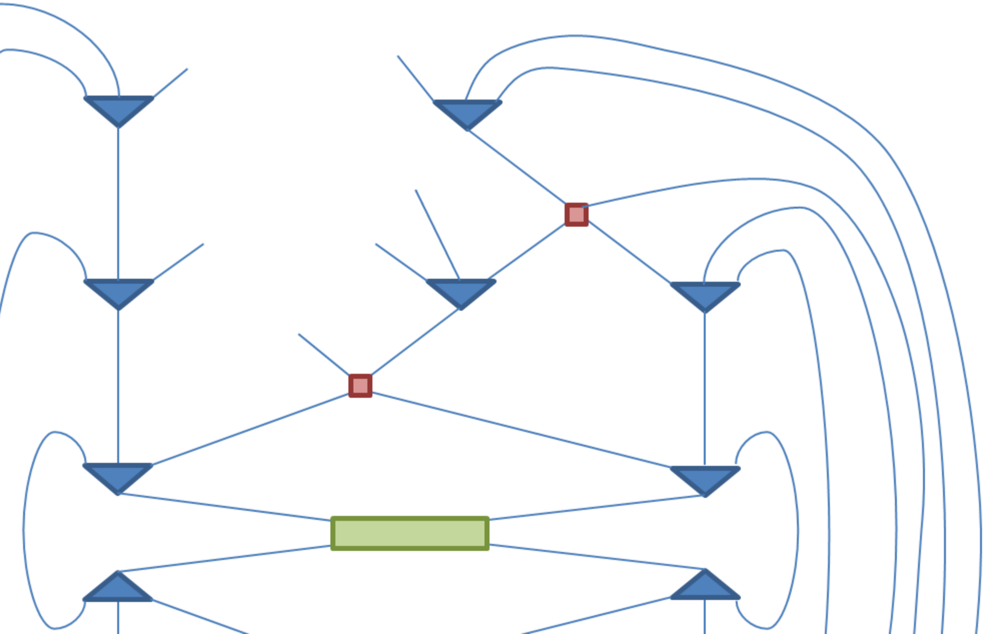
ENTANGLEMENT, MERA AND MODULAR HAMILTONIANS

TOWARDS ENTANGLEMENT AS EMERGENT SPACETIME

Work with Rob Myers, with help from Guifre Vidal and Lukasz Cincio
Work in progress



Wilke van der Schee
String seminar, IHEP, 3 Jun

OUTLINE

Introduction tensor networks and holography

- MERA: *multiscale entanglement renormalisation ansatz*
- MERA and entanglement entropy
- AdS/MERA: entanglement entropy in AdS/CFT

Reduced density matrix and modular Hamiltonian

- Results in Ising, Heisenberg, XX and Potts models

Outlook: emergent spacetime?

- MERA as a playground for (quantum) gravity…?

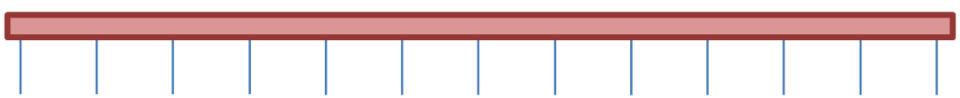
Brian Swingle, Entanglement Renormalization and Holography (2009)

TENSOR NETWORKS

Wave function one-dimensional spin chain is a tensor:

$$|\psi\rangle = \sum_{i_1\cdots i_N} c_{i_1\cdots i_N} |\psi_1\rangle \cdots |\psi_N\rangle$$

$$\cong$$

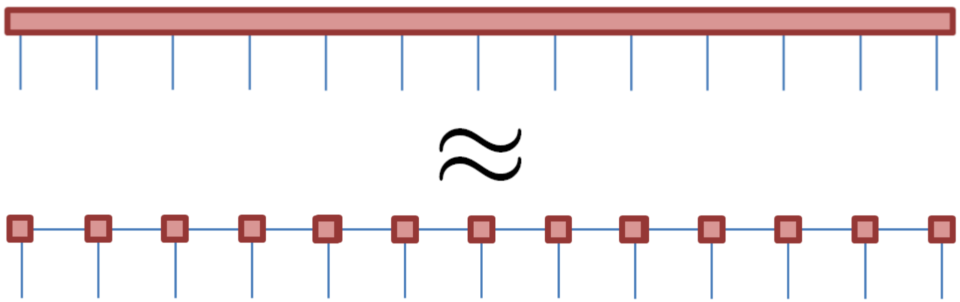


Hilbert space grows as 2^N

- Physics: typical ground state has only short range entanglement
- *Ground state lives in small corner of Hilbert space*

MATRIX PRODUCT STATE (MPS)

Approximate tensor, including local entanglement:



$$\cong$$

$$c_{i_1\cdots i_N} = \sum_{j_1\cdots j_{N-1}} m_{i_1}^{\;j_1} m_{i_2 j_1}^{\;j_2} \cdots m_{i_{N-1} j_{N-2}}^{\;j_{N-1}} m_{i_N j_{N-1}}$$

Total degrees of freedom: $N 2\chi^2 \ll 2^N$

χ **is dimension internal legs: *bond dimension***

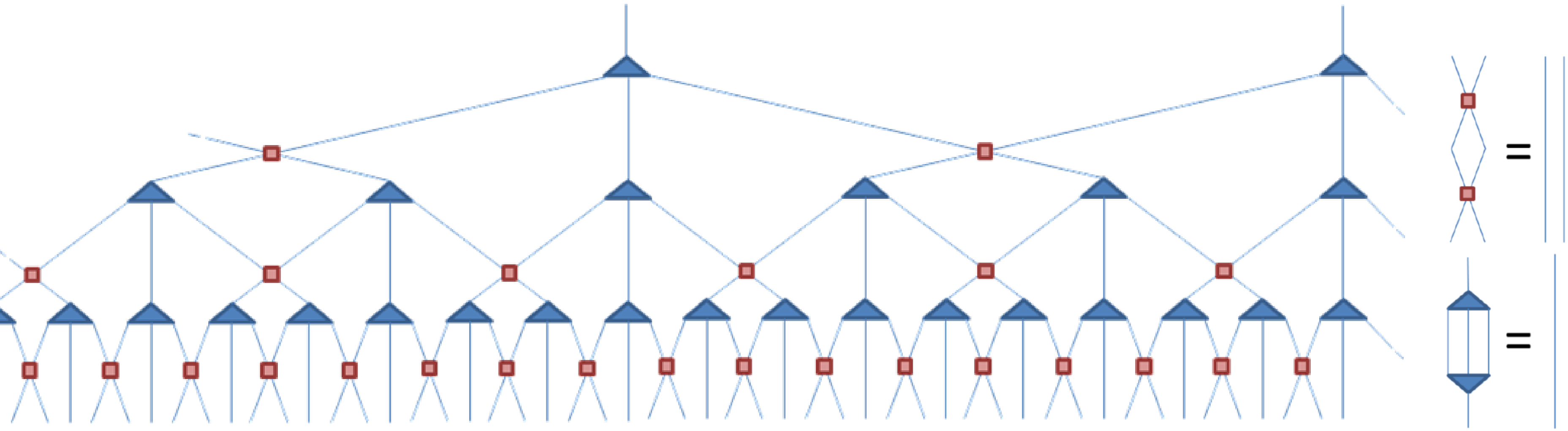
Note: approximation becomes exact for large enough χ

M. Fannes, B. Nachtergaele, and R.F. Werner, Finitely correlated states on quantum spin chains (1992)

MULTISCALE ENTANGLEMENT RENORMALISATION ANSATZ (MERA)

Long range correlations/entanglement require larger χ

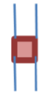
Choose different *ansatz* to incorporate RG flow:



