Fermionic phase transition induced by an effective impurity in holography

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In collaboration with Li-Qing Fang, Jian-Pin Wu and Bin Wang; JHEP 11 (2015) 134.

Outlines

- Review: application of Gauge/Gravity duality into CM
- Why impurity in holography
- A simple gravitational modle sources impurity
- Holographic Fermions with impurity
- Closing remarks

Review: Application of Gauge/Gravity duality into CMT

- **Holographic principle**

A d+2 dimensional theory of quantum gravity may be described by a d+1 dimensional quantum field theory without gravity.

(**Semi-**)**Classical** gravity in D+1-dim ⟺ **Large** N gauge theories in D-dim

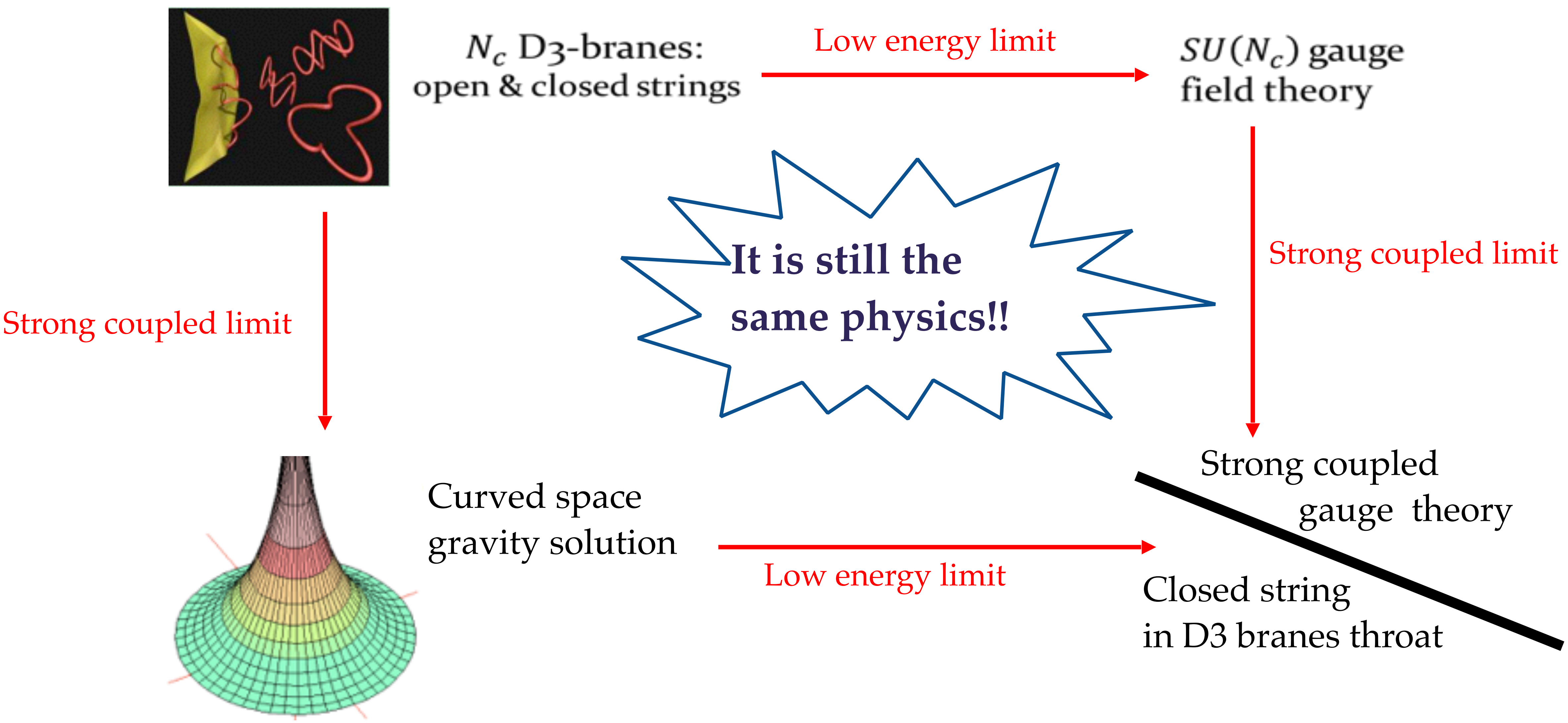
$$g^2 = 4\pi g_s$$
$$\lambda \equiv g^2 N = \frac{L^4}{l_s^4}$$
$$g_s \to 0, \frac{L}{l_s} \text{ fixed} \leftrightarrow N \to \infty, \lambda \text{ fixed}$$

$$g_s \to 0, \frac{L}{l_s} \to \infty \leftrightarrow N \to \infty, \lambda \to \infty$$

Weak/Strong duality

Review: Application of Gauge/Gravity duality into CMT

J. Maldacane, 1997



- AdS/CFT correspondence

$$dS^2 = \frac{L^2}{z^2}(dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu)$$

