Neutrino physics using quantum coherence

M. Yoshimura @ Okayama University, Japan
• Introduction: recent developments of neutrino physics

• Quantum coherence: an example of adiabatic Raman excitation

• Coherent quantum synchrotron

• Macro-coherent amplification of atomic process: RENP
Introduction

• What have been, and have not been, determined in neutrino experiments so far

• Remaining important questions on neutrino properties to probe physics beyond the standard theory and cosmology
Achievements of neutrino oscillation experiments

- Atomospheric, solar, accelerator and reactor neutrinos have been detected in water Cerenkov and liquid scintillator detectors placed underground
- Oscillations among 3 neutrinos have been observed
- Neutrinos have finite masses and are involved in weak interaction in their mixture

\[
\begin{align*}
    s_{12}^2 &= 0.31, \quad s_{23}^2 = 0.42, \quad s_{13}^2 = 0.024, \\
    \Delta m_{21}^2 &= 7.5 \times 10^{-5} \text{eV}^2, \quad |\Delta m_{31}^2| = 2.47 \times 10^{-3} \text{eV}^2.
\end{align*}
\]
Present status of neutrino physics

- Oscillation experiments
  - Finite mass
  - Flavor mixing
  - Only mass-squared difference can be measured.

\[ \nu_e |U_{e1}|^2 \]
\[ \nu_\mu |U_{\mu1}|^2 \]
\[ \nu_\tau |U_{\tau1}|^2 \]

\[ U = VP, \]
\[ \begin{bmatrix}
  c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
  -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
  s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{bmatrix}, \]

with \( c_{ij} = \cos \theta_{ij} \) and \( s_{ij} = \sin \theta_{ij} \). The diagonal unitary matrix \( P \) may be expressed by

\[ P = \text{diag}(1, e^{i\alpha}, e^{i\beta}), \]

\( \Delta m^2_{\text{atm}} = (50 \text{meV})^2 \) or \( \Delta m^2_{\text{atm}} = (10 \text{meV})^2 \)

(Mass)

\( \Delta m^2_{\text{sol}} = (50 \text{meV})^2 \)

\( \Delta m^2_{\text{sol}} = (10 \text{meV})^2 \)

Normal (NH)

Inverted (IH)

China, and Korea caught up
Important questions left in neutrino physics

- Absolute mass scale and the smallest mass (oscillation experiments are sensitive to mass squared differences alone)
- Majorana vs Dirac distinction
- CPV phase （Majorana case has 2 extra phases）

\[ \alpha, \beta, \delta \text{ (KM - type)} \]

These are relevant to explanation of matter-antimatter imbalance of universe.

We wish to experimentally achieve all of these goals.

SPAN project (discussed in the last part of this talk)

Our Okayama group has proposed an entirely new method using atoms, initiated R&D works and succeeded in establishing the huge rate enhancement in QED process, along with theoretical works.
Present status of nuclear target experiments

1. Majorana nature: neutrino-less double beta decay

2. Absolute mass

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Significance of Majorana neutrinos

• Suppose that neutral leptons consist of 4 components like all other quarks and leptons, the ordinary massless neutrino and the other 2-component neutral lepton having a much larger mass of Majorana-type than the Fermi scale

• -> Seesaw mechanism with a Dirac-type coupling via Higgs $\frac{m^2}{M}$

• Plausible scenario of lepto-genesis

  Heavy Majorana decay responsible for generation of lepton asymmetry, being converted to baryon asymmetry via strong electroweak B, L violation keeping B-L conserved.

  Prerequisite: ordinary neutrinos are also Majorana.

  New CPV sources related to heavy partners of mass $>>$ Fermi scale
Lepto-genesis

• Leading theory to explain the matter-antimatter imbalance of our universe
• Prerequisite: lepton number violation or Majorana type of mass, CP violation
• Sensitivity to low energy parameters Davidsson-Ibarra, NPB648, 345(2003)

CP asymmetry in leptogenesis

\[
\approx \frac{3y_1^2}{4\pi} \left( -2\left(\frac{m_3}{m_2}\right)^3 s_{13}^2 \sin 2(\delta + \alpha - \beta) + \frac{m_1}{m_2} \sin(2\alpha) \right)
\]

+ (high energy phases inaccessible in low energy experiments)

Ours are sensitive to $\alpha$, $\beta - \delta$; the same as in lepto-genesis
Majorana vs Dirac equations: chirally projected solutions

Majorana eq.: particle=antiparticle

\[(i\partial_t - i\vec{\sigma} \cdot \vec{\nabla})\varphi = i\hbar\sigma_2 \varphi^*\]

\[\varphi_{\vec{p},h}(x) = c(\vec{p},h)e^{-\imath p \cdot x} u(\vec{p},h) + c^\dagger(\vec{p},h)e^{\imath p \cdot x} \sqrt{\frac{E_p + h p}{E_p - h p}} \left( -i\sigma_2 \right) u^*(\vec{p},h),\]

\[u(\vec{p},h) = \frac{1}{2\sqrt{p E_p(p + h p_3)}} \begin{pmatrix} p + h p_3 \\ h(p_1 + i p_2) \end{pmatrix}.\]

2 neutrino wave functions are anti-symmetrized

Dirac eq.: degenerate 2 Majorana

\[(i\partial_t - i\vec{\sigma} \cdot \vec{\nabla})\varphi = m \chi, \quad (i\partial_t + i\vec{\sigma} \cdot \vec{\nabla})\chi = m \varphi\]

