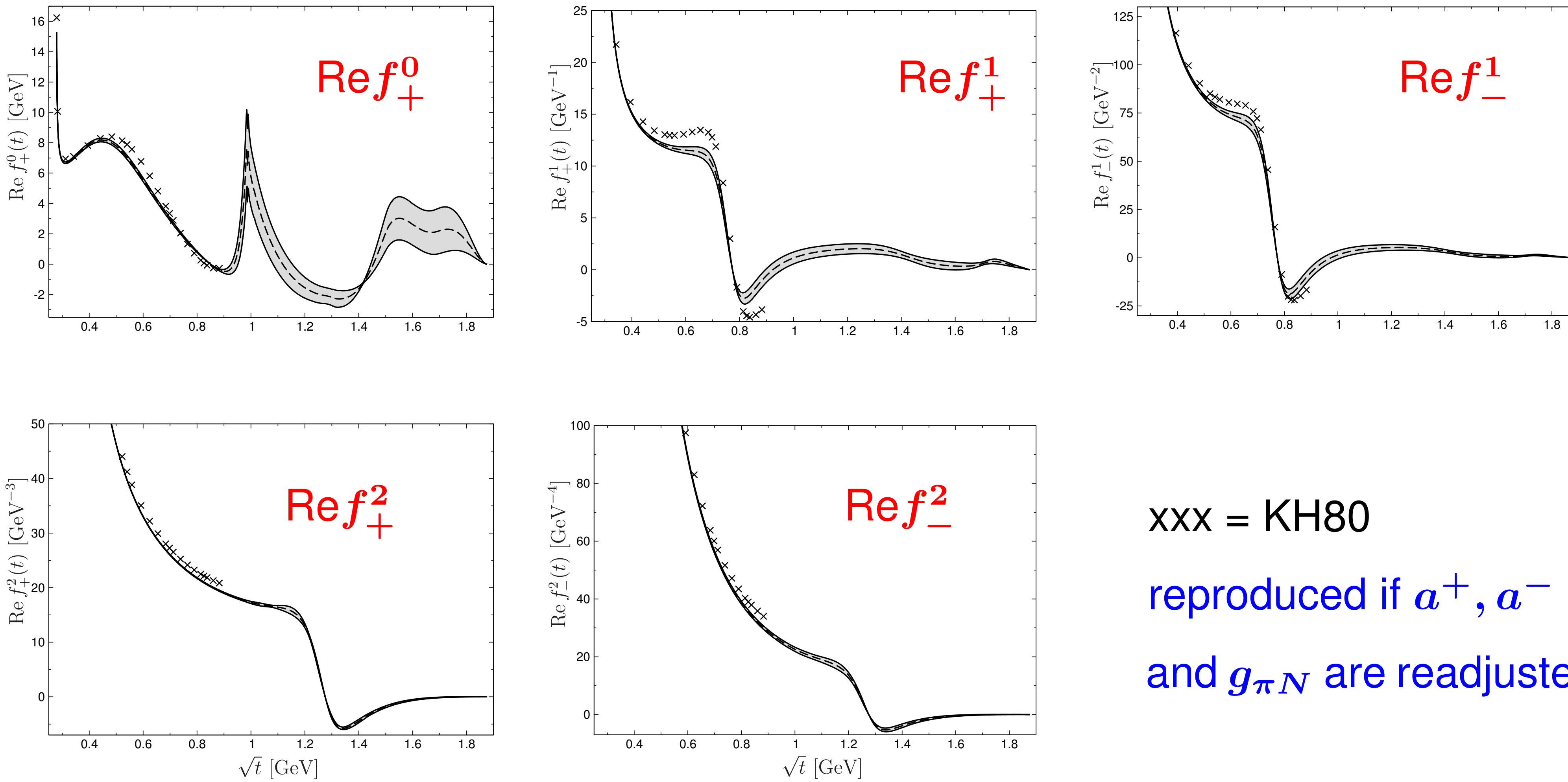
reproduced if a^+, a^-

and $g_{\pi N}$ are readjusted

$\sqrt{t}$ [GeV]

t-CHANNEL PARTIAL WAVES continued

- Real parts of the t-channel partial waves (cf KH80)