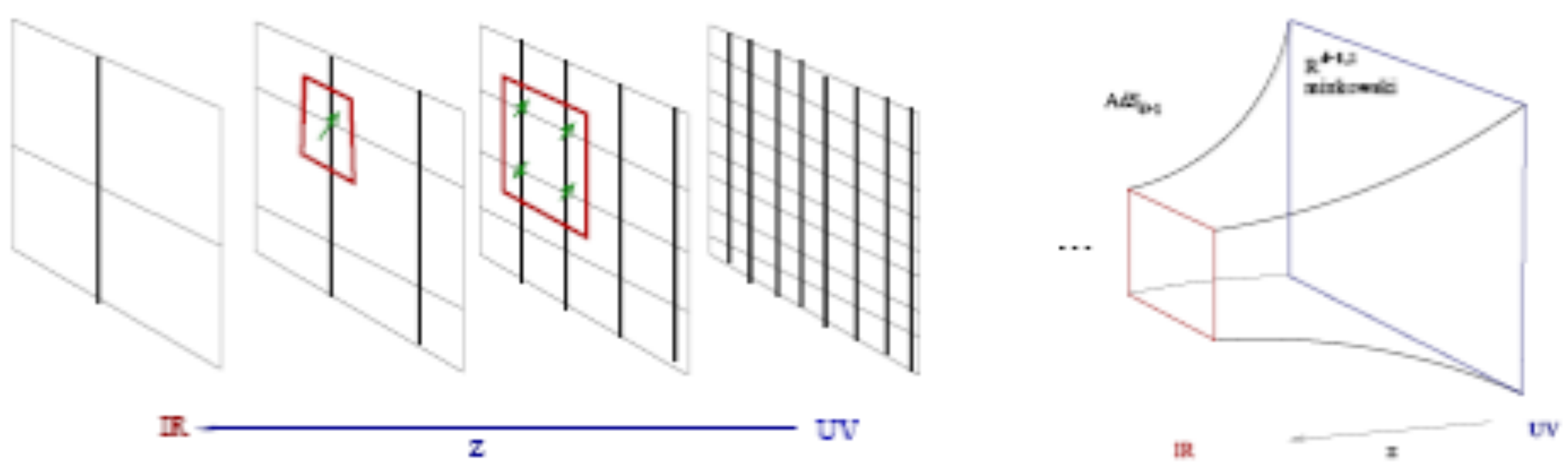


Figure 1: The extra ('radial') dimension of the bulk is the resolution scale of the field theory.

J. Mcgreevy arXiv:0909.0518

Review: Application of Gauge/Gravity duality into CMT

- **Recipe**

$$Z_{boundary}[J] = \int D\phi \, e^{iS[\phi_{cl}] + i\int d^d x J o}$$

$$G(x-y) = -i\langle T o(x) o(y)\rangle_{QFT} = -\frac{\delta^2 S[\phi_{cl}]}{\delta J(x)\delta J(y)}\bigg|_{\phi(z=0)=J}$$

Review: Application of Gauge/Gravity duality into CMT

- **Examples**

boundary		bulk
$T_{\mu\nu}$		g_{ab}
J_{μ}	⟷	A_a
O_B		ϕ
O_F		ψ

Shear viscosity:

$$G^R_{xy,xy}(\omega,0) = \int dt dx e^{i\omega t}\theta(t)\langle [T_{xy}(t,x), T_{xy}(0,0)]\rangle$$

$$\eta = -\lim_{\omega\to 0}\frac{1}{\omega}\mathrm{Im}\, G^R_{xy,xy}(\omega,0)$$

$$\frac{\eta}{s} \geq \frac{h}{4\pi}$$ KSS Bound

D. T. Son, etc.,hep-th/0405231

Conductivity:

$$G^R(\omega) = -\lim_{r\to\infty} g(r) r A_x A_x'$$

$$\sigma = \frac{1}{i\omega}G^R(\omega)$$

$$\frac{\omega_g}{T_c} \sim 8$$

G. T. Horowitz,arXiv:1002.1722

Holographic superconductor is a strongly coupled system.

Review: Application of Gauge/Gravity duality into CMT

- **More Progresses**
    - Holographic metal/superconductor, insulator/superconductor phase transition
    - Holographic (non-)Fermions, strange metals
    - Holographic Josephson junction
    - Holographic Lattice
    - Non-equilibrium condensation, dynamic
    - Holographic turbulence
        -
        -
        -

Why impurity??

