Holographic Description of BPS Wilson Loops in Flavored ABJM Theory

Mengqi Zhu

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Outline

- The Flavored ABJM Theory
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The Flavored ABJM Theory

The ABJM theory is a three-dimensional $\mathcal{N} = 6$ Chern-Simons-matter theory with gauge group $U_1(N) \times U_2(N)$ and Chern-Simons levels $(k, -k)$. One interesting generalization of the ABJM theory is to introduce flavors.

$$\mathcal{S}_{\text{mat}} = \int d^3x d^4\theta \, \mathrm{Tr}\left(-\bar{A}_i e^{-V_1} A_i e^{V_2} - \bar{B}_i e^{-V_2} B_i e^{V_1}\right) - \bar{Q}_i^r e^{-V_i} Q_i^r - \tilde{Q}_i^r e^{V_i} \bar{\tilde{Q}}_i^r$$

$$\mathcal{S}_{\text{CS}} = -i\frac{k}{4\pi} \int d^3x d^4\theta \int_0^1 dt \, \mathrm{Tr}\left(V_1 \bar{D}^\alpha (e^{tV_1} D_\alpha e^{-tV_1}) - V_2 \bar{D}^\alpha (e^{tV_2} D_\alpha e^{-tV_2})\right)$$

$$\mathcal{S}_{\text{pot}} = \int d^3x d^2\theta \, (W_{\text{ABJM}} + W_{\text{flavor}}) + c.c.$$

where

$$W_{\text{ABJM}} = -\frac{k}{8\pi} \mathrm{Tr}\,(\Phi_1^2 - \Phi_2^2) + \mathrm{Tr}\,(B_i \Phi_1 A_i) + \mathrm{Tr}\,(A_i \Phi_2 B_i)$$

$$W_{\text{flavor}} = \tilde{Q}_1^r \Phi_1 Q_1^r - \tilde{Q}_2^r \Phi_2 Q_2^r$$

The BPS Wilson Loop

- Construct BPS Wilson loop in three-dimensional $\mathcal{N} = 3$ supersymmetric Chern-Simons-matter theories ([Gaiotto and Yin, 07])

$$W = \frac{1}{\dim(\mathrm{R})} \mathrm{Tr}_R P \exp \left[\int d\tau (iA_\mu \dot{x}^\mu + \sum_{a=1}^{3} \phi_a s^a |\dot{x}|) \right]$$

where s_a are three constants satisfying $\sum (s^a)^2 = 1$.

- One-third of the supersymmetries are preserved. Without loss of generality, we focus on the case with $s^1 = s^2 = 0, s^3 = 1$

$$W_i = \frac{1}{\dim(\mathrm{R})} \mathrm{Tr}_R P \exp \left[\int d\tau (iA_{(i)\mu} \dot{x}^\mu + \sigma |\dot{x}|) \right]$$

The BPS Wilson Loop

In the flavored ABJM theory, the strong coupling limit of the VEV of this $1/3$-BPS Wilson loop in the fundamental representation was computed in [Santamaria et al., 2010] based on the supersymmetric localization.