2-component in weak process involved

\[\psi_D = (1 - \gamma_5) \psi / 2\]

\[\psi_D = b(\vec{p},h)e^{-\imath p \cdot x} u(\vec{p},h) + d^\dagger(\vec{p},h)e^{\imath p \cdot x} \sqrt{\frac{E_p + h p}{E_p - h p}} \left( -i\sigma_2 \right) u^*(\vec{p},h)\]

Particle annihilation

Anti-particle creation

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Coherent quantum beam
Neutrino pair beam and fundamental oscillation experiments

By N. Sasao and M. Yoshimura
Okayama University

References

Neutrino pair and gamma beams from circulating excited ions

arXiv: 1505.07572v2 [hep-ph]

Determination of CP violation parameter using neutrino pair beam

Uploaded to arXiv

Conventional neutrino sources: pi-, mu-, beta-decay

We shall use de-excitation of atoms/ions, producing pairs of neutrino and anti-neutrino.
Schwinger’s formula for synchrotron radiation

\[
\frac{d\mathcal{P}}{d\omega} = \frac{\omega^2}{4\pi^2} \Re \left( \int d\Omega dt' (\bar{j}(\vec{r}, t) \cdot (\bar{j}'(\vec{r}', t') - \rho(\vec{r}, t) \rho(\vec{r}', t')) e^{-i\omega(t' - t - \vec{k} \cdot (\vec{r}' - \vec{r})/\omega)} \right)
\]

\[
H_{int} = \sqrt{(\vec{p} - e\vec{A}_{ext})^2 + m^2 + e\Phi_{ext}}
\]

Fourier component of orbit function
In Green’s function

- Main results
- Exponential cutoff, both in energy and angular directions, only to keV region available
- Phase integral: same sign phase adds up
New features for excited ions

- Input of excitation energy in phase integral, leading to stationary points (positive and negative phases cancelling)

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ABSTRACT

We propose a new method of producing neutrino pair beam that consists of a mixture of neutrinos and anti-neutrinos of all flavors. The idea is based on a coherent neutrino pair emission from excited ions in circular motion. High energy gamma ray much beyond the keV range may also be produced by a different choice of excited level.
Intuitive understanding

• A kind of non-linear resonance: orbital energy balanced against internal ion energy, giving non-linear resonance oscillation. Its width around the stationary point gives a broad resonance-like behavior.

• Key concept for its success: quantum coherence typically realized by atomic system under laser irradiation, but may persist without phase relaxation. Both one-particle system and many-particle system are conceivable.

Simple example of quantum coherence: adiabatic Raman process
Preparation of initial coherence – Adiabatic Raman -

Two laser fields irradiates p-H2

Two photon Rabi frequency \( \Omega_{ge} \approx \frac{\Omega_g \Omega_e}{\Delta} \)

\( |g\rangle \) and \( |e\rangle \) are mixed with an angle

\[ \tan \theta \approx \frac{\Omega_{ge}}{\delta} \]

Non-degenerate Superposition States:

\[ |+\rangle = \cos \frac{\theta}{2} |g\rangle + e^{-i\varphi} \sin \frac{\theta}{2} |e\rangle \]

\[ |-\rangle = \cos \frac{\theta}{2} |g\rangle - e^{-i\varphi} \sin \frac{\theta}{2} |e\rangle \]

Coherence between \( |e\rangle \) and \( |g\rangle \)

\[ |\rho_{eg}| = \frac{1}{2} \sin \theta \]
\[ |\pm\rangle = |g\rangle \quad \theta = 0 \]

\[ |\pm\rangle = \cos \frac{\theta}{2} |g\rangle \pm e^{-i\varphi} \sin \frac{\theta}{2} |e\rangle \quad \theta \neq 0 \]

\[ |\pm\rangle = \frac{1}{\sqrt{2}} |g\rangle \pm e^{-i\varphi} \frac{1}{\sqrt{2}} |e\rangle \quad \theta = \frac{\pi}{2} \]

\[ |\pm\rangle = \cos \frac{\theta}{2} |g\rangle \pm e^{-i\varphi} \sin \frac{\theta}{2} |e\rangle \quad \theta \neq 0 \]

\[ \theta = 0 \quad |\pm\rangle = |g\rangle \]
How to calculate neutrino pair emission rate

- Semi-classical approximation: classical ion CM motion and quantum internal state

- Spin current dominance from valence electron transition

Source current

\[ J_{eg}^\alpha(x) = S^\alpha \frac{1}{\sqrt{\gamma}} \int \, dt \rho_{eg}(t) \delta^{(4)}(x - x_A(t)) \]

Hamiltonian

\[ H_w = \int d^3 x \frac{G_F}{\sqrt{2}} J_{eg}^\alpha(x) \cdot \sum_{i=e,\mu,\tau} \nu_i^\dagger(x) \sigma_\alpha \nu_i(x) \]

Coherence

\[ \rho_{eg}(t) = \rho_{eg}(0) \exp\left[-(i\epsilon_{eg} + \frac{1}{T_2}) \frac{t}{\gamma}\right] \]