- **A problem in holography conductivity:**

Because the boundary theory has translational symmetry

↓

No mechanism for momentum dissipates

↓

Infinite DC conductivity

↓

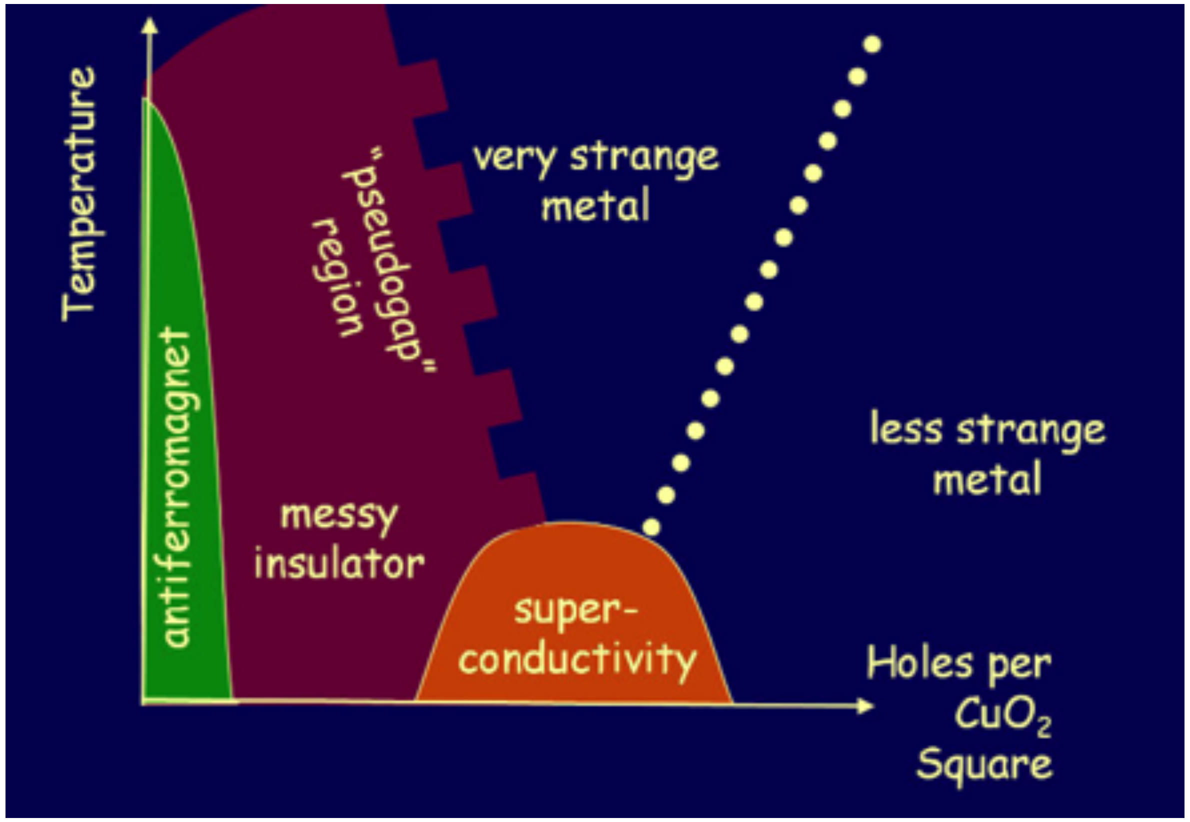
Not match the real world

We need to introduce “impurity” from gravity to break translational symmetry in the dual CFT.

Why impurity??

- **A challenge from experiment:**

Phase diagram in Cuprate $La_{2-x}Sr_xCuO_4$:



$$\rho \approx \rho_0 + BT + \ldots$$

Question: Is it possible to construct a gravitational solution dually describes this kind of field theory with impurity ??

A simple gravitational modle sources impurity

T. Andrade, etc., arxiv:1311.5157

Key points:

1.Neutral scalars in the bulk——>spatially dependent sources for operators in the dual CFT.

2.The bulk stress tensor and resulting black brane geometry are homogeneous and isotropic.

- **Action**

$$S_0 = \int_M \sqrt{-g}\left[R - 2\Lambda - \frac{1}{2}\sum_I^{d-1}(\partial\psi_I)^2 - \frac{1}{4}F^2\right] d^{d+1}x - 2\int_{\partial M}\sqrt{-\gamma}K d^d x.$$

- Make ψ massless, so that it only enters the bulk stress tensor through $\partial_\mu\psi$;
- Only turn on sources which are linear in the boundary coordinates, $\psi(0) \propto \alpha_i x^i$;
- Include a total of d – 1 scalar fields, ψ_I , then arrange their sources, $\alpha_{Ii}x^i$ such that the bulk metric is also isotropic.