- When $n_1 = N_f, n_2 = 0$, the leading exponential behavior of the VEV is

$$< W_i >\sim \exp\left[2\pi\sqrt{\frac{N}{2k+N_f}}\right]$$

- For the special case with $n_1 = N_f = k, n_2 = 0$,

$$< W_i >\sim \exp\left[2\pi\sqrt{\frac{N}{3k}}\right]$$

2. Holographic Description of BPS Wilson Loops

Brane Construction

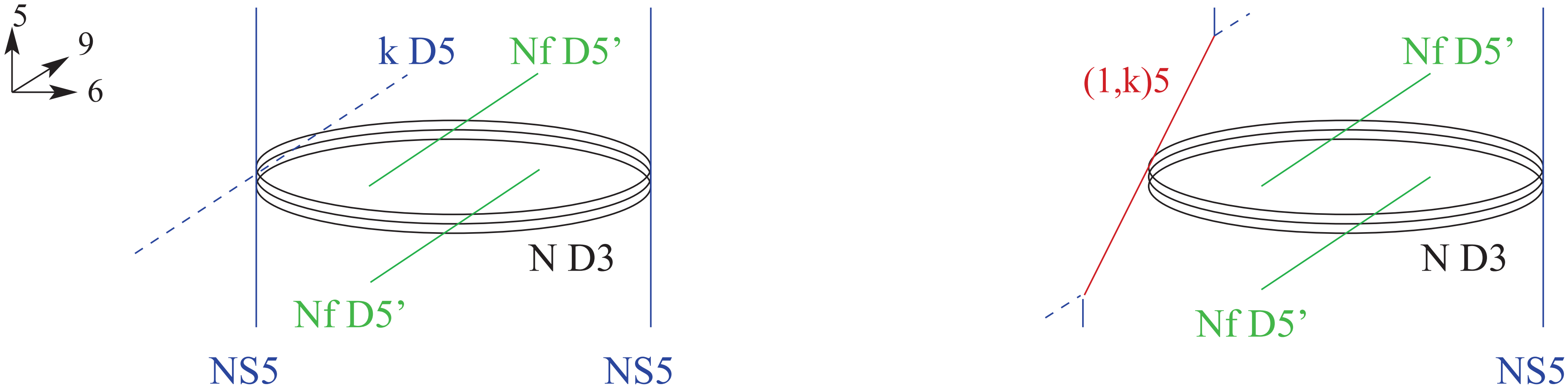

Figure 1: [Hohenegger and Kirsch, 09]

Lift to M-theory

- The resulting M-theory configuration will be a stack of N M2-branes located at the origin of a *toric hyperkähler manifold*.

- This is an eight-dimensional space $\mathcal{M}_8$ with $sp(2)$ holonomy and preserves $3/16$ of the supersymmetries of the eleven-dimensional supergravity, which is precisely the amount of supersymmetry expected for the dual of theories in $2{+}1$ dimensions with $\mathcal{N} = 3$ supersymmetry.

- Adding a stack of N M2-branes at the origin of $\mathcal{M}_8$ does not break any additional supersymmetry.

Lift to M-theory

The metric of $\mathcal{M}_8$ is given by ([Gauntlett et al., 97])

$$ds^2_{\mathcal{M}_8} \quad = U_{ij} d\vec{x}^i \cdot d\vec{x}^j + U^{ij}(d\varphi_i + A_i)(d\varphi_j + A_j)$$

with the following quantities

$$A_i = d\vec{x}^j \cdot \vec{\omega}_{ji} = dx^j_a \omega^a_{ji}\,, \qquad \partial_{x^j_a}\omega^b_{ki} - \partial_{x^k_b}\omega^a_{ji} = \epsilon^{abc}\partial_{x^j_c} U_{ki}$$

with $i, j = 1, 2$.

The two-dimensional matrix U_{ij} contains the information of the uplifted five-branes of the IIB setup

$$U = \mathbf{1} + \begin{pmatrix} h_1 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} h_2 & kh_2 \\ kh_2 & k^2 h_2 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & N_f^2 h_3 \end{pmatrix}\,,$$

with

$$h_1 = \frac{1}{2|\vec{x}_1|}\,, \qquad h_2 = \frac{1}{2|\vec{x}_1 + k\vec{x}_2|}\,, \qquad h_3 = \frac{1}{|N_f \vec{x}_2|}\,.$$

Lift to M-theory

An appropriate ansatz for N M2-branes at the origin of $\mathcal{M}_8$ is

$$ds^2 = H^{-2/3}(-dX_0^2 + dX_1^2 + dX_2^2) + H^{1/3} ds^2_{\mathcal{M}_8}$$
$$F = dX_0 \wedge dX_1 \wedge dX_2 \wedge dH^{-1}$$

The supergravity equations of motion require

$$\partial_\mu(\sqrt{g} g^{\mu\nu} \partial_\nu H) = 0$$

$\mathcal{M}_8$ was shown to be a particular hyperkahler quotient. $\mathcal{M}_8$ has $Sp(2)$ holonomy and it is a metric cone over 7-dimensional tri-Sasaki manifold. In this setup we are able to trace the M-theory circle ([Gaiotto and Jafferis, 09]) but it is very hard to solve the Killing spinor equation.

For ABJM Theory

As a comparison, the resulting M-theory configuration is a stack of N M2-branes probing a C^4/Z_k singularity. Consider the back reaction of the M2-branes, the near horizon geometry is $AdS_4 \times S^7/Z_k$. ([Aharony et al., 08])

The Duality

- The flavored ABJM theory with the Chern-Simons levels $(k, -k)$ and N_f flavor is dual to M-theory on $AdS_4 \times M_7(N_f, N_f, k)$, where the Eschenburg space $M_7(N_f, N_f, k)$ is a special 3-Sasakian manifold.