$\sqrt{t}$ [GeV]

xxx = KH80

reproduced if a^+, a^-

and $g_{\pi N}$ are readjusted

THRESHOLD PARAMETERS

- Threshold parameters from: $$\text{Re}\, f^I_{\ell\pm}(s) = q^{2\ell}\left(a^I_{\ell\pm} + b^I_{\ell\pm} q^2 + \ldots\right)$$

	RS	KH80
$a^{1/2}_{0+}$	169.8 ± 2.0	173 ± 3
$a^{3/2}_{0+}$	-86.3 ± 1.8	-101 ± 4
$a^{1/2}_{1+}$	-29.4 ± 1.0	-30 ± 2
$a^{3/2}_{1+}$	211.5 ± 2.8	214 ± 2
$a^{1/2}_{1-}$	-70.7 ± 4.1	-81 ± 2
$a^{3/2}_{1-}$	-41.0 ± 1.1	-45 ± 2
$b^{1/2}_{0+}$	-35.2 ± 2.2	-18 ± 12
$b^{3/2}_{0+}$	-49.8 ± 1.1	-58 ± 9

	RS	KH80
a^{+}_{0+}	-0.9 ± 1.4	-9.7 ± 1.7
a^{-}_{0+}	85.4 ± 0.9	91.3 ± 1.7
a^{+}_{1+}	131.2 ± 1.7	132.7 ± 1.3
a^{-}_{1+}	-80.3 ± 1.1	-81.3 ± 1.0
a^{+}_{1-}	-50.9 ± 1.9	-56.7 ± 1.3
a^{-}_{1-}	-9.9 ± 1.2	-11.7 ± 1.0
b^{+}_{0+}	-45.0 ± 1.0	-44.3 ± 6.7
b^{-}_{0+}	4.9 ± 0.8	13.3 ± 6.0

- In units of $10^{-3}/M_\pi$ or $10^{-3}/M_\pi^3$, respectively
- As a^+ is very sensitive to isospin breaking and PWAs measure $\pi^\pm p$, use $(a_{\pi^-p} + a_{\pi^+p})/2 = (-0.9 \pm 1.4) \cdot 10^{-3}/M_\pi$
- Most striking difference to KH80: S-wave scattering lengths! (here: input)

RESULTS for the SIGMA-TERM

- Basic formula:

$$\sigma_{\pi N} = F_\pi^2(d_{00}^+ + 2M_\pi^2 d_{01}^+) + \Delta_D - \Delta_\sigma - \Delta_R$$

- Subthreshold parameters output of the RS equations:

$$d_{00}^+ = -1.36(3)M_\pi^{-1} \qquad \text{[KH: } -1.46(10)M_\pi^{-1}\text{]}$$

$$d_{01}^+ = \quad 1.16(3)M_\pi^{-3} \qquad \text{[KH: } \quad 1.14(2)\ M_\pi^{-3}\text{]}$$

- $\Delta_D - \Delta_\sigma = (1.8 \pm 0.2)$ MeV — Hoferichter, Ditsche, Kubis, UGM (2012)

- $\Delta_R \lesssim 2$ MeV — Bernard, Kaiser, UGM (1996)

- Isospin breaking in the CD theorem shifts $\sigma_{\pi N}$ by $+3.0$ MeV

$\Rightarrow$ Final result:

$$\sigma_{\pi N} = (59.1 \pm 1.9_{\text{RS}} \pm 3.0_{\text{LET}})\ \text{MeV} = (59.1 \pm 3.5)\ \text{MeV}$$

[NB: recover $\sigma_{\pi N} = 45$ MeV if KH80 scattering lengths are used]