A simple gravitational modle sources impurity

T. Andrade, etc., arxiv:1311.5157

- **Solution**

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2\delta_{ab}dx^a dx^b, \qquad A = A_t(r)dt, \qquad \psi_I = \alpha_{Ia}x^a,$$

where a labels the d – 1 spatial x^a directions, I is an internal index that labels the d – 1 scalar fields and α_{Ia} are real arbitrary constants.

$$f = r^2 - \frac{\alpha^2}{2(d-2)} - \frac{m_0}{r^{d-2}} + \frac{(d-2)\mu^2}{2(d-1)}\frac{r_0^{2(d-2)}}{r^{2(d-2)}},$$

$$A_t = \mu\left(1 - \frac{r_0^{d-2}}{r^{d-2}}\right),$$

with $\alpha^2 \equiv \frac{1}{d-1}\sum_{a=1}^{d-1}\vec{\alpha}_a \cdot \vec{\alpha}_a,$ and $\vec{\alpha}_a \cdot \vec{\alpha}_b = \alpha^2\delta_{ab} \quad \forall a, b.$

$$T = \frac{f'(r_0)}{4\pi} = \frac{1}{4\pi}\left(dr_0 - \frac{\alpha^2}{2r_0} - \frac{(d-2)^2\mu^2}{2(d-1)r_0}\right). \qquad s = 4\pi r_0^{d-1}.$$

A simple gravitational modle sources impurity

Conductivity

Consider linearized fluctuations of the form

$$\delta A_x = e^{-i\omega t} a_x(r), \qquad \delta g_{tx} = e^{-i\omega t} r^2 H_{tx}(r) \qquad \delta\psi = e^{-i\omega t}\alpha^{-1}\chi(r)$$

$$\alpha_{Ia} = \alpha\delta_{Ia}$$

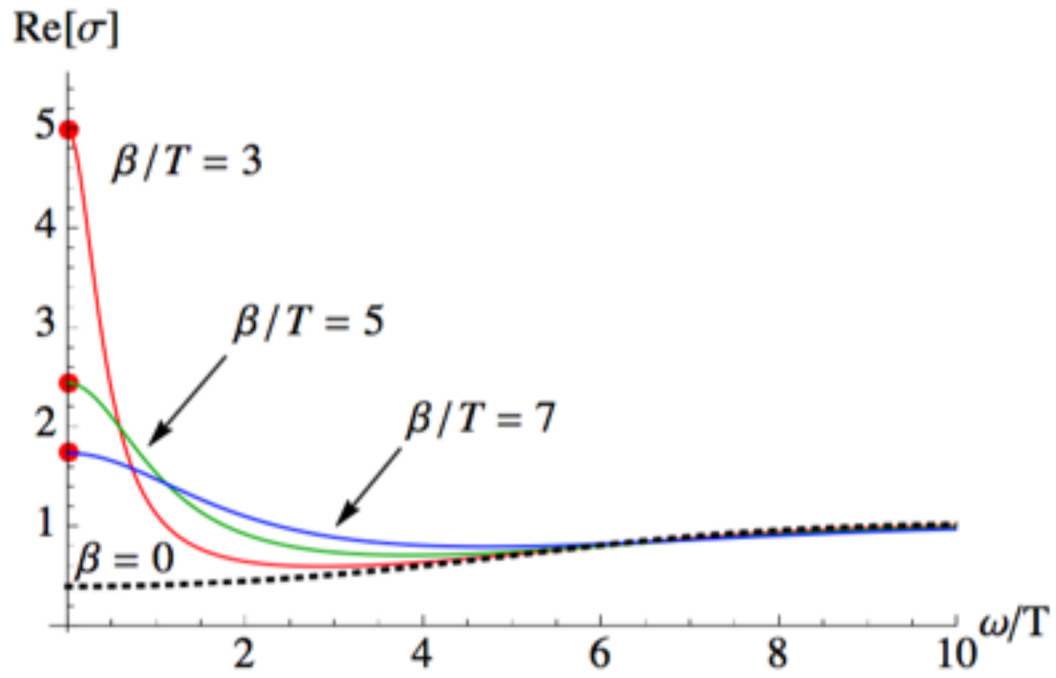
Analytic:

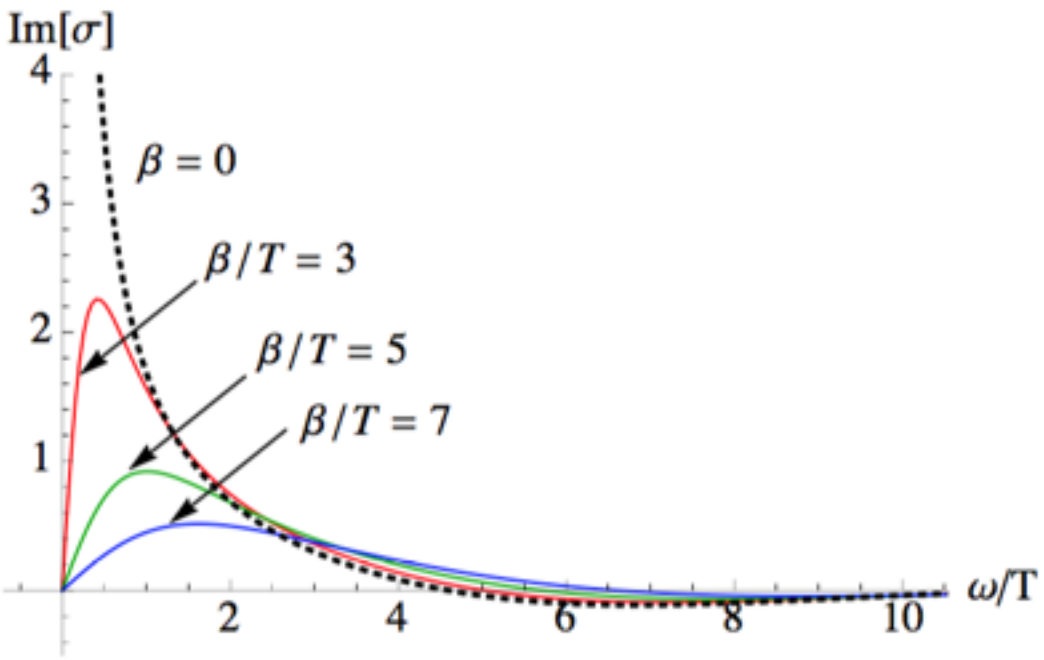
$$\frac{\sigma_{DC}}{\mu^{d-3}} = a_d\left(\frac{\alpha}{\mu}\right) + b_d\left(\frac{\alpha}{\mu}\right)\frac{T}{\mu} + O\left(\frac{T}{\mu}\right)^2, \qquad T \ll \mu$$