- The flavored ABJM theory with the Chern-Simons levels $(1, -1)$ and a flavor is dual to the $d = 11$ supergravity (M-theory) on $AdS_4 \times N(1, 1)$. ([Fujita,11])

- The flavored ABJM theory with the Chern-Simons levels $(k, -k)$ and k flavors is dual to the $d = 11$ supergravity (M-theory) on $AdS_4 \times N(1, 1)/Z_k$.

The Duality

- BPS Wilson loop in the fundamental representation is believed to be dual to the BPS M2-brane in the dual theory. Based on the experience in the ABJM theory, we believe the M2-brane is of worldvolume $AdS_2 \times S^1$, where $AdS_2 \subset AdS_4$, $S^1 \subset N(1,1)$. And the S^1 should be along the M-theory circle.

- The leading exponential behavior of the VEV of the Wilson loop in the strong coupling limit is captured by the regulated action of the membrane configuration.

The Duality

- As mentioned before, it is very hard to solve the Killing spinor the metric obtain in GGPT. Fortunately, mathematically we know that $N(1,1)$ is just a coset space $SU(3)/U(1)$([Page and Pope, 1984]). Using this coset description, the metric of $N(1,1)$ can be expressed in terms of a coordinate system which is more suitable for solving the Killing spinor equations.

- A price to pay: we lost the trace of M-theory circle.

- However, M-theory circle generated by a supersymmetry-preserving Killing vectors such that the configuration has the chance to preserve the largest amount of supersymmetries. We find two such Killing vectors.

3. Background and Killing Spinors

Background Metric

The metric of the background $AdS_4 \times N(1,1)$ is

$$ds^2 = R^2(\frac{1}{4}ds^2_{AdS_4} + ds^2_{N(1,1)}),$$

with

$$ds^2_{AdS_4} = \cosh^2 u(-\cosh^2 \rho dt^2 + d\rho^2) + du^2 + \sinh^2 u d\phi^2,$$

and

$$\begin{aligned} ds^2_{N(1,1)} &= \frac{1}{2}(d\alpha^2 + \frac{1}{4}\sin^2\alpha(\sigma_1^2 + \sigma_2^2) + \frac{1}{4}\sin^2\alpha\cos^2\alpha\sigma_3^2 + \frac{1}{2}(\Sigma_1 - \cos\alpha\sigma_1)^2 \\ &+ \frac{1}{2}(\Sigma_2 - \cos\alpha\sigma_2)^2 + \frac{1}{2}(\Sigma_3 - \frac{1}{2}(1+\cos^2\alpha)\sigma_3)^2) \end{aligned}$$

Background Metric

where σ_i and Σ_i are right invariant one-forms on $SO(3)$ and $SU(2)$ respectively

$$
\begin{aligned}
\sigma_1 &= \sin\phi_1 d\theta_1 - \cos\phi_1 \sin\theta_1 d\psi_1, \\
\sigma_2 &= \cos\phi_1 d\theta_1 + \sin\phi_1 \sin\theta_1 d\psi_1, \\
\sigma_3 &= d\phi_1 + \cos\theta_1 d\psi_1, \\
\Sigma_1 &= \sin\phi_2 d\theta_2 - \cos\phi_2 \sin\theta_2 d\psi_2, \\
\Sigma_2 &= \cos\phi_2 d\theta_2 + \sin\phi_2 \sin\theta_2 d\psi_2, \\
\Sigma_3 &= d\phi_2 + \cos\theta_2 d\psi_2.
\end{aligned}
$$

Flux

The volume of $N(1,1)$ of unit radius is

$$\mathrm{vol}(N(1,1)) = \frac{\pi^4}{8}.$$

Now the flux quantization gives

$$R = 2\pi l_p \left(\frac{N}{6 \cdot \mathrm{vol}(N(1,1))} \right)^{1/6} = l_p \left(\frac{2^8 \pi^2 N}{3} \right)^{1/6}.$$

The background four-form field strength is

$$H = -\frac{3}{8} R^3 \cosh^2 u \cosh \rho \sinh u dt \wedge d\rho \wedge du \wedge d\phi.$$