RESULTS for the SCALAR NUCLEON COUPLINGS

- WIMP scattering off nuclei sensitive to scalar nucleon couplings:

$$\langle N|m_q\bar{q}q|N\rangle = f_q^N\, m_N \qquad (N = p, n;\ q = u, d, s)$$

- Include isospin breaking corrections and use $m_u/m_d = 0.46$

$$\Rightarrow\ f_u^p = (20.8 \pm 1.5)\cdot 10^{-3}\,, \qquad f_d^p = (41.1 \pm 2.8)\cdot 10^{-3}$$

$$f_u^n = (18.9 \pm 1.4)\cdot 10^{-3}\,, \qquad f_d^n = (45.1 \pm 2.7)\cdot 10^{-3}$$

$$\sum_{q=u,\ldots,t} f_q^N = \frac{2}{9} + \frac{7}{9}\left(f_u^N + f_d^N + f_s^N\right) = 0.305 \pm 0.009$$

– sizeable reduction in uncertainties of $f_{u,d}^N$ due to the precise σ-term

– combination of couplings relevant for Higgs-mediated interactions

– f_s^N from Lattice QCD Junnarkar, Walker-Loud (2013)

RESULTS for the LECs

- Chiral expansion expected to work best at the subthreshold point (polynomial, maximal distance to singularities)

- Express subthreshold parameters in terms of LECs → invert system

- LECs c_i of the dimension two chiral effective πN Lagrangian:

LEC	RS	KGE 2012	UGM 2005
c_1 [GeV^{-1}]	-1.11 ± 0.03	$-1.13 \ldots -0.75$	$-0.9^{+0.2}_{-0.5}$
c_2 [GeV^{-1}]	3.13 ± 0.03	$3.49 \ldots 3.69$	3.3 ± 0.2
c_3 [GeV^{-1}]	-5.61 ± 0.06	$-5.51 \ldots -4.77$	$-4.7^{+1.2}_{-1.0}$
c_4 [GeV^{-1}]	4.26 ± 0.04	$3.34 \ldots 3.71$	$-3.5^{+0.5}_{-0.2}$

Krebs, Gasparyan, Epelbaum, Phys. Rev. C85 (2012) 054006
UGM, PoS LAT2005 (2006) 009

- also results for pertinent dimension three and four LECs

Comparison with recent results from lattice QCD

Hoferichter, Ruiz de Elvira, Kubis, UGM, arXiv:1602.07688

RESULTS for the SIGMA-TERM

- Recent results from various LQCD collaborations:

collaboration	$\sigma_{\pi N}$ [MeV]	reference	tension to RS
BMW	$38(3)(3)$	Dürr et al. (2015)	3.8σ
χQCD	$44.4(3.2)(4.5)$	Yang et al. (2015)	2.2σ
ETMC	$37.22(2.57)\left(^{+0.99}_{-0.63}\right)$	Abdel-Rehim et al. (2016)	4.9σ
CRC 55	$35(6)$	Bali et al. (2016)	4.0σ

- We seem to have a problem - do we? [we = RS folks]

- Robust prediction of the RS analysis:

$$\sigma_{\pi N} = (59.1 \pm 3.1)\,\mathrm{MeV} + \sum_{I_s} c_{I_s}\left(a^{I_s} - \bar{a}^{I_s}\right) \quad \left(I_s = \tfrac{1}{2}, \tfrac{3}{2}\right)$$