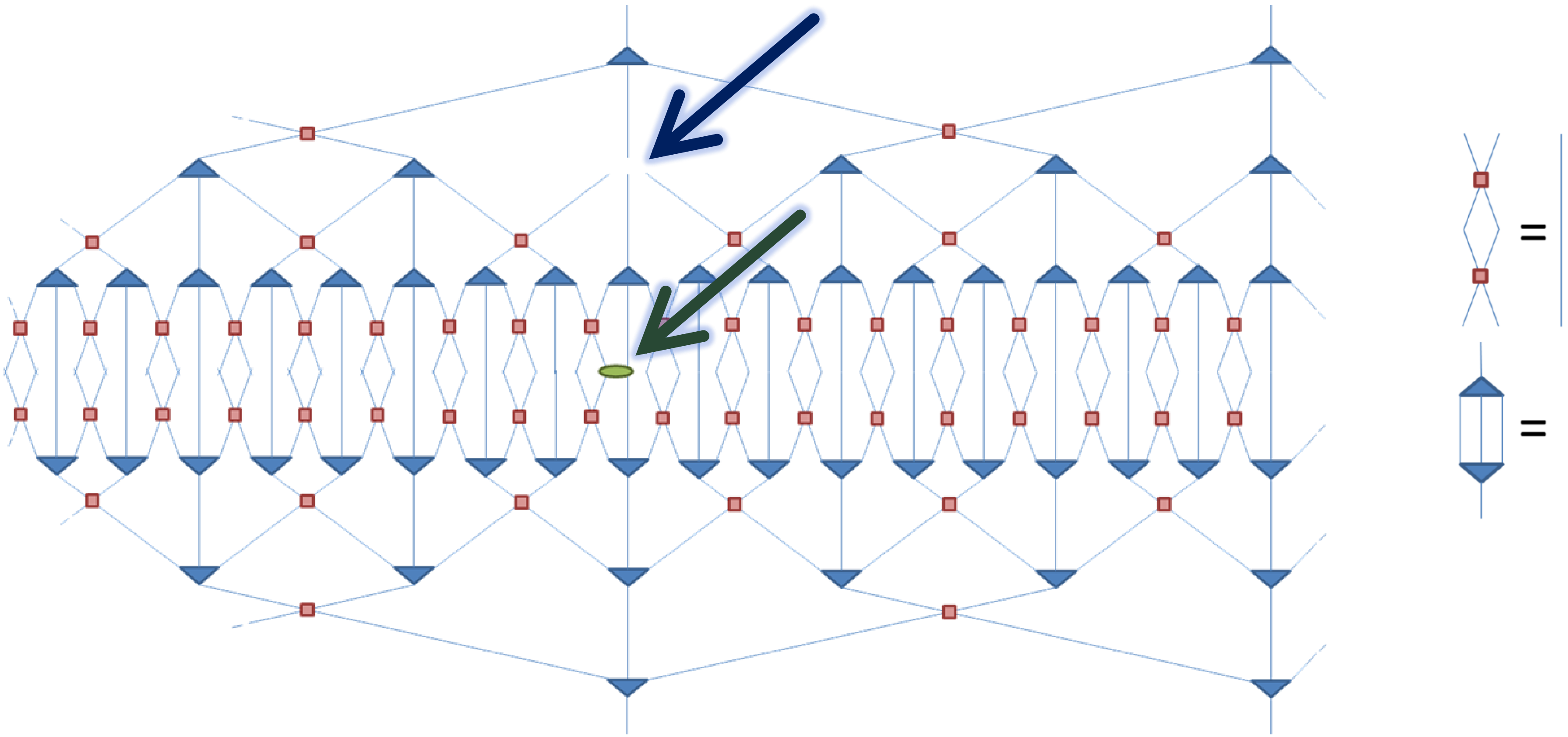
Disentanglers and coarse grainers (ternary)

Extra advantage: scale invariance is very natural!

G. Vidal, Entanglement Renormalization (2007)

TASK: FIND  ,  TO MINIMISE ENERGY

Sum contributions of Hamiltonian to `environment' of tensor:

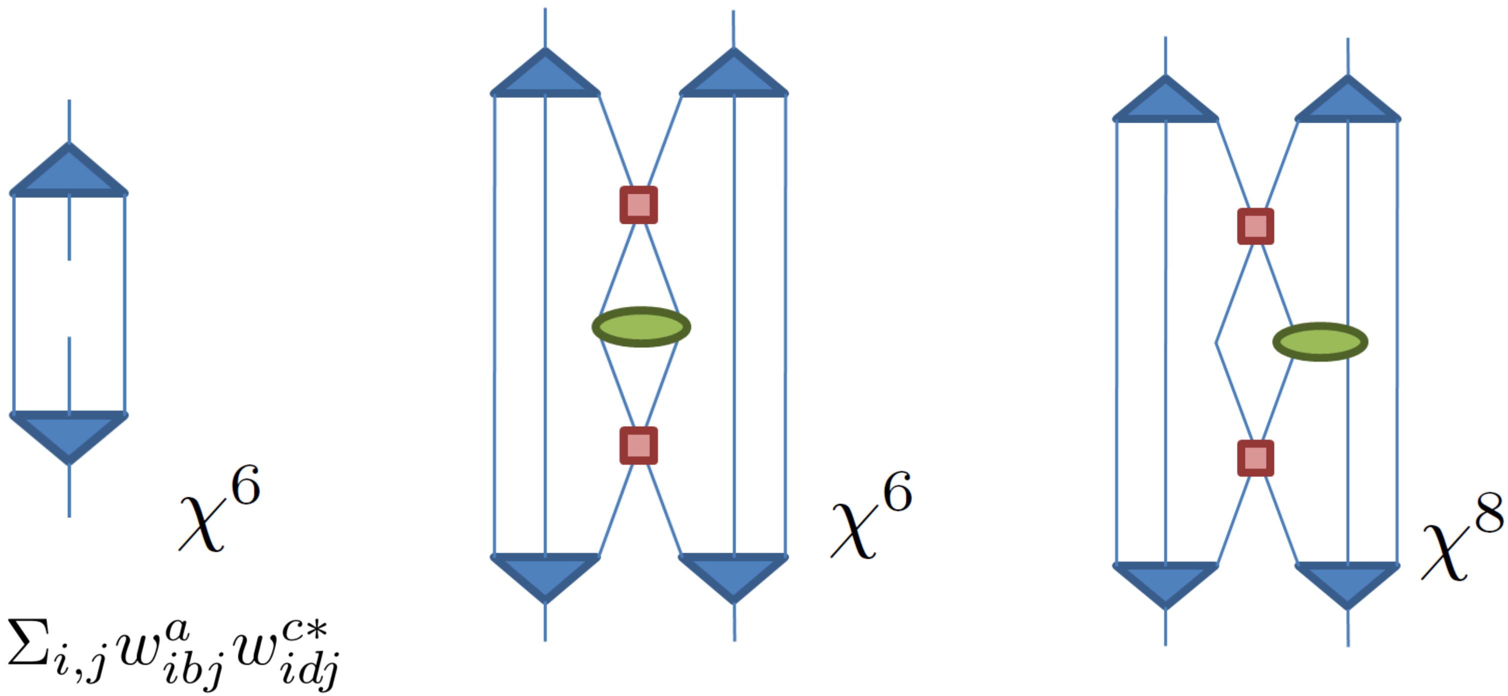


Use some tricks, repeat often and (hopefully) converge ☺

G. Evenbly and G. Vidal, Algorithms for entanglement renormalization (2007)

FUN FACT: TENSOR CONTRACTIONS NP COMPLETE

Algorithm depends crucially on `efficiently contractible'