Finite

T. Andrade, etc., arxiv:1311.5157

Numeric:



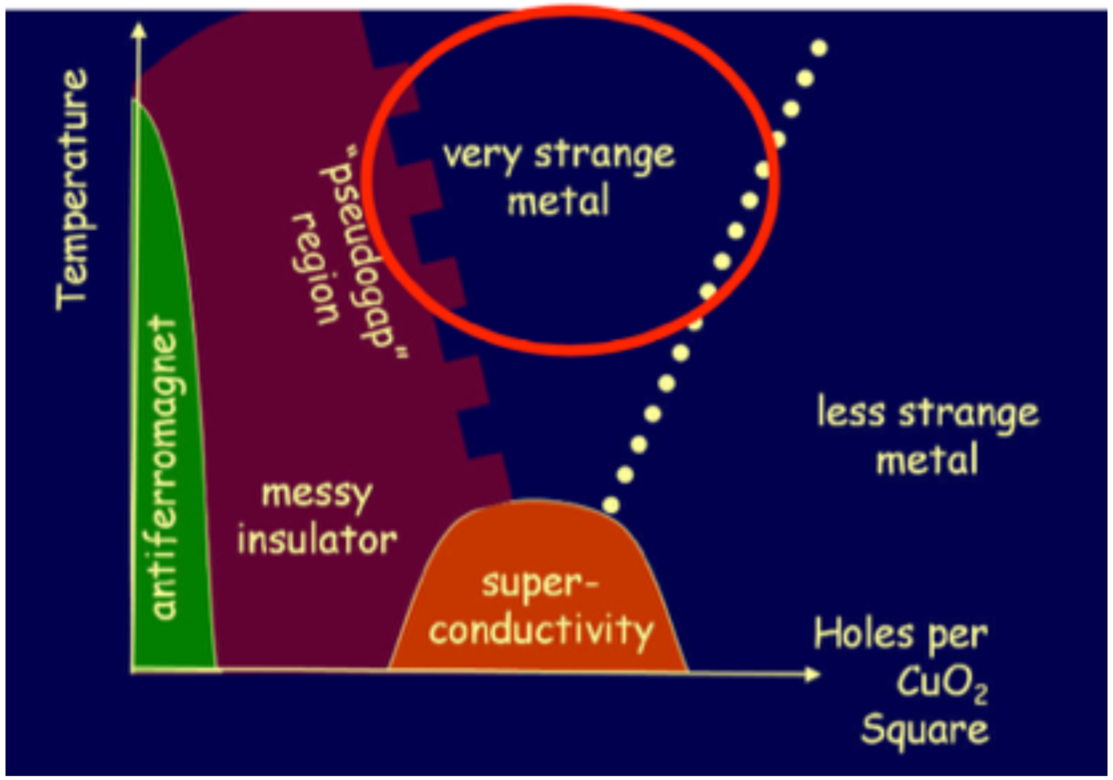


Kim, etc., arxiv:1409.8346

Question: Is it possible to construct a gravitational solution dually describes this kind of field theory??

YES!!

What's we asked: If we consider the Dirac spinor in this gravity background with impurity, in this holographic Fermionic system, can we find a Fermi surface? What's the low energy excitation?