Vielbeins

Corresponding to the metric, the vielbeins could be chosen to be

$$
\begin{aligned}
e^{\underline{0}} &= \frac{R}{2}\cosh u \cosh \rho dt, \qquad e^{\underline{1}} = \frac{R}{2}\cosh u d\rho, \\
e^{\underline{2}} &= \frac{R}{2}du, \qquad e^{\underline{3}} = \frac{R}{2}\sinh u d\phi, \\
e^{\underline{4}} &= \frac{R}{\sqrt{2}}d\alpha, \qquad e^{\underline{5}} = \frac{R}{2\sqrt{2}}\sin\alpha\sigma_1, \\
e^{\underline{6}} &= \frac{R}{2\sqrt{2}}\sin\alpha\sigma_2, \qquad e^{\underline{7}} = \frac{R}{2\sqrt{2}}\sin\alpha\cos\alpha\sigma_3, \\
e^{\underline{8}} &= \frac{R}{2}(\Sigma_1 - \cos\alpha\sigma_1), \qquad e^{\underline{9}} = \frac{R}{2}(\Sigma_2 - \cos\alpha\sigma_2), \\
e^{\underline{\sharp}} &= \frac{R}{2}(\Sigma_3 - \frac{1}{2}(1+\cos^2\alpha)\sigma_3).
\end{aligned}
$$

The spin connection with respect to these vielbeins is obtained from the Cartan structure equation, but we refrain to report them here.

The Killing spinor equation

In terms of the vielbeins, the four-form field strength can be written as

$$H = -\frac{6}{R} e^{\underline{0}} \wedge e^{\underline{1}} \wedge e^{\underline{2}} \wedge e^{\underline{3}}.$$

The Killing spinor equation in $AdS_4 \times N(1,1)$ is

$$\nabla_{\underline{M}}\eta + \frac{1}{576}(3\Gamma_{\underline{NPQR}}\Gamma_{\underline{M}} - \Gamma_{\underline{M}}\Gamma_{\underline{NPQR}})H^{\underline{NPQR}}\eta = 0.$$

Our convention about the product of eleven Γ matrices is

$$\Gamma_{\underline{0123456789\sharp}} = 1.$$

The Integrability Condition

The integrability condition gives

$$C^{\underline{abcd}}\Gamma_{\underline{ab}}\eta = 0,$$

where $C^{\underline{abcd}}$ is the Weyl tensor of $N(1,1)$.

It gives the projection condition

$$\Gamma_{\underline{4567}}\eta = -\eta.$$

Killing Spinor

The solutions of the above Killing spinor equations are

$$\eta = e^{\frac{\phi_2}{2}(\Gamma_{\underline{47}}+\Gamma_{\underline{89}})} e^{\frac{\theta_2}{2}(\Gamma_{\underline{46}}-\Gamma_{\underline{8\sharp}})} e^{\frac{\psi_2}{2}(\Gamma_{\underline{47}}+\Gamma_{\underline{89}})} e^{-\frac{u}{2}\Gamma_{\underline{2}}\hat{\Gamma}} e^{-\frac{\rho}{2}\Gamma_{\underline{1}}\hat{\Gamma}} e^{-\frac{t}{2}\Gamma_{\underline{0}}\hat{\Gamma}} e^{\frac{\phi}{2}\Gamma_{\underline{23}}}\eta_0,$$

with η_0 satisfying the following projection conditions

$$\Gamma_{\underline{4567}}\eta_0 = -\eta_0, \qquad (\Gamma_{\underline{58}} + \Gamma_{\underline{69}} + \Gamma_{\underline{7\sharp}} - \Gamma_{\underline{4}}\hat{\Gamma})\eta_0 = 0.$$

There are 12 supercharges . This is consistent with the duality with 3d $\mathcal{N} = 3$ superconformal field theory.