Spin factor

\[ (S^\alpha) = (\gamma \vec{\beta} \cdot \vec{S}_e, \vec{S}_e + \frac{\gamma^2}{\gamma + 1} (\vec{\beta} \cdot \vec{S}_e) \vec{\beta}) \sim \gamma (\vec{\beta} \cdot \vec{S}_e, (\vec{\beta} \cdot \vec{S}_e) \vec{\beta}) \]

\[ \rightarrow \quad \gamma^2 \frac{S_e^2}{3} \left( 1 + \frac{1}{3} \frac{p_1 \cdot p_2}{E_1 E_2} - \frac{m_1 m_2}{2E_1 E_2} \delta_M \right) \]

Ion trajectory

\[ \vec{r}_A(t) = \rho \left( \sin \frac{vt}{\rho}, 1 - \cos \frac{vt}{\rho}, 0 \right) \]
Difference from usual synchrotron radiation

Airy integral \( Ai(\epsilon - x) = \frac{1}{\pi} \int_0^\infty \cos \left( - (\epsilon - x)t + \frac{1}{3} t^3 \right) \)

\( \epsilon > 0 \) quantum mixture case

Cancellation of positive and negative phases
Energy input leads to resonance-like behavior
Energy spectrum

Our gamma beam

Synchrotron radiation
\[
\frac{d^4 \Gamma_{ij}}{dE_1 dE_2 d\Omega_1 d\Omega_2} = \frac{4G_F^2}{2^{7/4} \cdot 3\sqrt{3\pi}(2\pi)^6} |C_{ij}|^2 S_e^2 N |\rho_{eg}(0)|^2 \sqrt{\rho} E_1^2 E_2^2 F^{-1/4},
\]

\[
F = (E_1 + E_2)(\frac{\epsilon_{eg}}{\gamma} - \frac{E_1 + E_2}{2\gamma^2}) - \frac{1}{2}(E_1^2 \psi_1^2 + E_2^2 \psi_2^2) - \frac{E_1 E_2}{2}(\theta_1 - \theta_2)^2 - \frac{\epsilon_{eg}}{2\gamma}(E_1 \theta_1^2 + E_2 \theta_2^2).
\]

\[
\Delta \psi = O \left( \frac{1}{\gamma} \sqrt{\frac{2(E_m - 2E)}{E}} \right), \quad \Delta \theta = O \left( \sqrt{\frac{E_m - 2E}{E_m}} \right), \quad E_m = 2\gamma \epsilon_{eg}.
\]

\[
\Delta |\theta_1 - \theta_2| < O \left( \frac{1}{\gamma} \sqrt{\frac{(E_1 + E_2)(E_m - E_1 - E_2)}{E_1 E_2}} \right) \quad 100 \mu\text{radian} \quad 10^4 / \gamma
\]

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Differential and total production rates

\[ \Gamma = \sum_i \Gamma_i \sim 3.1 \times 10^{21} \text{Hz} \left( \frac{\rho}{4 \text{km}} \right)^{1/2} \frac{S_e^2 N |\rho_{eg}(0)|^2}{10^8} \left( \frac{\gamma}{10^4} \right)^4 \left( \frac{\epsilon_{eg}}{50 \text{keV}} \right)^{11/2}, \]

with \( E_m = 2\epsilon_{eg}\gamma = 1 \text{GeV} \frac{\epsilon_{eg}}{50 \text{keV}} \frac{\gamma}{10^4} \).
Scaling law

\[ \Gamma \propto \frac{1}{\gamma} \cdot N |\rho_{eg}(0)|^2 \cdot G_F^2 E_m^5 \cdot \sqrt{\rho \epsilon_{eg}} \propto \gamma^{4.11/2} \epsilon_{eg} \]

Equi-rate, equi-energy curves
Competition with QED photon emission

an example of He-like ion, $\text{Pb}^{80+}$

$|e\rangle = \left((2s)(1s)\right)^3_{j=1} \text{ (a spin triplet state described in } jj \text{ coupling scheme)}$

level spacing $\epsilon_{eg} \sim 70 \text{ keV}$

M1 photon emission \hspace{1cm} \Gamma_{\gamma} = \gamma_{M1} N \rho_{ee}(0)$

$\gamma_{M1} \sim 3.4 \times 10^{13} \text{ Hz}$

\[ -\rightarrow \quad |\rho_{eg}(0)|^2 > O(0.1) \rho_{ee}(0) \left( \frac{\gamma}{10^4} \right)^{-4} \]
Neutrino oscillation
Detection of neutrino pair away from synchrotron

\[ \sum_b \left( \frac{G_F}{\sqrt{2}} \right)^2 \bar{v}_{a \alpha} (1 - \gamma_5) l_a J^\alpha \bar{c} \gamma_\beta (1 - \gamma_5) \nu_c (J^\beta)^\dagger \langle \bar{a} | e^{-iH_L} | b \rangle \langle c | e^{-iH_L} | b \rangle \mathcal{P}_{bb}(1, 2) \]

\[ H = U \begin{pmatrix} \frac{m_1^2}{2E} & 0 & 0 \\ 0 & \frac{m_2^2}{2E} & 0 \\ 0 & 0 & \frac{m_3^2}{2E} \end{pmatrix} U^\dagger \mp \sqrt{2} G_F n_e \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \]