Holographic Fermions with impurity

X.Kuang, etc., arxiv:1507.03121

- **Setup**

$$S_F = \int d^{d+1}x\sqrt{-g}i\bar{\psi}\Big(\Gamma^a D_a - m\Big)\psi,$$

$$\boxed{\Gamma^a D_a\psi - m\psi = 0}$$

$$D_a = \partial_a + \frac{1}{4}(\omega_{\mu\nu})_a\Gamma^{\mu\nu} - iqA_a \qquad (\omega_{\mu\nu})_a = (e_\mu)_b \nabla_a (e_\nu)^b$$

Background:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2\sum_{i=1}^{d-1} dx^i dx^i,$$

$$A = \mu\left(1 - \frac{r_0^{d-2}}{r^{d-2}}\right)dt, \qquad \Psi_I = \alpha_{Ii}x^i = \alpha\delta_{Ii}x^i$$

Remark: $T \to 0$ and $r \to r_0$,the geometry is $AdS_2 \times \mathbb{R}^{d-1}$

Holographic Fermions with impurity

X.Kuang, etc., arxiv:1507.03121

- **Explicit Dirac equation**

$$\Gamma^a D_a \psi - m\psi = 0$$

$$\psi = \int d\omega dk e^{-i\omega t + ik_i x^i} (-g g^{rr})^{-\frac{1}{4}} \phi, \qquad \phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

$$\frac{\sqrt{g_{xx}}}{\sqrt{g_{rr}}} \partial_r \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} + \sqrt{g_{xx}} m \sigma^3 \otimes \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \frac{\sqrt{g_{xx}}}{\sqrt{g_{tt}}} (\omega + qA_t) i\sigma^2 \otimes \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \mp k\sigma^1 \otimes \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

$$\phi_I = \begin{pmatrix} y_I \\ z_I \end{pmatrix} \qquad \xi_I = \frac{y_I}{z_I}$$

Flow equation:

$$\left(\frac{\sqrt{g_{xx}}}{\sqrt{g_{rr}}} \partial_r + 2\sqrt{g_{xx}} m\right) \xi_I = \left[\frac{\sqrt{g_{xx}}}{\sqrt{g_{tt}}} (\omega + qA_t) + (-1)^I k\right] + \left[\frac{\sqrt{g_{xx}}}{\sqrt{g_{tt}}} (\omega + qA_t) - (-1)^I k\right] \xi_I^2$$

Holographic Fermions with impurity

X.Kuang, etc., arxiv:1507.03121

- **Boundary conditions**

Boundary: $\phi_I \xrightarrow{r\to\infty} a_I r^{-m} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + b_I r^m \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

- **Read off the Green function**

Recipe → $G_{II} = \frac{a_I}{b_I}$

$$G(\omega, k) = \lim_{r\to\infty} r^{2m} \begin{pmatrix} \xi_1 & 0 \\ 0 & \xi_2 \end{pmatrix}$$

So, after solving the flow equation, we can extract the Green function of holographic Fermionic system.

Horizon: $\xi_I \overset{r\to r_0}{=} i$ Infalling

Holographic Fermions with impurity

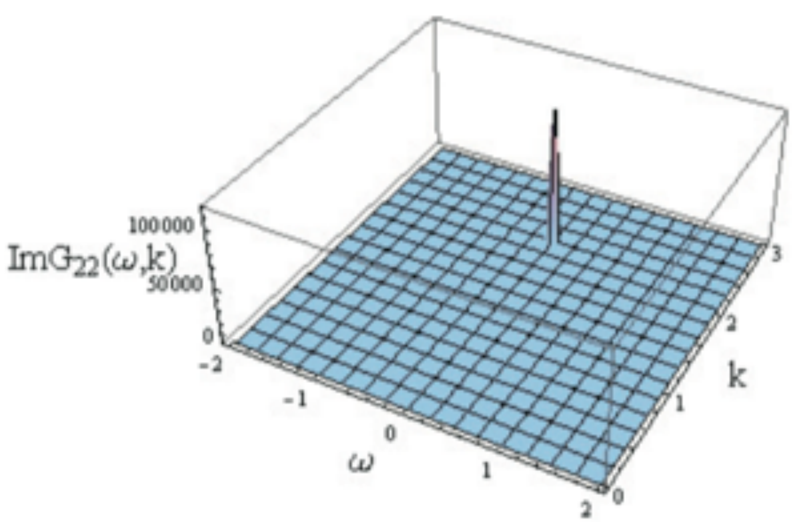
X.Kuang, etc., arxiv:1507.03121

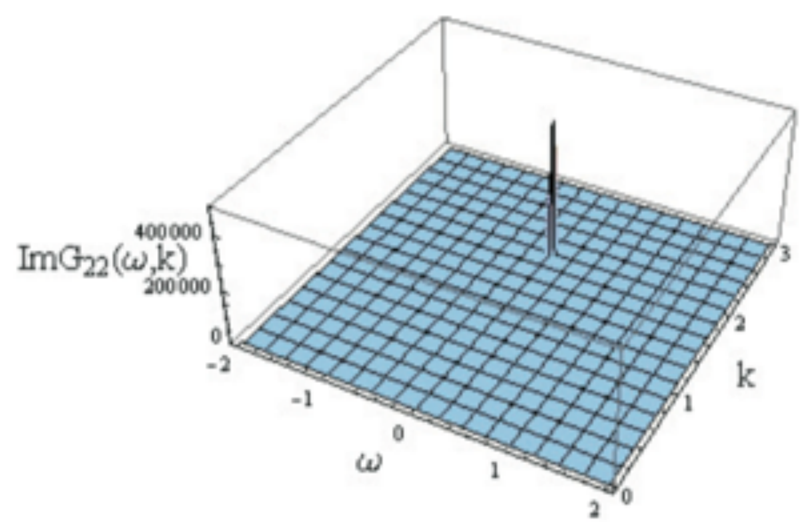
- ## Numerical Result (Fermi Surface)