4. BPS M2-branes

Killing Vectors

The tangent vector of the M-theory circle should be a supersymmetry-preserving Killing vector $\hat{K}$

$$\mathcal{L}_{\hat{K}}\eta \equiv \hat{K}^{\underline{M}}\nabla_{\underline{M}}\eta + \frac{1}{4}(\nabla_{\underline{M}}\hat{K}_{\underline{N}})\Gamma^{\underline{MN}}\eta = 0$$

We find the following two Killing vectors

$$\begin{aligned} \hat{K}_1 &= \partial_{\psi_1} \\ \hat{K}_2 &= \partial_{\phi_1} + \partial_{\phi_2} \end{aligned}$$

Probe M2-branes

The bosonic part of the $M2$-brane action is

$$S_{M2} = T_2 \left(\int d^3\xi \sqrt{-\mathrm{det} g_{mn}} - \int P[C_3] \right)$$

The gauge choice for the background 3-form gauge potential C_3 is

$$C_3 = \frac{R^3}{8}(\cosh^3 u - 1) \cosh \rho dt \wedge d\rho \wedge d\phi$$

Membrane equation of motion is

$$\begin{aligned} & \frac{1}{\sqrt{-g}} \partial_m \left(\sqrt{-g} g^{mn} \partial_n X^{\underline{N}} \right) G_{\underline{MN}} + g^{mn} \partial_m X^{\underline{N}} \partial_n X^{\underline{P}} \Gamma^{\underline{Q}}_{\underline{NP}} G_{\underline{QM}} \\ = \quad & \frac{1}{3! \sqrt{-g}} \epsilon^{mnp} H_{\underline{MNPQ}} \partial_m X^{\underline{N}} \partial_n X^{\underline{P}} \partial_p X^{\underline{Q}} \end{aligned}$$

BPS Condition

The supercharges preserved by the M2-brane are determined by the following equation

$$\Gamma_{M2}\eta = \eta,$$

with

$$\Gamma_{M2} = \frac{1}{\sqrt{-g}}\partial_\tau X^{\underline{M}}\partial_\xi X^{\underline{N}}\partial_\sigma X^{\underline{P}} e_{\underline{M}}^{\underline{A}} e_{\underline{N}}^{\underline{B}} e_{\underline{P}}^{\underline{C}} \Gamma_{\underline{ABC}}$$

First Ansatz

The first ansatz for M2-brane is

$$t = \tau, \quad \rho = \xi, \quad \psi_1 = \sigma, \quad \sigma \in [0, 2\pi]$$

the S^1 is generated by the Killing vector $\hat{K}_1$.

The equations of motion give the constraints that

$$(u, \alpha) = (0, 0),$$

or

$$(u, \alpha, \theta_1) = (0, \frac{\pi}{2}, 0),$$

or

$$(u, \alpha, \theta_1) = (0, \frac{\pi}{2}, \frac{\pi}{2}).$$

On-shell action of M2-brane

$$S_{M2} = \frac{T_{M2}R^3}{4}\int d^3\sigma \cosh^2 u \cosh\rho \times \left[\frac{1}{256}(45 + 20\cos 2\alpha - \cos 4\alpha - 8\cos 2\theta_1 \sin^4\alpha)\right]^{1/2},$$

Switch to the Euclidean AdS_4 with the following metric

$$ds_4^2 = \frac{1}{4}(\cosh^2 u(d\rho^2 + \sinh^2\rho d\psi^2) + du^2 + \sinh^2 u d\phi^2).$$

After adding boundary terms to regulate the action, we get

$$S_{M2} = -4\pi\sqrt{\frac{N}{3}\left(\frac{1}{256}(45 + 20\cos 2\alpha - \cos 4\alpha - 8\cos 2\theta_1 \sin^4\alpha)\right)}$$

On-shell action of M2-brane

For the M2-brane put at $\alpha = 0$, $(\alpha, \theta_1) = (\pi/2, 0)$, or $(\alpha, \theta_1) = (\pi/2, \pi/2)$, the on-shell action is repsectively

$$-2\pi\sqrt{\frac{N}{3}}, \quad -\pi\sqrt{\frac{N}{3}}, \quad -\pi\sqrt{\frac{2N}{3}}.$$

The BPS M2-brane

$$\Gamma_{M2} = \frac{1}{\sqrt{-g}} \frac{R^2}{4} \cosh^2 u \cosh \rho \Gamma_{\underline{01}} \tilde{\Gamma},$$

with

$$\begin{aligned}
\tilde{\Gamma} &= -\frac{R}{2\sqrt{2}} \sin\alpha \cos\phi_1 \sin\theta_1 \Gamma_{\underline{5}} + \frac{R}{2\sqrt{2}} \sin\alpha \sin\phi_1 \sin\theta_1 \Gamma_{\underline{6}} \\
&+ \frac{R}{2\sqrt{2}} \sin\alpha \cos\alpha \cos\theta_1 \Gamma_{\underline{7}} + \frac{R}{2} \cos\alpha \cos\phi_1 \sin\theta_1 \Gamma_{\underline{8}} \\
&- \frac{R}{2} \cos\alpha \sin\phi_1 \sin\theta_1 \Gamma_{\underline{9}} - \frac{R}{4}(1 + \cos^2\alpha) \cos\theta_1 \Gamma_{\underline{\sharp}}.
\end{aligned}$$