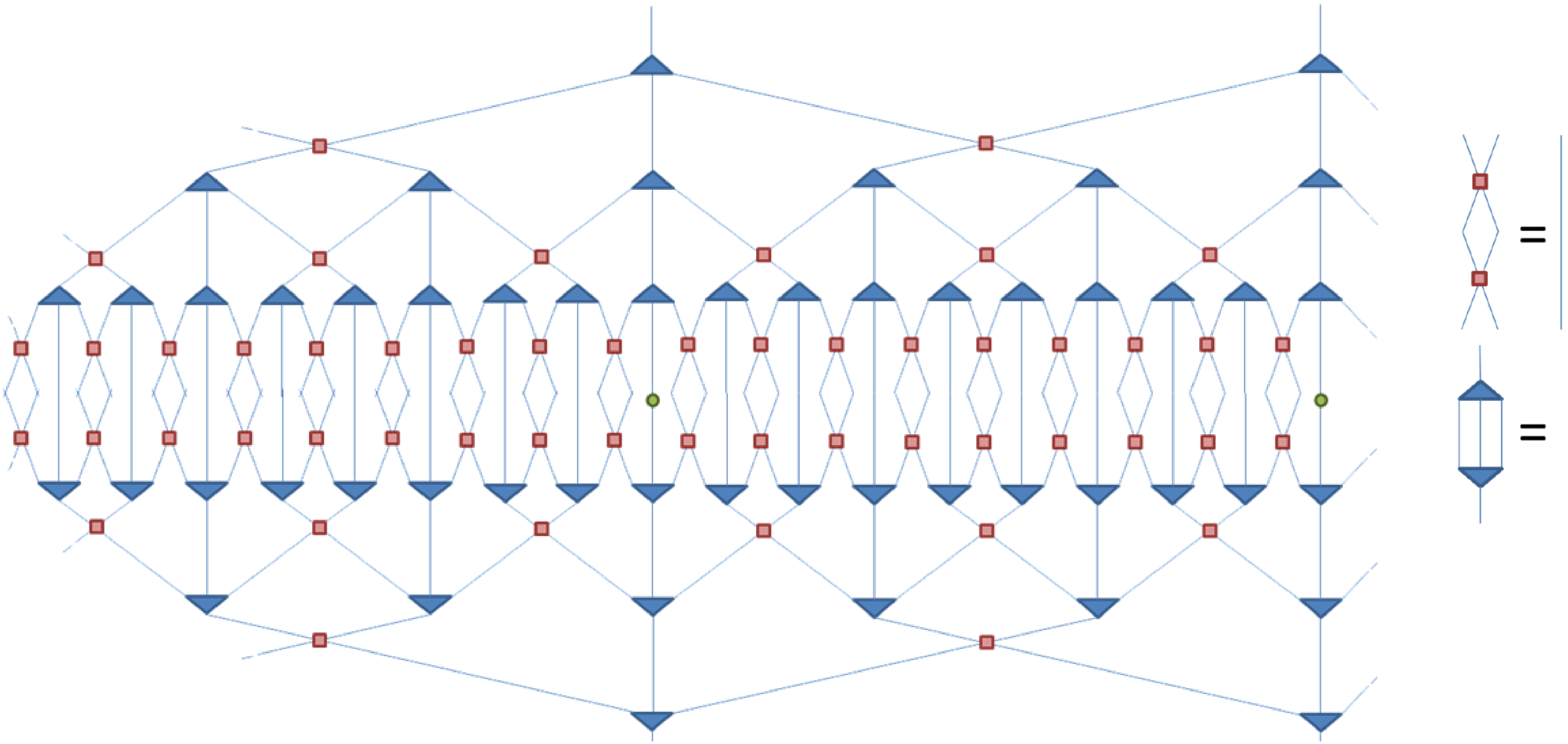


Much harder for 2 dimensions (i.e. χ^{16} or χ^{23})

R.N.C. Pfeifer, J. Haegeman and F. Verstraete, Faster identification of optimal contraction sequences for tensor networks (2013)

EXAMPLE: CORRELATORS IN MERA

Choose operators at smart locations

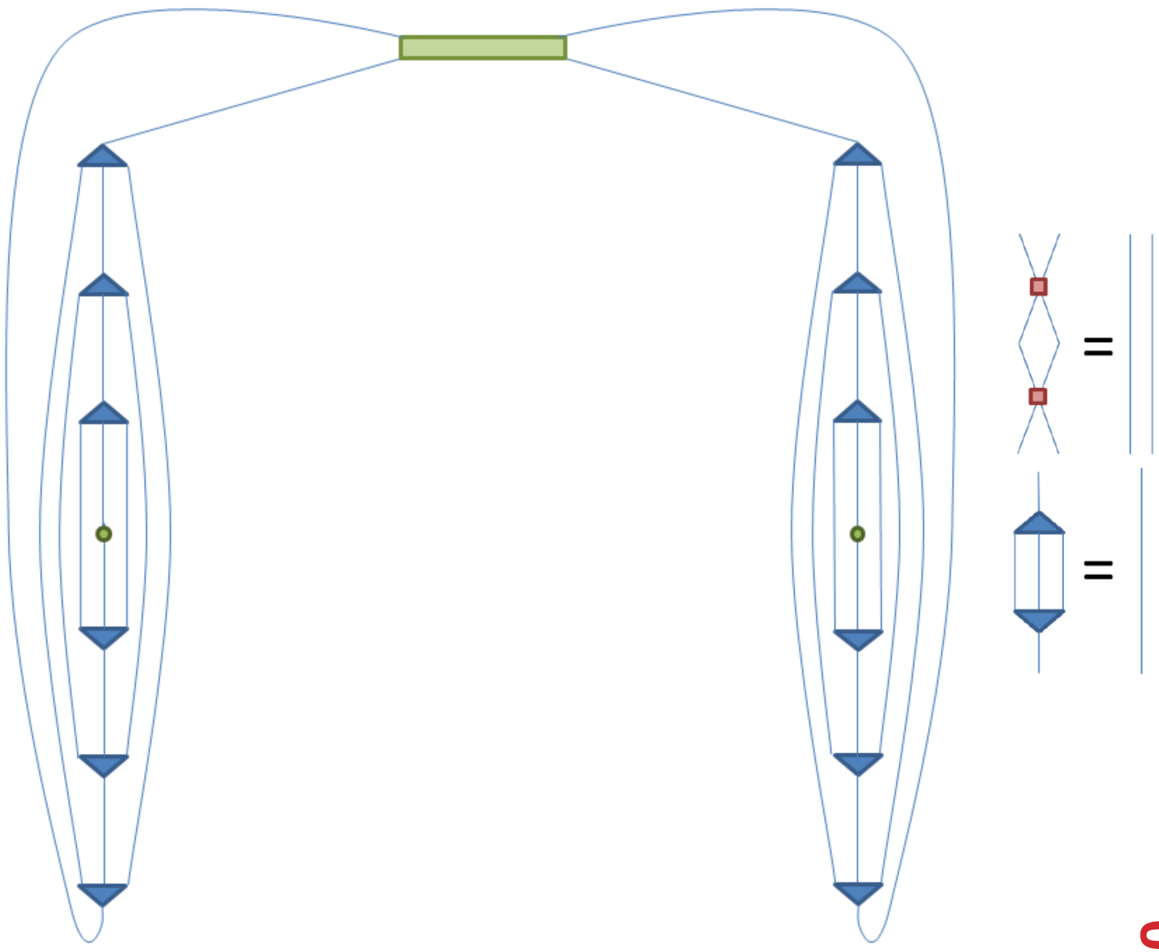


Simplify ☺

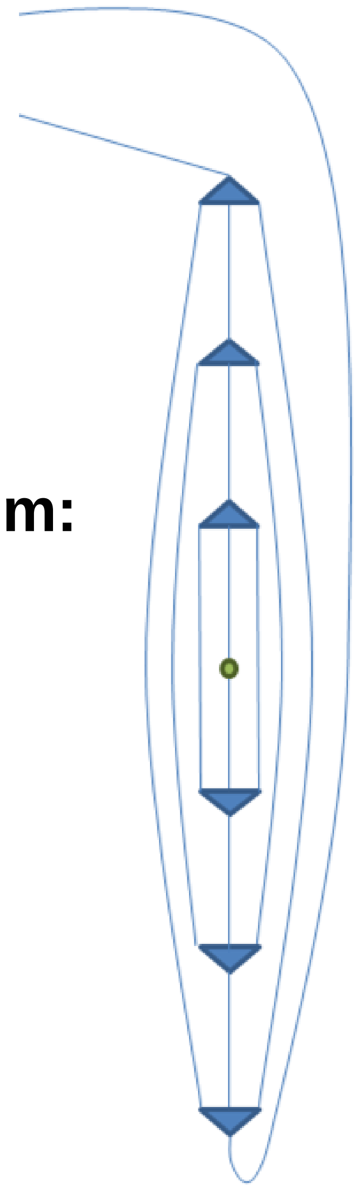
R.N.C. Pfeifer, G. Evenbly and G. Vidal, Entanglement renormalization, scale invariance, and quantum criticality (2009)