• Single neutrino detection eliminates oscillation pattern

\[ \langle c | e^{-iH_L} | b \rangle = \sum_i V_{ci}^* V_{bi} e^{-i\lambda_i L}, \quad \langle \bar{a} | e^{-iH_L} | \bar{b} \rangle = \sum_i \bar{V}_{ai} \bar{V}_{bi} e^{-i\tilde{\lambda}_i L}, \]

\[ \sum_b \langle \bar{a} | e^{-iH_L} | \bar{b} \rangle \langle c | e^{-iH_L} | b \rangle c_b = \sum_{ij} V_{ci}^* \bar{V}_{aj} \xi_{ij} e(\lambda_j, \lambda_i), \quad (c_b) = \frac{1}{2} (1, -1, -1), \]

\[ \sum_c \left| \sum_{ij} V_{ci}^* \bar{V}_{\mu j} \xi_{ij} e(\lambda_j, \lambda_i) \right|^2 = \sum_{ijkl} \sum_c V_{ci}^* V_{ck} \bar{V}_{\mu j} \bar{V}_{\mu l}^* \xi_{ij} \xi_{kl}^* e(\lambda_j, \lambda_i) e^*(\lambda_l, \lambda_k) \]

\[ = \sum_j \bar{V}_{\mu j} \bar{V}_{\mu j} e(\lambda_j, \lambda_i) e^*(\lambda_l, \lambda_i) \sum_i \xi_{ij} \xi_{il}^* = \frac{1}{4} \sum_{jl} \bar{V}_{\mu j} \bar{V}_{\mu l} e(m_j, m_i) e^*(m_l, m_i) \delta_{jl} = \frac{1}{4}, \]
Short baseline experiments

\[ \sqrt{2G_F n_e L} \sim 1 \times \frac{n_e}{1.4 \times 6 \times 10^{23}\text{cm}^{-3}} \frac{L}{1860\text{km}} \]

\[ \left| \sum_{ij} V_{ci}^{*} V_{\mu j}^{*} \xi_{ij} e(\bar{\lambda}_j, \lambda_i) \right|^2 \frac{d^4 \Gamma}{dE_1 dE_2 d\Omega_1 d\Omega_2} \frac{d^2 \sigma}{dE_+ d\sin \psi_+} \frac{d^2 \sigma}{dE_- d\sin \psi_-} \]

Plotted quantities

\[ P_{\bar{a}c} = \left| \sum_{ij} U_{ci}^{*} U_{aj} \xi_{ij} e^{-iL(m_j^2/E_1 + m_i^2/E_2)} \right|^2 \]

\[ A(\delta) = \frac{d\Gamma(\delta : G_F) - d\Gamma(-\delta : -G_F)}{d\Gamma(\delta : G_F) + d\Gamma(-\delta : -G_F)} . \]

10 ~ 100 mHz for a 100 kt class

\[ \sigma n_{NL} \sim 10^{-11} \sim 10^{-10} \text{ for a single detection} \]
Figure 1: Oscillation pattern given by $P_{\mu e}$ of eq.(13) (in solid black) and asymmetry (in dashed red) at various distances for $\bar{\nu}_\mu \nu_e$ CC double events. $\delta = \pi/4$, $E_{\bar{\nu}_\mu} = 500$MeV, $E_{\nu_e} = 5$MeV.
How short baseline exp. became effective

• Two factors of L/E, one 10km/10MeV instead of 500km/500 MeV giving the same oscillation pattern
Figure 2: Asymmetry at various distances for $\bar{\nu}_\mu \nu_e$ CC double events. $\delta = \pi/4$, $E_{\bar{\nu}_\mu} = 500\text{MeV}$ and $E_{\nu_e} = 5\text{MeV}$ in solid black, $50 \text{MeV}$ in dashed red, and $500 \text{MeV}$ in dash-dotted blue (much smaller than the other two cases). NH of smallest mass zero is assumed.
Figure 3: Asymmetry vs electron neutrino energy for $\bar{\nu}_\mu \nu_e$ CC double events. $E_{\bar{\nu}_\mu} = 500$MeV and $\delta = \pi/6$ in solid black, $\pi/4$ in dashed red, and $\pi/2$ in dash-dotted blue. NH of smallest mass zero is assumed.
Figure 4: Asymmetry vs CPV $\delta$ for $\bar{\nu}_\mu \nu_e$ CC double events. $E_{\bar{\nu}_\mu} = 500\text{MeV}, E_{\nu_e} = 5\text{MeV}$ at 10 km away in solid black, 50km in dashed red, and 100km in dash-dotted blue. NH of smallest mass zero is assumed.
Figure 5: NH vs IH distinction at 10 km away from the synchrotron, given by asymmetric energy combinations: $P_{\mu e}$ is plotted for $E_{\nu_\mu} = 500, 200\text{MeV}$, fixed and variable $E_{\nu_e}$. NH in blacks, 500 MeV in solid and 200 MeV in dotted lines, and IH in colored, 500 MeV, in dashed red and 200 MeV in dash-dotted blue. $\delta = 0$. 
Figure 6: Effects of earth matter on oscillation patterns given by $P_{\bar{\mu}e}$. Oscillation without matter effect in solid black, with matter effect of earth-model made of pure SiO$_2$ in dashed red, and its electron number density 20% made larger in dash-dotted blue. $\delta = \pi/4$ and energy combination $(E_{\bar{\nu}_\mu}, E_{\nu_e}) = (500$ MeV, 50MeV) for $\bar{\mu}e$ events.
Features of pair beam

- Double detection required for oscillation experiments
- Short baseline experiments recommended to avoid the earth matter effect
- Excellent opportunity for $\Delta$ and NH/IH
Conclusion on neutrino pair beam

• Excited ions with a high coherence can produce mixture of neutrino pairs of high energies and high intensities, with a good collimation.