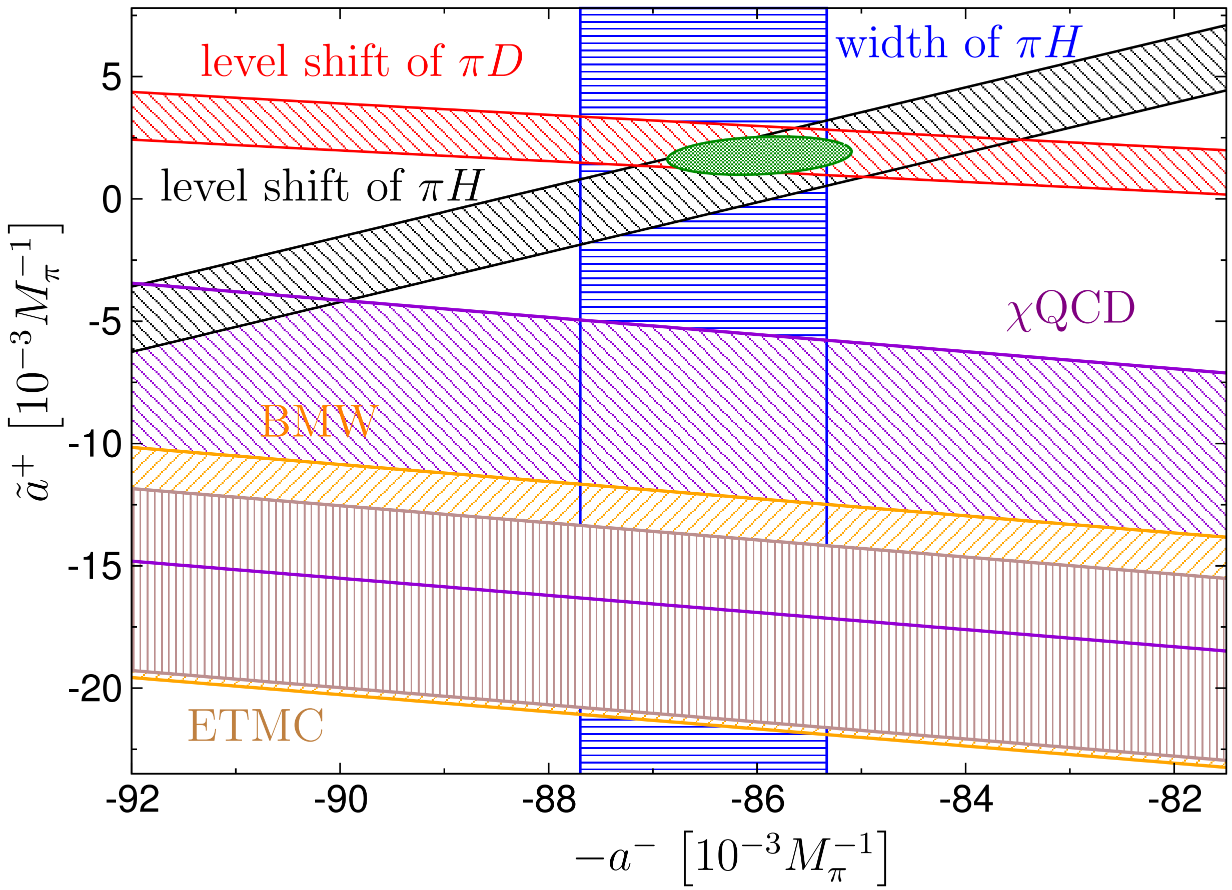
$$c_{1/2} = 0.242\,\mathrm{MeV} \times 10^3 M_\pi, \quad c_{3/2} = 0.874\,\mathrm{MeV} \times 10^3 M_\pi$$

$$\bar{a}^{1/2} = (169.8 \pm 2.0) \times 10^{-3} M_\pi^{-1}, \quad \bar{a}^{3/2} = (-86.3 \pm 1.8) \times 10^{-3} M_\pi^{-1}$$

$\rightarrow$ expansion around the reference values from πH and πD

RESULTS for the SIGMA-TERM

- Apply this linear expansion to the lattice data:



$\Rightarrow$ Lattice results clearly at odds with empirical information on the scattering lengths!