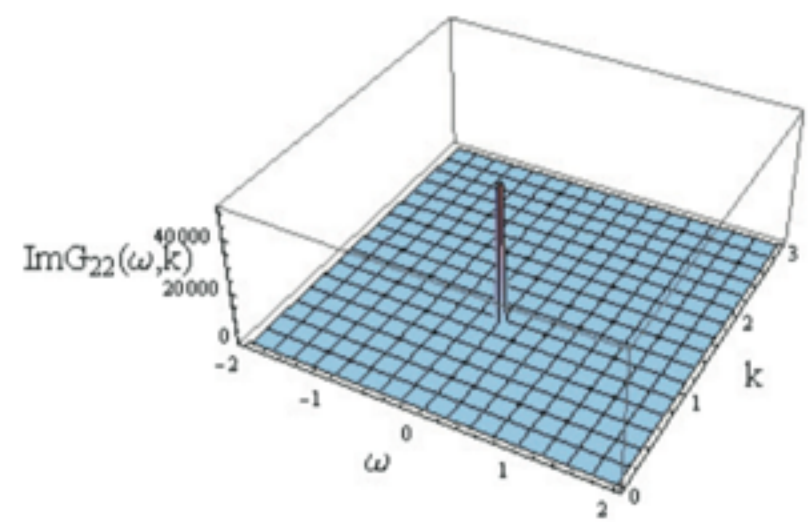
Remark: 1) Fermi surface

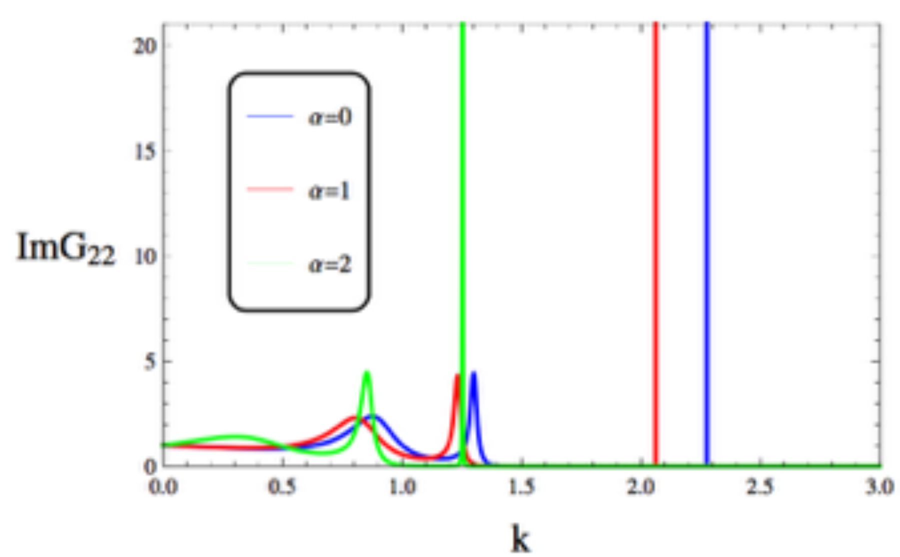
$$G^R(\omega,k)=\frac{Z}{\omega-v_F(k-k_F)+\Sigma(\omega,k)} \qquad \Sigma(\omega,k)\sim\omega^2$$

$\omega=0,\ k=k_F,\ G^R(\omega,k_F)$ has a pole. Fermi surface is the sphere with radius kF.









- (non-)Fermi Liquid

$$G^R(\omega,k)=\frac{Z}{\omega-v_F(k-k_F)+\Sigma(\omega,k)}$$

Matching solutions:

$$G_R(k,\omega) \approx \frac{b_+^{(0)}(k_F)}{\partial_k a_+^{(0)}(k_F)k_\perp + \omega a_+^{(1)}(k_F) + a_-^{(0)}(k_F)\mathcal{G}_{k_F}(\omega)}$$

$$= \frac{h_1}{k_\perp - \frac{1}{v_F}\omega - h_2 e^{i\gamma_{k_F}}\omega^{2\nu_{k_F}}}$$

ν_F, h_1 and h_2 can be determined by numerical boundary datas.

$$\omega_*(k) \propto k_\perp^z \quad \text{with} \quad z = \begin{cases} \frac{1}{2\nu_{k_F}} & \nu_{k_F} < \frac{1}{2} \\ 1 & \nu_{k_F} > \frac{1}{2} \end{cases}$$