• Their short baseline oscillation experiments can provide a tool of high precision CPV parameter determination and NH/VH distinction.
Macro-coherent RENP
Collaborators

Staff:
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Students:
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 Relevant atomic process to us
Radiative Emission of Neutrino Pair (RENP) from metastable atomic levels

• Process undoubtedly existing in standard theory, assuming finite neutrino masses

• Possible to amplify otherwise small rates by developing macro-coherence of a twin process
Expected RENP rate

\[ I(\omega) \]

\[ \Gamma_{\gamma 2\nu}(\omega,t) = \Gamma_0 I(\omega) \eta_\omega(t) \]

Overall rate

spectral factor (\nu-parameter)

activity factor in target (macroscopic pol. x field)

Weak interaction process in atomic transition

Neutral current

\[ \nu_i \]

\[ \nu_j \]

\[ |e\rangle \]

\[ |g\rangle \]

\[ \nu \]

Neutrino pair from nucleus

Large rate enhancement for heavy atoms

\[ \Gamma_0 \sim 40 \text{Hz} \cdot \left( \frac{n}{7 \times 10^{19} \text{ cm}^{-3}} \right)^3 \cdot \left( \frac{V}{10^2 \text{ cm}^3} \right) \]
M/D distinction and CPV phases

• M-case has interference terms caused by identical particle effect due to the Pauli principle, hence rates differ from D-case

• Majorana CPV phases arise from anti-symmetrized 2 neutrino wave functions.

Access to independent phase combination, unlike its linear combination in neutrino-less double beta decay

\[
\left| \sum_{i} m_i U_{ei}^2 \right|^2 = m_3^2 s_{13}^4 + m_2^2 s_{12}^4 c_{13}^4 + m_1^2 c_{12}^4 c_{13}^4 + 2m_1m_2 s_{12}^2 c_{12}^2 c_{13}^4 \cos(2\alpha) \\
+ 2m_1m_3 s_{13}^2 c_{12}^2 c_{13}^2 \cos(2(\beta - \delta)) + 2m_2 m_3 s_{13}^2 s_{12}^2 c_{13}^2 \cos 2(\alpha - \beta + \delta),
\]
Majorana phase dependence

- Pair emission current at cross thresholds

\[
\begin{align*}
\langle (ip_1 h_1, jp_2 h_2) | j_\nu | 0 \rangle &= \xi_i^* \xi_j e^{i(p_1 + p_2) \cdot x} v_1^\dagger \sigma u_2 - \xi_i^* \xi_j e^{i(p_1 + p_2) \cdot x} v_2^\dagger \sigma u_1 \\
&= e^{i(p_1 + p_2) \cdot x} \left( i \Im \xi_i^* \xi_j (v_1^\dagger \sigma u_2 + v_2^\dagger \sigma u_1) + \Re \xi_i^* \xi_j (v_1^\dagger \sigma u_2 - v_2^\dagger \sigma u_1) \right)
\end{align*}
\]

\[
\xi_i^* \xi_j = U_{ei}^* U_{ej} = c_{ij}^{(0)}, \quad U_{e1} = c_{12} c_{13}, \quad U_{e2} = s_{12} c_{13} e^{i\alpha}, \quad U_{e3} = s_{13} e^{i\beta}
\]

Unless \((v_1^\dagger \sigma u_2 + v_2^\dagger \sigma u_1)\) and \((v_1^\dagger \sigma u_2 - v_2^\dagger \sigma u_1)\) are orthogonal, T-reversal violation \(\propto \Im \xi_i^* \xi_j \Re \xi_i^* \xi_j\) can be measured, and all Majorana phases \(\alpha, \beta\) are measurable. Non-orthogonality holds for \(i \neq j\), or \(m_i \neq m_j\).

\[
\cos(2\alpha), \quad \cos 2(\beta - \delta), \quad \text{at (12), (13), (23) thresholds}
\]

Unlike its linear combination in neutrino-less double beta

\[
\left| \sum_i m_i U_{ei}^2 \right|^2 = m_{3}^2 s_{13}^4 + m_{2}^2 s_{12}^4 c_{13}^4 + m_{1}^2 c_{12}^4 c_{13}^4 + 2m_{1} m_{2} s_{12}^2 c_{12}^2 c_{13}^4 \cos(2\alpha)
\]

\[
+ 2m_{1} m_{3} s_{13}^2 c_{12}^2 c_{13}^2 \cos 2(\beta - \delta) + 2m_{2} m_{3} s_{13}^2 s_{12}^2 c_{13}^2 \cos 2(\alpha - \beta + \delta),
\]
Rate amplification by macroscopic coherence

• Super-radiance coherent volume (Dicke)
  – In case of SR, coherent volume is proportional to $\lambda^2 L$.
  – Phase decoherence time ($T_2$) must be longer than $T_{SR}$

$$\text{Rate} \propto \left| \sum_j e^{i \vec{k} \cdot \vec{r}_j} M_{\text{atm}} \right|^2 \propto N^2 \quad \left( \text{for } |r_j - r_i| \leq \lambda \right)$$

• For a process with plural outgoing particles
  - Phase matching condition (momentum conservation) is satisfied.
  - Coherent volume is not limited by $\lambda$, can be macroscopic.

$$\text{Rate} \propto \left| \sum_j e^{i (\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \cdot \vec{r}_j} M_{\text{atm}} \right|^2 \propto N^2 \quad \left( \text{for } \vec{k}_1 + \vec{k}_2 + \vec{k}_3 = 0 \right)$$
Superradiance: 2 level and 1 photon case