EXAMPLE: CORRELATORS IN MERA

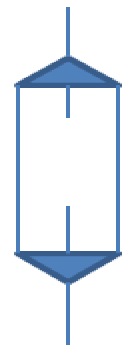
Add reduced density matrix (green)



EXAMPLE: CORRELATORS IN MERA

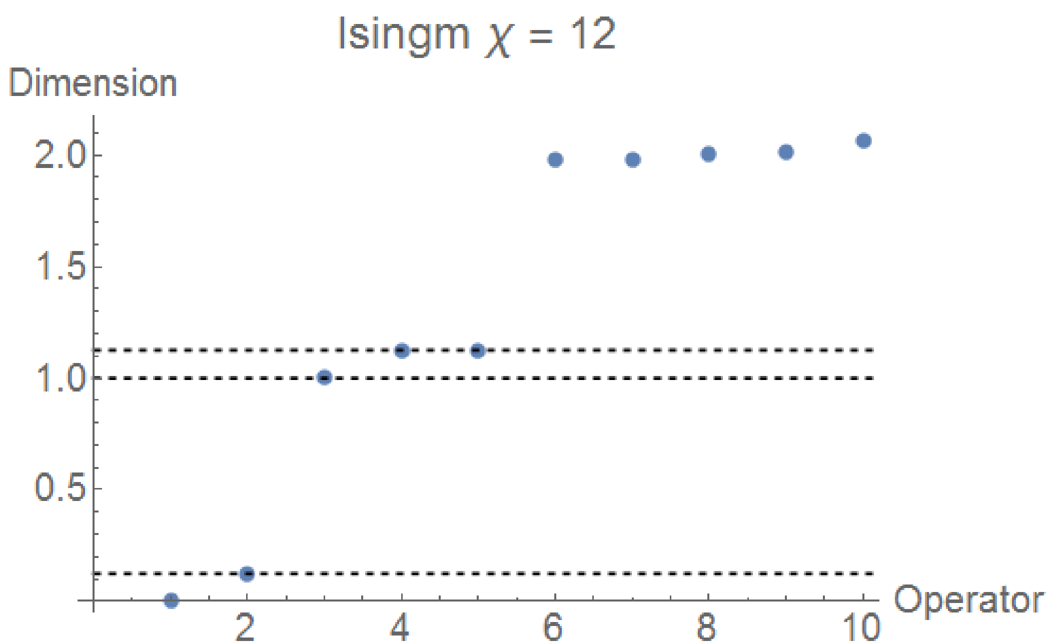


Scaling superoperator:



Conformal dimensions are given by spectrum:

$$\Delta_\alpha = -\log_3(\lambda_\alpha)$$



GRAVITY AS AN EMERGENT FORCE, ADS/MERA

Field theory without gravity ⟺ string theory with gravity

Holographic: gravity has one extra dimension, RG scale

→ Propose connection between MERA and gravity

Caveat: gravity `emerges' only for specific field theories
(large N, strong coupling)

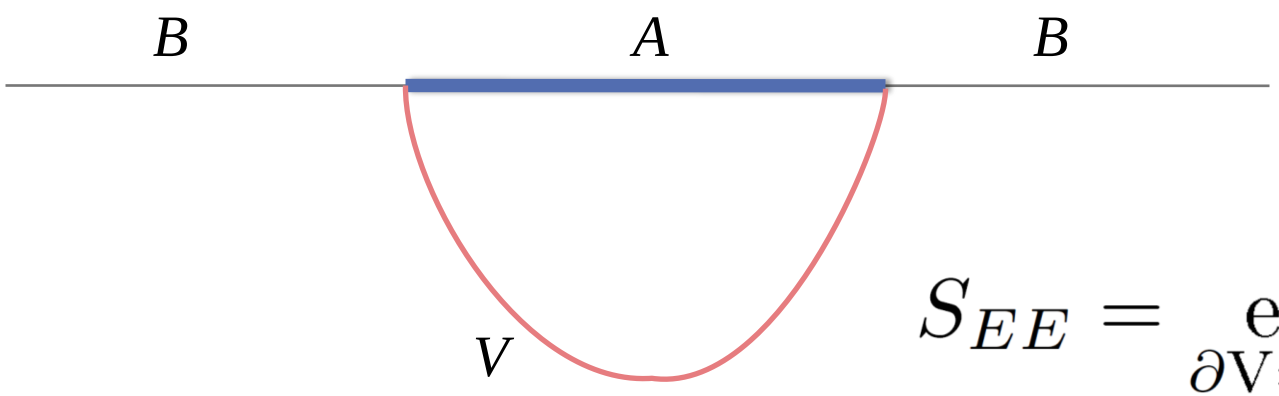
Brian Swingle, Entanglement Renormalization and Holography (2009)

ENTANGLEMENT = GEOMETRY (PICTURE)

Entanglement: trace out part of space → mixed state (entropy!)

Remarkable statement (Ryu+Takayanagi):

entanglement entropy = area extremal surface in AdS



$$S_{EE} = \underset{\partial V = \partial A}{\text{ext}} \frac{A_V}{4G_N}$$

S. Ryu, T. Takayanagi, Holographic derivation of entanglement entropy from the anti–de sitter space/conformal field theory correspondence (2006

ENTANGLEMENT ENTROPY

Reduced density matrix: $\rho_{red,L} = \mathrm{tr}(\rho)$

Obtain mixed state with probabilities: $p_\rho = \mathrm{eig}\left(\rho_{red,L}\right)$

- **Has entropy:** $S_{EE} = -\sum_i p_{\rho,i} \log(p_{\rho,i}) = \frac{c}{3}\log(L) + \mathcal{O}(1)$
- **I.e. ground state → excited state!**