The BPS M2-brane

Only the M2 branes put at

$$(u, \alpha, \theta_1, \theta_2) = (0, 0, 0, 0),$$

or

$$(u, \alpha, \theta_1, \theta_2) = (0, \pi/2, 0, 0)$$

are BPS. They are all $1/3$-BPS. Among them, the 1/3-BPS M2-brane put at

$$(u, \alpha, \theta_1, \theta_2) = (0, 0, 0, 0),$$

gives the dominant contributions to the VEV of the $1/3$-BPS Wilson loops. The holographic prediction for the VEV of this loop is

$$< W > \sim \exp(2\pi \sqrt{\frac{N}{3}})$$

Second Ansatz

The second ansatz for M2-brane is

$$t=\tau,\quad \rho=\xi,\quad \phi_1=2\sigma,\quad \phi_2=\phi_0+2\sigma,$$

with $\sigma\in[0,2\pi]$ and ϕ_0 a constant. This corresponds to the case that S^1 is generated by $\hat{K}_2$.

The equations of motion require

$$u=0,\qquad \alpha=\pi/2.$$

The regulated on-shell action

$$S=-2\pi\sqrt{\frac{N}{3}}.$$

The BPS M2-brane

$$\Gamma_{M2} = \frac{1}{\sqrt{-g}} \frac{R^2}{4} \cosh^2 u \cosh \rho \Gamma_{\underline{01}} \tilde{\Gamma},$$

with

$$\tilde{\Gamma} = \frac{R}{\sqrt{2}} \sin\alpha \cos\alpha \Gamma_{\underline{7}} + \frac{R}{2} \sin^2 \alpha \Gamma_{\underline{\sharp}}.$$

only the M2-brane put at $(u, \alpha, \theta_2) = (0, \pi/2, 0)$ is BPS, and it is 1/3-BPS.

The prediction for the VEV of the dual $1/3$-BPS Wilson loop is

$$< W > \sim \exp(2\pi \sqrt{\frac{N}{3}})$$

Generalize to $AdS_4 \times N(1,1)/Z_k$

If we consider M-theory on $AdS_4 \times N(1,1)/Z_k$

- the flux quantization now gives

$$R = 2\pi l_p \left(\frac{N}{6\mathrm{vol}(N(1,1)/Z_k)} \right)^{1/6} = l_p \left(\frac{2^8 \pi^2 N k}{3} \right)^{1/6},$$

- the length of the σ direction of the M2-brane worldvolume is reduced by a factor $1/k$.

- Taking these two effects into account,

$$< W > \sim \exp(2\pi \sqrt{\frac{N}{3k}}).$$

- This prediction matches exactly with the result derived from the supersymmetric localization method.

Discussion

- Decompose the Killing spinors in $AdS_4 \times N(1,1)$ as

$$\eta = \epsilon \otimes \alpha$$

- Decompose the projection condition

$$\Gamma_{M2}^{AdS_4} \epsilon_\pm = \pm \epsilon_\pm, \quad \Gamma_{M2}^{N(1,1)} \alpha_\pm = \pm \alpha_\pm$$

- $\epsilon_+ \otimes \alpha_+$ or $\epsilon_- \otimes \alpha_-$

  the dimension of the solution space should be $4n$ with n an integer.

- No M2-brane with more supersymmetries than 1/3-BPS!

Conclusion and Discussion

- We discussed the holographic dual of BPS Wilson loop operators in flavored ABJM theory.
- We found the 1/3-BPS membrane configurations and show the regulated action of the membrane is exactly consistent with the strong coupling behavior of the VEV of the Wilson loop.
- Which 1/3-BPS membrane corresponds to the 1/3-BPS Wilson loop?
- We conjecture that there are no BPS Wilson loops preserving more than 1/3 supersymmetries in $\mathcal{N} = 3$ Chern-Simons-matter theories.

Thank You !