Rate enhanced by N

Delayed enhanced signal accompanied by ringing

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Paired Super-Radiance (PSR) important to develop large rates for RENP (also to prove the principle of macro-coherence)

- Macro-coherent amplification
  - A new type of coherent phenomena
  - Should be established experimentally
- Two photon emission process

- Paired Super-Radiance
  - QED instead of weak process
  - Good experimental signature; i.e. back-to-back radiations with same color.

\[ |e\rangle \rightarrow |g\rangle + \gamma + \gamma \]
Some details on simulations of macro-coherence development
Effective 2-level model for trigger and medium evolution

2 level interaction with field

\[ \frac{d}{dt} \begin{pmatrix} c_e \\ c_g \end{pmatrix} = -i \mathcal{H} \begin{pmatrix} c_e \\ c_g \end{pmatrix}, \quad \mathcal{H} = 2 \begin{pmatrix} \mu_{ee} & 2e^{i\epsilon_{eg}} \mu_{ge} \\ 2e^{-i\epsilon_{eg}} \mu_{ge} & \mu_{gg} \end{pmatrix} E^2 \]

Stark shifts: 2 × 2 Hamiltonian

**Ba**

\[
\begin{pmatrix}
6.0 & 2.1 \\
2.1 & 16
\end{pmatrix}
\text{GHz} \frac{|E|^2}{10^6 \text{Wmm}^{-2}}
\]

\[ \mu_{ee} = 2 \sum_j \frac{d_{je}^2 E_{je}}{E_{je}^2 - \omega^2}, \quad \mu_{gg} = 2 \sum_j \frac{d_{jg}^2 E_{jg}}{E_{jg}^2 - \omega^2}, \quad d_{ij} = \sqrt{3\pi \gamma_{ij} \frac{E_{ij}^3}{E_{ij}^2}} \]

**Yb**

\[
\begin{pmatrix}
-1.2 \times 10^{-7} & 1.7 \times 10^{-4} \\
1.7 \times 10^{-4} & 6.6
\end{pmatrix}
\text{GHz} \frac{|E|^2}{10^6 \text{Wmm}^{-2}}
\]

\[ \mu_{eg} = \sum_j \frac{d_{je} d_{jg}}{E_c - \delta \omega}, \quad \mu_{ge} = \sum_j \frac{d_{je} d_{jg}}{E_c + \delta \omega} \]

**Xe**

\[
\begin{pmatrix}
-8.4 \times 10^{-11} & 1.5 \times 10^{-5} \\
1.5 \times 10^{-5} & 0.22
\end{pmatrix}
\text{GHz} \frac{|E|^2}{10^6 \text{Wmm}^{-2}}
\]

**Ca⁺**

\[
\begin{pmatrix}
10 & 2.5 \\
2.5 & 4.5
\end{pmatrix}
\text{GHz} \frac{|E|^2}{10^6 \text{Wmm}^{-2}}
\]

**pH₂**

\[
\begin{pmatrix}
0.27 & 0.16 \\
0.16 & 0.22
\end{pmatrix}
\text{GHz} \frac{|E|^2}{10^6 \text{Wmm}^{-2}}
\]

M. Yoshimura @ IHEP, 2015/06/29
Maxwell-Bloch equation for PSR simulations: 1+1 dim

Bloch equation for medium

\[ \ddot{R} = \text{tr} \rho \sigma = \langle \psi | \sigma | \psi \rangle \]

\[ \partial_t R_1 = (\mu_{ee} - \mu_{gg}) E^+ E^- R_2 - i \mu_{ge} (e^{i \epsilon_{eg} E^+ E^+} - e^{-i \epsilon_{eg} E^- E^-}) R_3 - \frac{n_1}{T_2}, \]

\[ \partial_t R_2 = -(\mu_{ee} - \mu_{gg}) E^+ E^- R_1 + \mu_{ge} (e^{i \epsilon_{eg} E^+ E^+} + e^{-i \epsilon_{eg} E^- E^-}) R_3 - \frac{R_2}{T_2}, \]

\[ \partial_t R_3 = \mu_{ge} (i(e^{i \epsilon_{eg} E^+ E^+} - e^{-i \epsilon_{eg} E^- E^-}) R_1 - (e^{i \epsilon_{eg} E^+ E^+} + e^{-i \epsilon_{eg} E^- E^-}) R_2) - \frac{R_3 + n}{T_1}. \]

T\_2 phase relaxation time, T\_1 population decay time

Field equation

\[ (\partial_t^2 - \nabla^2) \vec{E} = \nabla^2 D \vec{E}, \]

\[ -D \vec{E}^+ = \left( \frac{\mu_{ee} + \mu_{gg}}{2} n + \frac{\mu_{ee} - \mu_{gg}}{2} R_3 \right) \vec{E}^+ + \mu_{ge} e^{-i \epsilon_{eg} t} (R_1 - i R_2) \vec{E}^- . \]

SVEA (Slowly Varying Envelope Approximation)

\[ E = \frac{1}{2} \left( e^{-i \omega_1 (t-x)} E_R + e^{-i \omega_2 (t+x)} E_L + \text{(h.c.)} \right) , \quad \omega_1 + \omega_2 = \epsilon_{eg} \]

complex amplitudes \( E_R(x,t), E_L(x,t) \) slowly varying in 1+1 spacetime

• Coupled system of field and medium polarization highly non-linear

M. Yoshimura @ IHEP, 2015/06/29
PSR simulations for two counter-propagating modes

Explosive PSR with initial coherence

para-H$_2$ $n = 1 \times 10^{21}$ cm$^{-3}$, $L = 30$ cm, $T_1 = 1$ $\mu$s, $T_2 = 10$ ns
Coherent initial state: $r_1 = 1$, $r_2 = r_3 = 0$

Explosive event:
Most energy stored in upper level is released in $< 1$ns

Figure 6: Spacetime profile of $r_1$ for the 1 Wmm$^{-2}$ case of Fig(3).