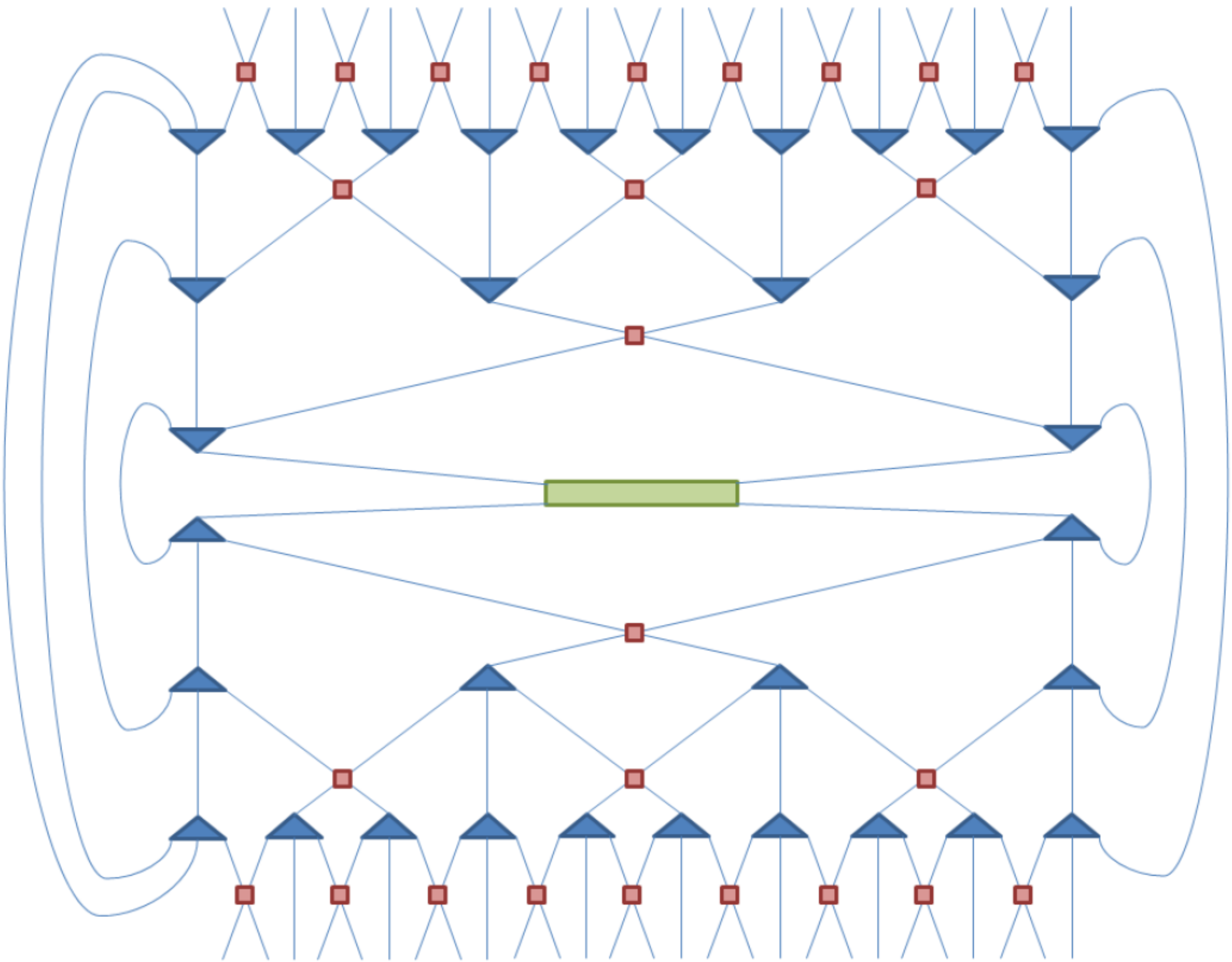
Ising model: $H_{\mathrm{Ising}} = -\sum_r \left(\lambda\sigma_z^{[r]} + \sigma_x^{[r]}\sigma_x^{[r+1]}\right)$

- **Energy:** $e_0 = -\frac{2}{L\sin(\pi/2L)} \approx -\frac{4}{\pi} - \frac{\pi}{6L^2}$
- **Central charge 1/2**

P. Calabrese and J. Cardy, Entanglement entropy and quantum field theory (2004)

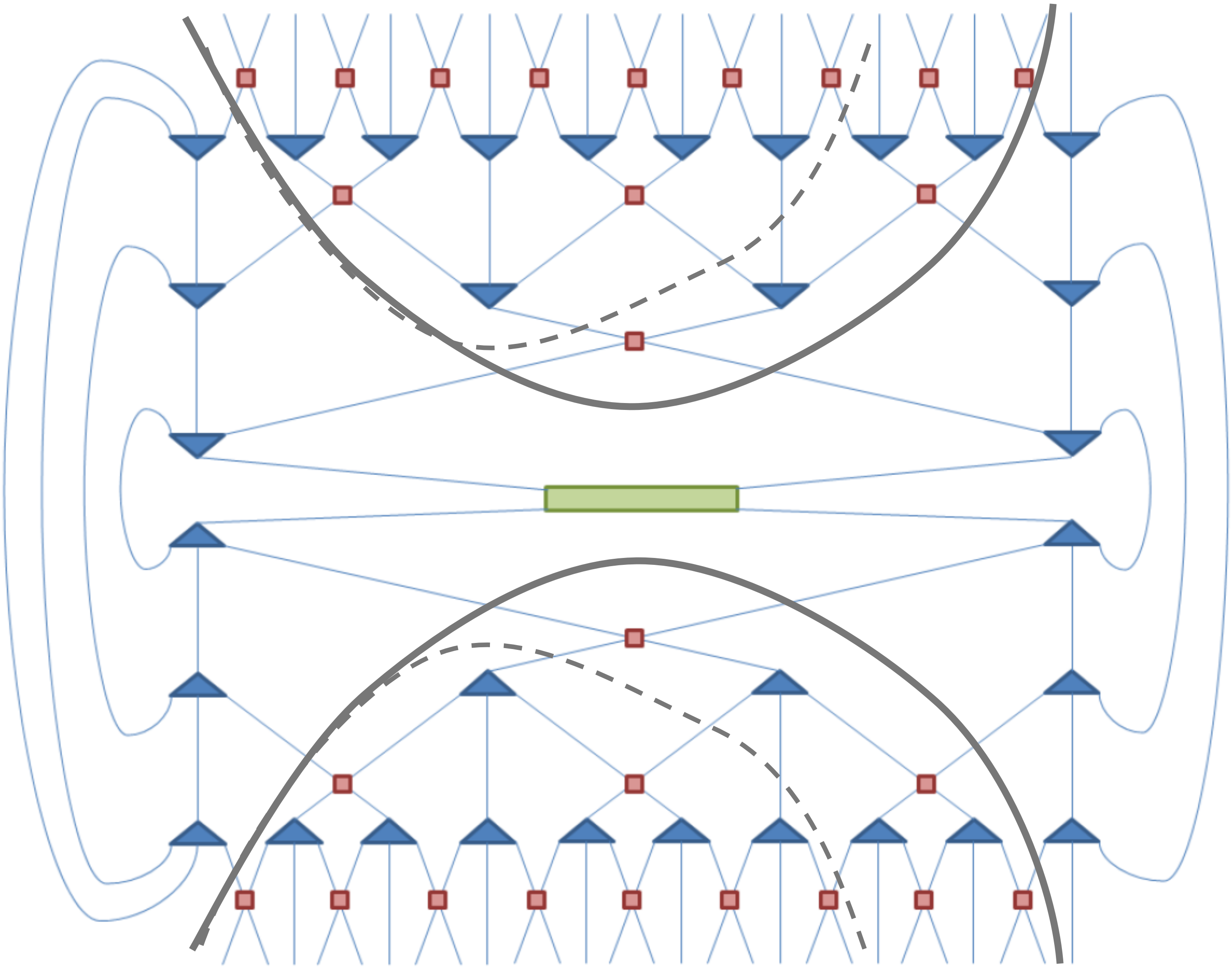
Wilke van der Schee, MIT

REDUCED DENSITY MATRIX IN MERA



$\rho_{red,L} = \mathrm{tr}(\rho)$

`GEODESIC' IN ADS SPACETIME (SWINGLE)