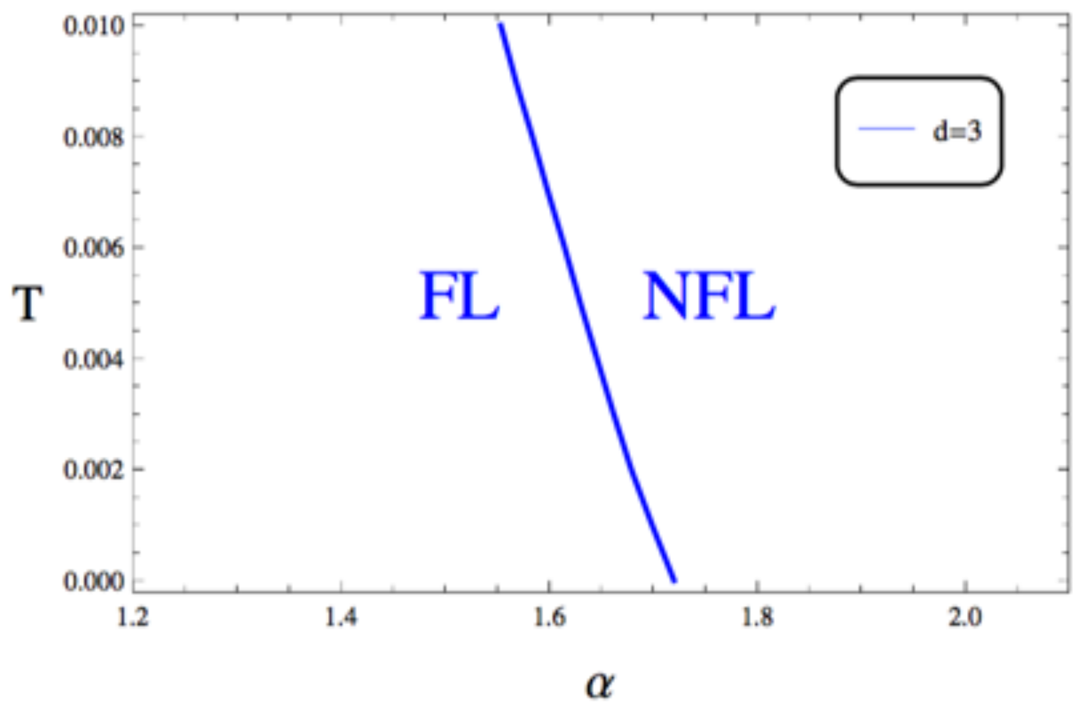
$$\Gamma(k) \propto k_\perp^\delta \quad \text{with} \quad \delta = \begin{cases} \frac{1}{2\nu_{k_F}} & \nu_{k_F} < \frac{1}{2} \\ 2\nu_{k_F} & \nu_{k_F} > \frac{1}{2} \end{cases} .$$

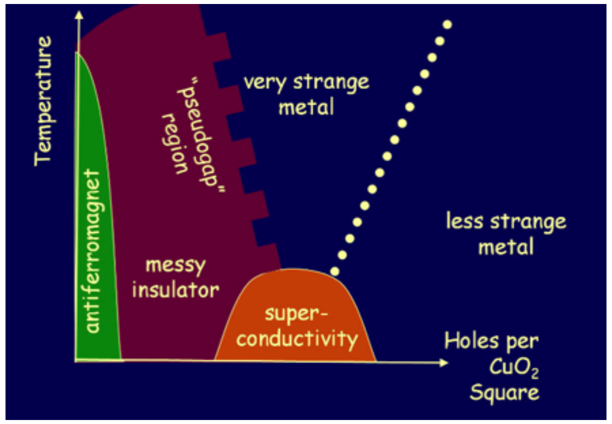
Fermi Liquid: $z = 1$ and $\delta = 1$

Holographic Fermions with impurity

X.Kuang, etc., arxiv:1507.03121

- **Numerical Result (Phase diagram)**





Remark: α should not be considered as doping in cuprate because the phase diagram is different.

What's kind of impurity it describes? disorder??

α provides a new mechanism introducing impurity from holography!! We expect that phase diagram can be observed in some real materials in the future.

1504.05561： Phases of holographic superconductors with broken translational symmetry.

X.Kuang, etc., arxiv:1507.03121

- **Other Properties**

1. Effect of dimension: Fermi liquid phase is forbidden in the d=4 and d=5 geometry.

2. Larger q suppresses the effect of impurity, making the phase transition occurs at stronger impurity.

Closing remarks

Summery:

- Review the recipe of AdS/CFT.
- Answer why we need impurity for holography and introduce a simple gravitational model describing the CFT with impurity.
- Study the holographic Fermions and check the Fermi liquid type; Find an interesting phase diagram.

Prospects:

- It is interesting to study the existence of Anderson localization in this model to check if this impurity play the role of disorder.
- We expect that such a phase diagram can be observed in some real materials in the future.
- What we will see if the coupling between Dirac field and the scalar field is considered??

非常感谢!!