Figure 7: Spatial profile of $r_1$ at the latest time, 0.3 ns after trigger irradiation, of Fig(6).

Figure 8: Spacetime profile of $r_3$ for the 1 Wmm$^{-2}$ case of Fig(3).

Figure 9: Spatial profile of $r_3$ at the latest time, 0.3 ns after trigger irradiation, of Fig(8).
Weak linear regime of PSR

- In reality,
  finite target size
  short relaxation
  time target number density not large enough etc.

May lead to termination of complete macro-coherence development. In this case its rate given by

\[ \propto n\rho_{eg} \]
RENP process from developed macro-coherence

• RENP rate amplified by both medium polarization and two frequency fields, stored by large amount

• RENP occurs perturbatively under this circumstance, hence reliably calculable except the activity factor, $\eta$
Radiative emission of neutrino pair (RENP)

\[ \Gamma = \Gamma_0 \ I(\omega) \ \eta(t) \]

Two pair production thresholds of mass eigen states

\[ \omega_{ij} = \frac{\epsilon_{eg}}{2} - \left( \frac{m_i + m_j}{2\epsilon_{eg}} \right)^2 \]

Due to energy and momentum conservation
Resolved by precision of trigger lasers

M. Yoshimura @ IHEP, 2015/06/29
Nuclear monopole rates

• Pair emission from nucleus (monopole) gives the largest rates

\[ Q_w = \frac{Z^2}{2} \]
\[ Q_w \sim N - 0.044Z \]

Nuclear coherence effect

Spectrum rates for gas Xe

50 Hz
Dirac vs Majorana & CP phases

We need to go to the lower energy (smaller level spacing) to see M/D distinction and CPV phases.

\[ E_{eg} = 0.429 \text{ eV} \]
\[ E_{pg} = 0.446 \text{ eV} \]
Detection of relic neutrinos of 1.9 K

Recent work with N. Sasao and M. Tanaka
arXiv: 1409.3648

- Direct remnant at a few seconds after the big bang
- Prove that neutrinos were in thermal equilibrium, giving the important basis of light element synthesis such as 4He
- $T$ differs from 2.7K of microwave, because electron-positron annihilation occurred after the neutrino decoupling at a few MeV, heating up matter in equilibrium
- Prediction is firm: $(4/11)^{(1/3)} \times 2.7 \text{ K} = 1.9 \text{ K}, \ 110\text{cm}^{-3}$
• Spectrum distortion by the Pauli blocking caused by ambient relic neutrinos

Neutrino distribution function

\[
f(p) = \frac{1}{\zeta e^{p^2+m^2/(z_d+1)^2/T} + 1} \approx \frac{1}{\zeta e^{p/T} + 1}
\]

\[
\zeta = e^{-\mu_d/T_d}, \quad z_d = O(10^{10})
\]

Blocking given by \(1-f(p)\)

\[
F_{ij}^A(\omega; T_{\nu}) = \frac{1}{8\pi \omega} \int_{E_{\nu}}^{E_{\nu}^+} dE_1 g_{ij}^A(E_1) \cdot \left(1 - f(\sqrt{E_1^2 - m_i^2})\right) \left(1 - f(\sqrt{(\epsilon_{eg} - \omega - E_1)^2 - m_j^2})\right),
\]

\[
g_{ii}^M(E) = -E^2 + (\epsilon_{eg} - \omega)E + \frac{1}{2}m_i^2 - \frac{1}{4}\epsilon_{eg}(\epsilon_{eg} - 2\omega) + \delta_M \frac{m_i^2}{2},
\]

\[
g_{ij}^S(E) = -\frac{1}{3}E^2 + \frac{1}{3}(\epsilon_{eg} - \omega)E + \frac{1}{12}\epsilon_{eg}(\epsilon_{eg} - 2\omega) - \frac{1}{12}(m_i^2 + m_j^2) - \delta_M \frac{m_i m_j}{2},
\]

\[
E_\pm = \frac{1}{2} \left( (\epsilon_{eg} - \omega)(1 + \frac{m_i^2 - m_j^2}{\epsilon_{eg}(\epsilon_{eg} - 2\omega)}) \pm \omega \Delta_{ij}(\omega) \right), \quad \Delta_{ij}(\omega) = \left\{ \left(1 - \frac{(m_i + m_j)^2}{\epsilon_{eg}(\epsilon_{eg} - 2\omega)}\right) \left(1 - \frac{(m_i - m_j)^2}{\epsilon_{eg}(\epsilon_{eg} - 2\omega)}\right) \right\}^{1/2}.
\]
Temperature measurement possible for RENP?