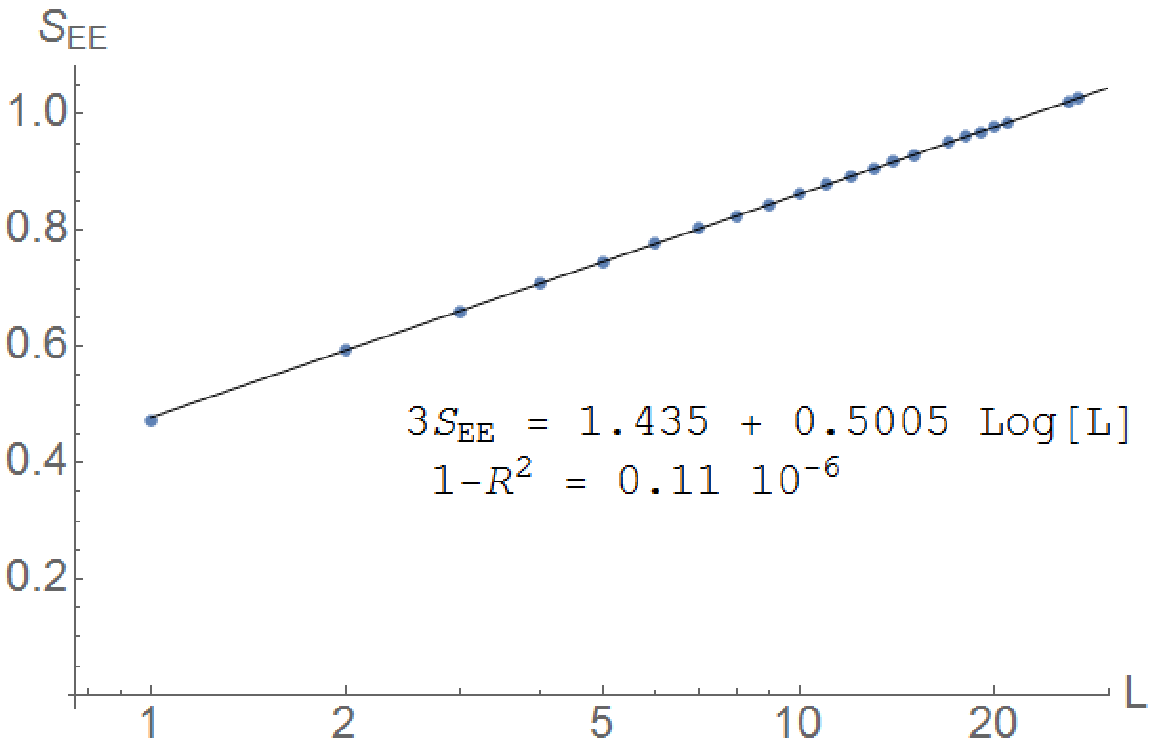


Brian Swingle, Entanglement Renormalization and Holography (2009)

$$S_{EE} = \frac{c}{3}\log(L) + \mathcal{O}(1)$$

EQUIVALENT REDUCED DENSITY MATRIX

Resulting entanglement entropies:

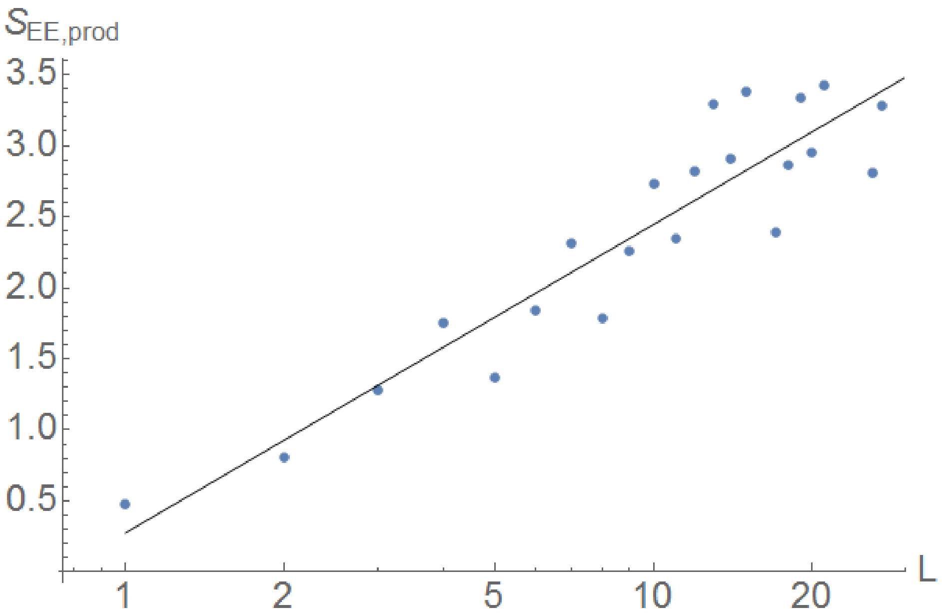


QUESTION: WHAT GIVES A GEOMETRIC PICTURE?

Reduced density matrix gives a local `slice' in `AdS'

Hard to formalise: legs do not in general decouple

- Do they decouple with large *c*?
- If so, then entanglement entropy = sum over entropy/leg
- What about dual of Ising model?



$3S_{EE} = 0.8247 + 2.828\ \mathrm{Log}[L]$
$R^2 = 0.69$

MODULAR HAMILTONIAN

Reduced density matrix: $\rho_{red,L} = \mathrm{tr}(\rho)$

Modular Hamiltonian: $\rho_{red,L} = e^{-H}$

- **Generates flow in causal domain**
- **Usually very non-local**

Sphere

- **Casini, Huerta, Myers:** $H_{\mathcal{D}} = 2\pi \int_V d^{d-1}x \frac{(R^2 - r^2)}{2R} T^{00}(x) + c'$

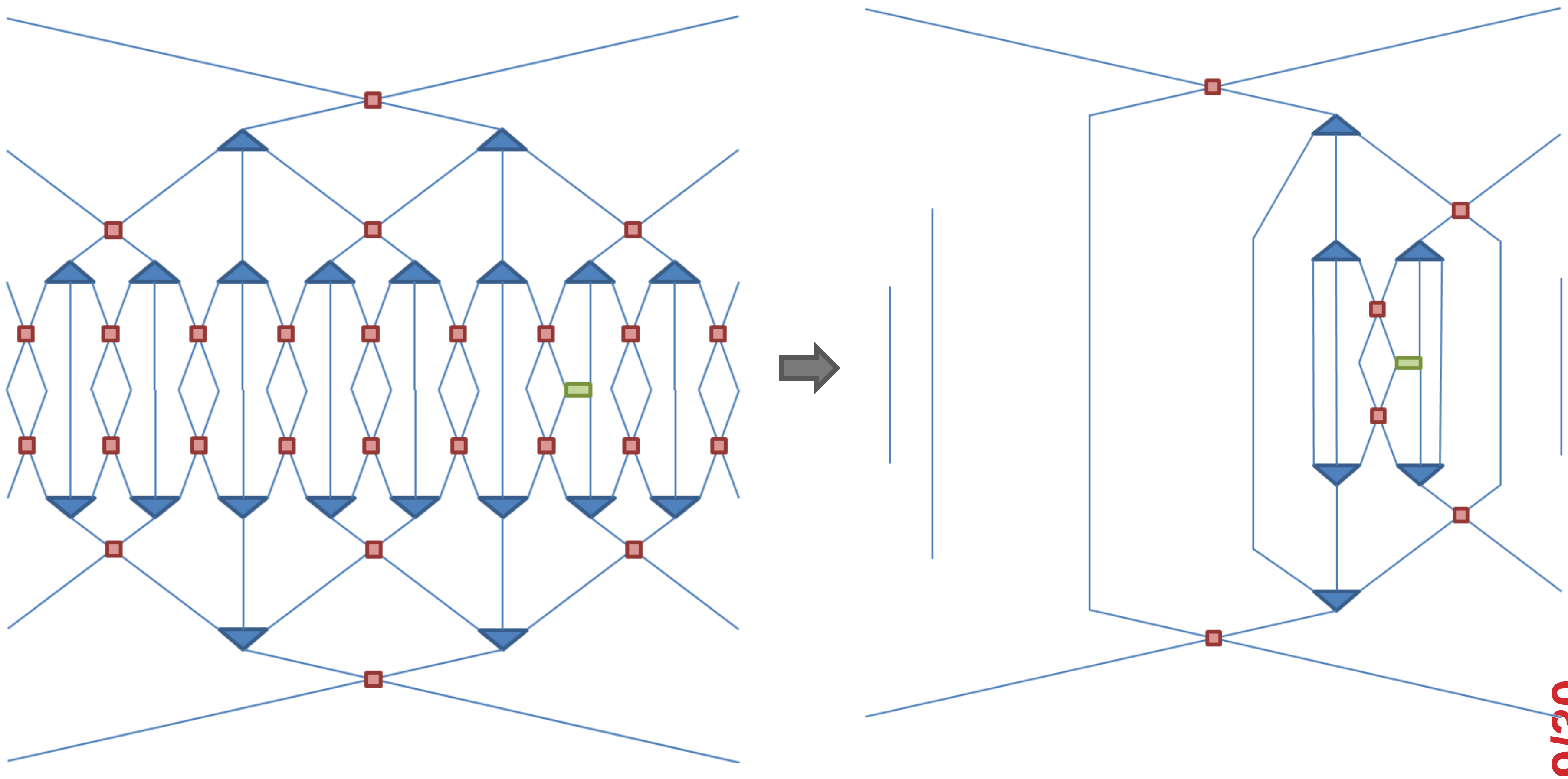
$$\Rightarrow \quad H_{\text{modular}} = \beta \frac{2\pi}{L} \left(\sum_{r=1}^{L} (r - \frac{1}{2})(L - r + \frac{1}{2}) \sigma_z^{[r]} - \sum_{r=1}^{L-1} (r)(L - r) \sigma_x^{[r]} \sigma_x^{[r+1]} \right) + c'$$