$\Rightarrow$ scattering lengths to $[\mathbf{5 \ldots 10}]\%$ $\rightarrow$ $\boldsymbol{\delta\sigma_{\pi N} = [5.0 \ldots 8.5]}$ MeV

SUMMARY & OUTLOOK

- Derived closed system of RS equations (PWHDRs) for $\pi N \to \pi N$
- Solved t-channel MO problem for the one- and two-channel approximation
- Numerical solution of the full system of RS equations
  $\hookrightarrow$ KH80 self-consistent, but at odds with hadronic atom phenomenology
- Complete error analysis (first time!)
- Precise determination of the pion-nucleon σ-term: $\sigma_{\pi N} = (59.1 \pm 3.5)$ MeV
- Precise determination of threshold parameters, scalar couplings & CHPT LECs
- New determination of the nucleon isovector spectral functions (in the works)
- Open ends:
  $\hookrightarrow$ lattice determinations of $\sigma_{\pi N}$ at odds with modern scattering lengths
  $\hookrightarrow$ strangeness content $\sim \langle N | m_s \bar{s} s | N \rangle$

SPARES

SUBTHRESHOLD EXPANSION

- expand around $\nu = t = 0$:

$$\boxed{X(\nu,t) = \sum_{m,n} x_{mn}\,\nu^{2m}\,t^n} \;,\quad X \in \left\{\bar{A}^+, \frac{\bar{A}^-}{\nu}, \frac{\bar{B}^+}{\nu}, \bar{B}^-, \bar{D}^+, \frac{\bar{D}^-}{\nu}\right\}$$

  - ⋆ subtraction of pseudovector Born terms $\to \bar{X}$
  - ⋆ $\boxed{x_{mn}}$ are the **subthreshold parameters** $\to$ can be calculated via sum rules

- low-energy expansion of the pion-nucleon scattering amplitude ($D = A + \nu B$):

$$A^+(\nu,t) = \frac{g^2_{\pi N}}{m_N} + d^+_{00} + d^+_{01}t + \mathcal{O}\left(\nu^2, \nu^2 t, t^2\right)$$

$$A^-(\nu,t) = \nu d^-_{00} + \mathcal{O}\left(\nu^3, \nu t\right)\;, \quad B^+(\nu,t) = g^2_{\pi N}\frac{4m_N\nu}{M^2_\pi} + \nu b^+_{00} + \mathcal{O}\left(\nu^3, \nu t\right)$$

$$B^-(\nu,t) = -\frac{g^2_{\pi N}}{M^2_\pi}\left[2 + \frac{t}{M^2_\pi}\right] - \frac{g^2_{\pi N}}{2m^2_N} + b^-_{00} + b^-_{01}t + \mathcal{O}\left(\nu^2, \nu^2 t, t^2\right)$$

t-CHANNEL MO PROBLEM

- One-channel MO problem with finite matching point t_m

$$f(t) = \Delta(t) + \frac{1}{\pi}\int\limits_{4M_\pi^2}^{t_m} dt' \frac{T(t')^\star f(t')}{t'-t} + \frac{1}{\pi}\int\limits_{t_m}^{\infty} dt' \frac{\mathrm{Im}\, f(t')}{t'-t}$$

$\hookrightarrow$ $\Delta(t)$: pole terms, s-channel imag. parts, other t-channel PWs

$\hookrightarrow$ solve for $f(t)$ in $4M_\pi^2 \le t \le t_m$ requires

- $\mathrm{Im}\, f(t)$ for $t \ge t_m$
- $T(t)$ for $4M_\pi^2 \le t \le t_m$

- Solution via once-subtracted Omnès function (w/ $\Omega(0) = 1$):

$$\Omega(t) = \exp\left\{\frac{t}{\pi}\int\limits_{4M_\pi^2}^{t_m} \frac{dt'}{t'}\frac{\delta(t')}{t'-t}\right\} = |\Omega(t)| \exp\{i\delta(t)\theta(t-4M_\pi^2)\theta(t_m-t)\}$$