Ratio of rates: with to without Pauli blocking

with/without Pauli blocking

Level spacing 11 meV
Smallest mass 5 meV

Difference of distortions for 1.9 and 2.7 K
10% level

For small level spacing, temperature measurement possible.
Experimental result (wait for Sasao)

- Linear growth region for two pair modes in the same direction
- Exponential time growth prop. to $n^*$ coherence, cut by de-coherence $T_2$
Twin process and controlled switching

RENP uses large medium polarization and stored fields by PSR, but two processes have different selection rules

\[ \text{RENP: } (E_0 \text{ or } M1) \times E1 \]
\[ \text{PSR: } E1 \times E1 \]

PSR-RENP switching is achieved by application of modulated \( E \)
Ideal state for RENP after PSR activity

Soliton-condensates stable against two-photon emission, unstable for RENP

Analogue of stopped light polariton in cavity QED

Realized by two counter-propagating trigger PSR modes
Experimental strategy towards neutrino mass spectroscopy

• 1\textsuperscript{st} stage: proof of macro-coherence principle using QED (PSR): we already achieved this stage, almost but not completely

• 2\textsuperscript{nd} stage: control of PSR and soliton formation, switching between PSR and RENP modes, study of solid targets

• 3\textsuperscript{rd} stage: discovery of the RENP process, measurements of mass matrix
Summary on RENP

- Systematic neutrino mass spectroscopy is made possible when macro-coherence is realized. Controlled PSR by formation of soliton-condensate should be achieved.

- Since the macro-coherent QED process (PSR) has been experimentally observed (next talk), we go to the next stage towards RENP R&D.
Summary of this talk

• We should maximally exploit quantum coherence towards the ultimate, systematic clarification of mysteries of neutrino.

• Coherent quantum heavy ion synchrotron is ideal for CPV parameter measurements and ***

• Macro-coherent RENP provides small-scale laboratory experimental means, including MD distinction and relic detection.
Backup
RENP using pair beam

• New work in progress
• Absolute mass determination and MD distinction expected
• Kinematics different from SPAN: energy and momentum conservation not obeyed, and only the sum of photon and two neutrinos limited
• Rate scales with gamma^6
\[ \frac{d\Gamma}{d\omega} = RF\left(\frac{\omega}{\omega_m}\right), \quad \omega_m = 2\epsilon_{eg}\gamma, \]

\[ R = \frac{\sqrt{\pi}}{2\sqrt{3}(2\pi)^8} v_5 G_F^2 e^2 r_{ep}^2 \gamma^6 N|\rho_{eg}(0)|^2 \sqrt{\rho_{eg}}^{19/2} \frac{1}{\epsilon_{pe}^2} \]

\[ \sim 0.99 \times 10^{12}\text{Hz} \frac{N|\rho_{eg}(0)|^2}{10^8} \frac{\gamma_{pe}}{100\text{MHz}} \sqrt{\frac{\rho}{4\,\text{km}}} \left(\frac{\gamma}{10^4}\right)^6 \left(\frac{\epsilon_{eg}}{1\text{keV}}\right)^{15/2}, \]

\[ v_5 = \int dV_5 (1 - r^2)^{-1/4} \sim 9.1 \times 10^{-6}, \quad F(y) = \int_{m_i/\epsilon_{eg}}^1 dx_1 dx_2 H(y, x_1, x_2), \]

\[ H(y, x_1, x_2) = y^{5/2} \left(1 + \frac{2\epsilon_{eg}}{\epsilon_{pe}} y\right)^{-2} x_1 x_2 (x_1 + x_2 + y)^{1/4} G(x_1, x_2, y)^{9/4} \Theta(G(x_1, x_2, y)), \]

\[ G(x_1, x_2, y) = 1 - x_1 - x_2 - y - \frac{1}{4\epsilon_{eg}^2} \left(\frac{m_1^2}{x_1} + \frac{m_2^2}{x_2}\right). \]
Pair emission build-up time should be shorter than 2nu production time.

\[ \Delta t < \frac{1}{\Gamma_{2\nu}} \]

\[ \Delta t = \sqrt{\frac{3\xi}{2}} \frac{E_1 + E_2}{F}, \quad \sqrt{\xi} = \sqrt{\frac{2\sqrt{2}}{3}} \sqrt{\rho(E_1 + E_2)} \left( \frac{F}{(E_1 + E_2)^2} \right)^{3/4} \]

\[ \Gamma_{2\nu} \sim 3.1 \times 10^{13} \text{Hz} \left( \frac{\rho}{4 \text{ km}} \right)^{1/2} \left( \frac{E_{\gamma}}{10^4} \right)^4 \left( \frac{\varepsilon_{eg}}{50 \text{keV}} \right)^{11/2} \]
Difference from usual synchrotron radiation

Ground state ion

\[ \int_0^\infty dx h(x) \cos \xi \left( \frac{1}{2} x^3 + \frac{3}{2} x \right) \rightarrow \sqrt{\frac{\pi}{6}} e^{-\xi} \frac{h(0)}{\sqrt{\xi}}. \]

\[ \xi = \rho(E_1 + E_2) \times \text{a function of} \left( \frac{E_1}{E_2}, \frac{\varepsilon_{eg}}{E_1 + E_2}, \gamma, \text{angles} \right) \]

• Always the same sign phase added, leading to exponential damping

Excited ion with coherence

\[ \int_0^\infty dx h(x) \cos \xi \left( \frac{1}{2} x^3 - \frac{3}{2} x \right) \rightarrow \sqrt{\frac{2\pi}{3}} \cos (\xi - \frac{\pi}{4}) \frac{h(1)}{\sqrt{\xi}}. \]

Cancellation of positive and negative phases
Energy input leads to resonance-like behavior