Entanglement entropy = Thermal (Rindler) entropy ??

H. Casini, M. Huerta and R. C. Myers, Towards a derivation of holographic entanglement entropy (2011)

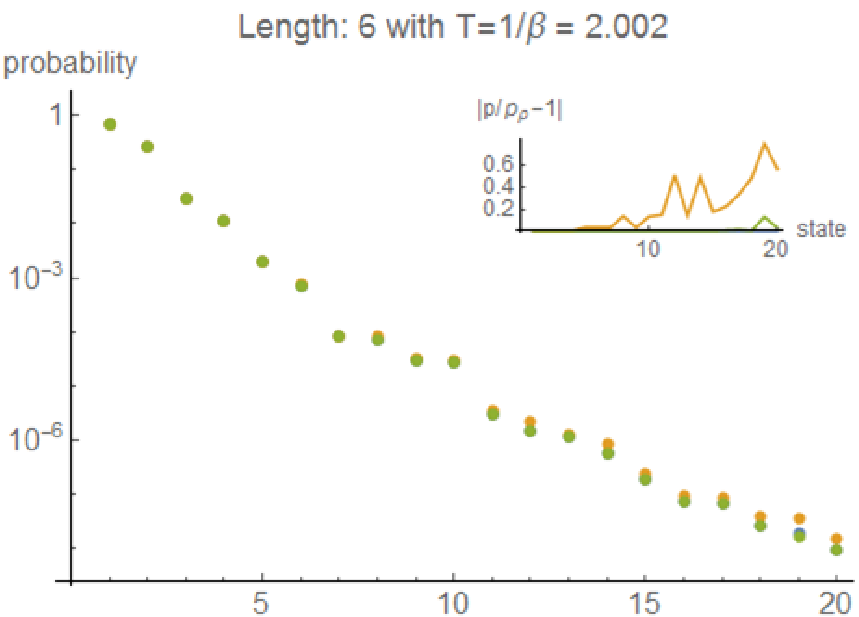
(MODULAR) HAMILTONIAN

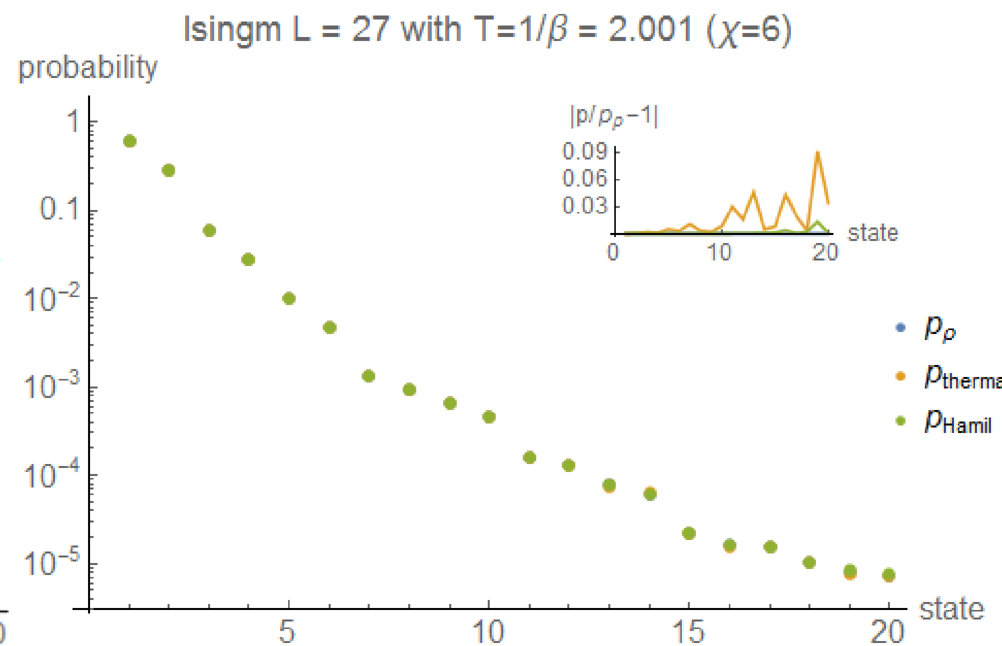
Sum over individual terms of modular Hamiltonian:



MODULAR HAMILTONIAN

Compare spectra:





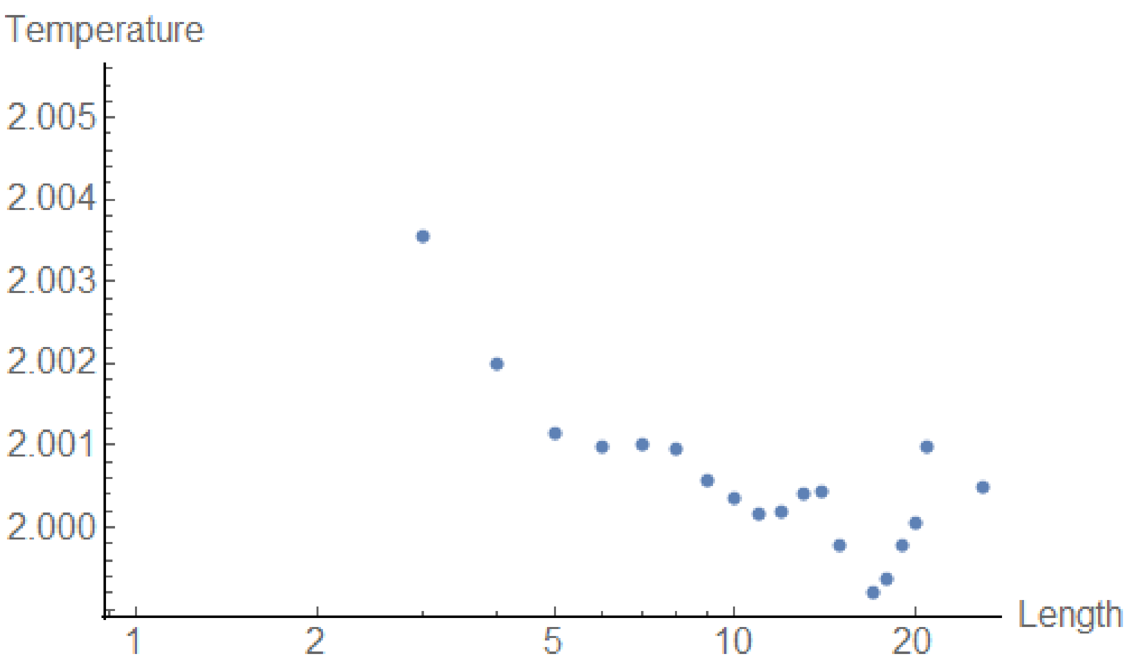
$$p_\rho = \text{eig}(\rho_{red,L}),$$
$$p_{thermal} = e^{-\beta E_i},$$
$$p_{hamil} = \langle\psi_i|\rho_{red,L}|\psi_i\rangle,$$

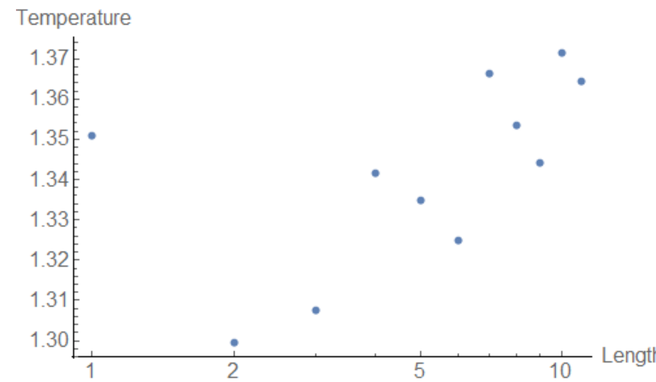
Casini, Huerta, Myers

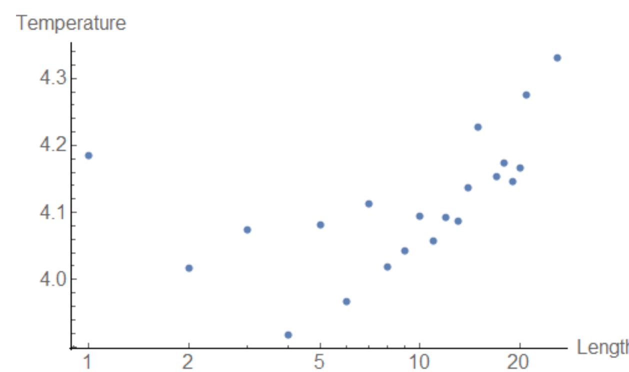
$$H_{\text{red}}|\psi_i\rangle = E_i|\psi_i\rangle$$

PUZZLE: WHAT DETERMINES TEMPERATURE?

(Fitted) temperature varies with model.. UV cut-off, but how?



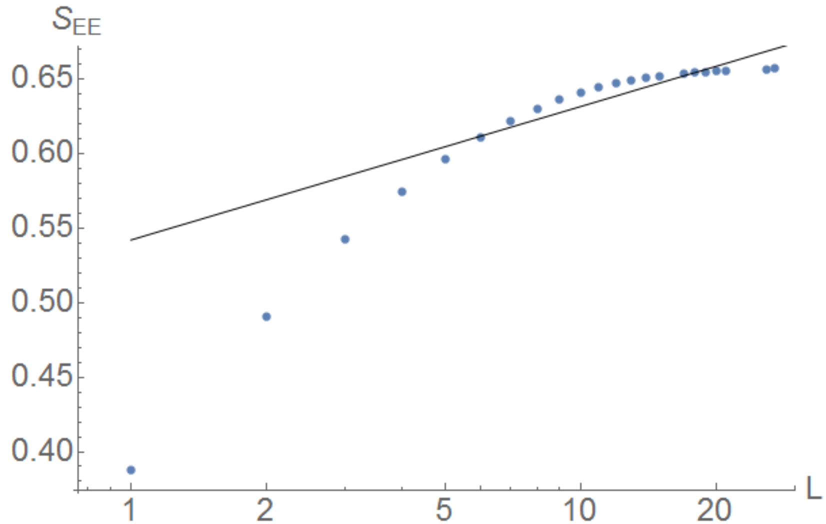




$$H_{\text{Ising}} = -\sum_r \left(\lambda \sigma_z^{[r]} + \sigma_x^{[r]} \sigma_x^{[r+1]} \right)$$

CHECK: NON-CONFORMAL THEORIES

Ising with λ=1.1:

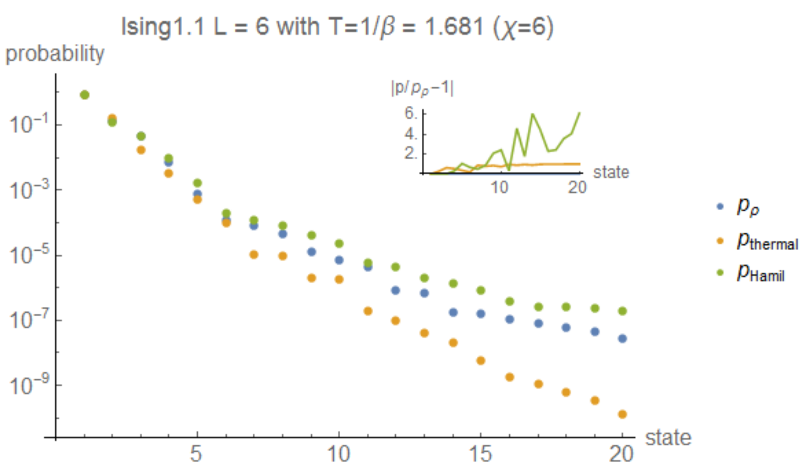


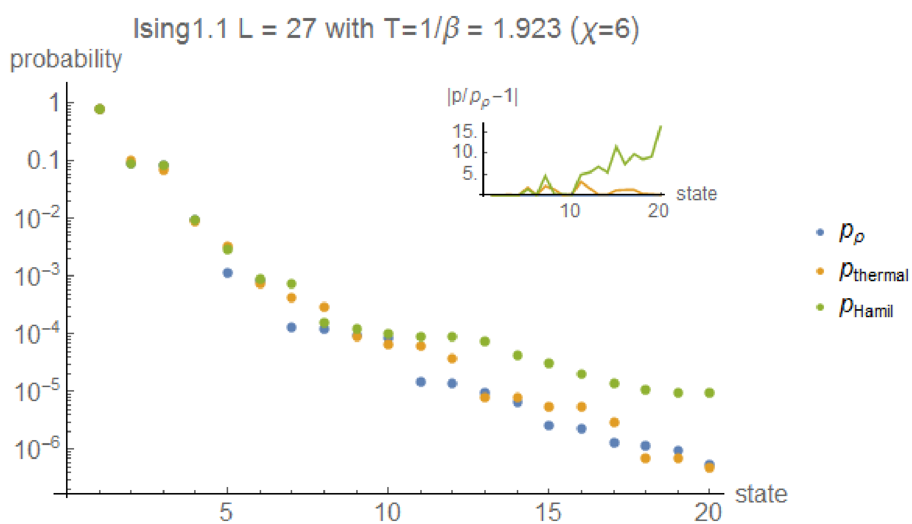
$3S_{EE}$ = 1.627 + 0.1168 Log[L]

$1-R^2$ = 5

1.4 10 10^{-6}

Probabilities:





OUTLOOK

Hardly the beginning

- Computed (equivalent) reduced density matrices
- Shown known local modular Hamiltonian also works in UV
- Discrete check of `entanglement entropy = thermal entropy' (sphere)
- Emergent translational invariance in AdS?

Entanglement `lives' on narrow slice of MERA

- Emergent geometry?
- Classical limit? Large N limit? Strong coupling?
- → chance of studying AdS in highly quantum regime

Other uses of MERA (Mathematica package to appear ☺)

- Mutual information, Renyi entropies, negativity, non-conformal …
- One of few tools to study holography from field theory side!