Thermalization process in far-from equilibrium systems

Soeren Schlichting
Brookhaven National Lab

Based on work in collaboration with Berges, Boguslavski & Venugopalan

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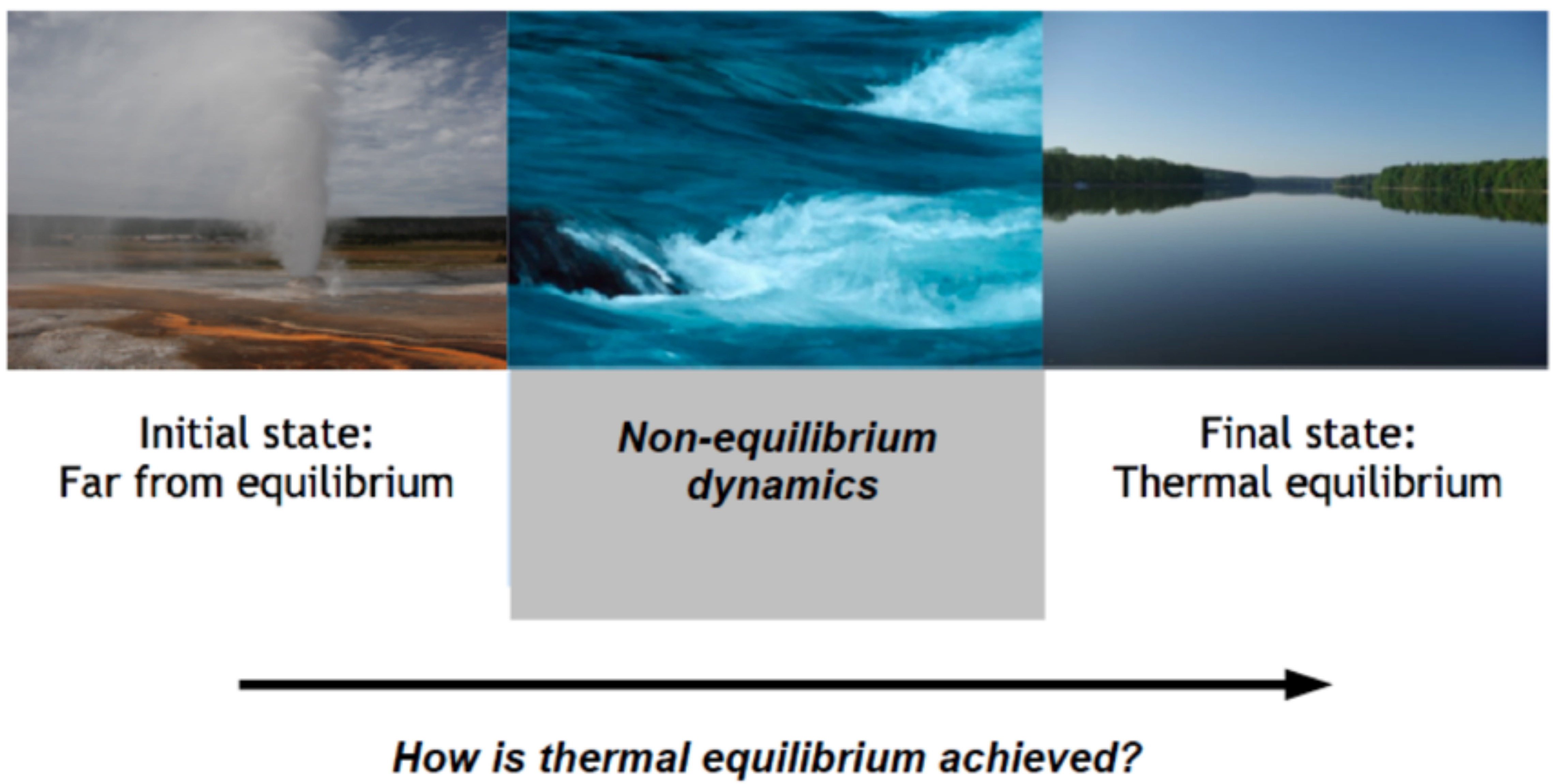
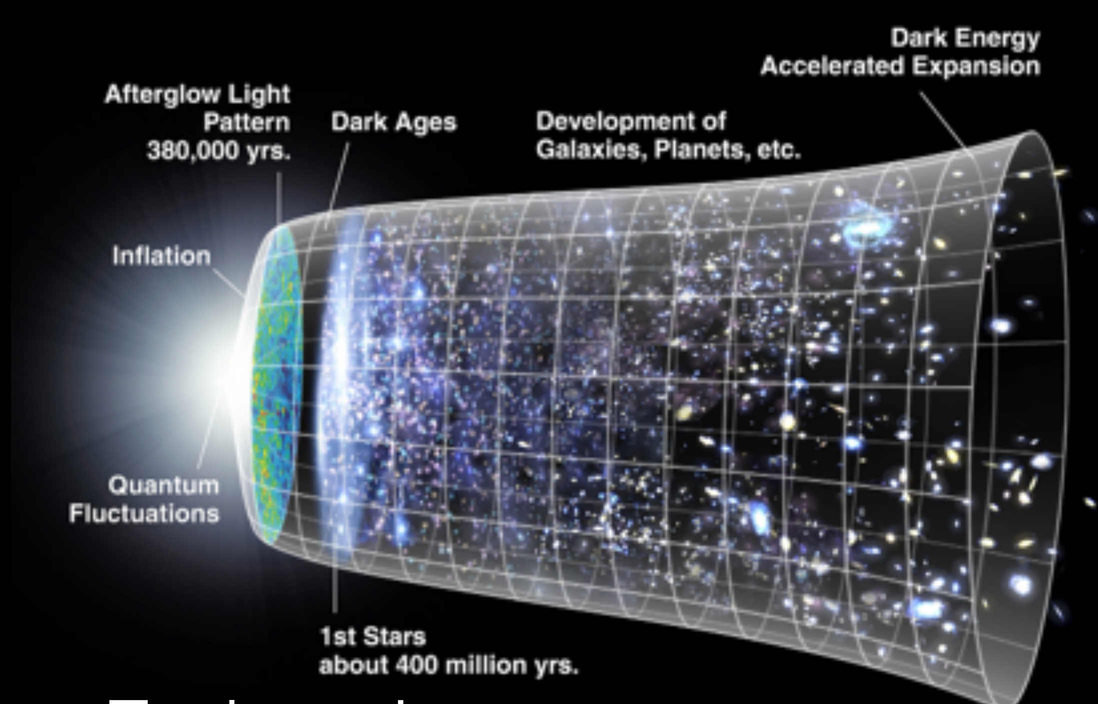
Thermalization process

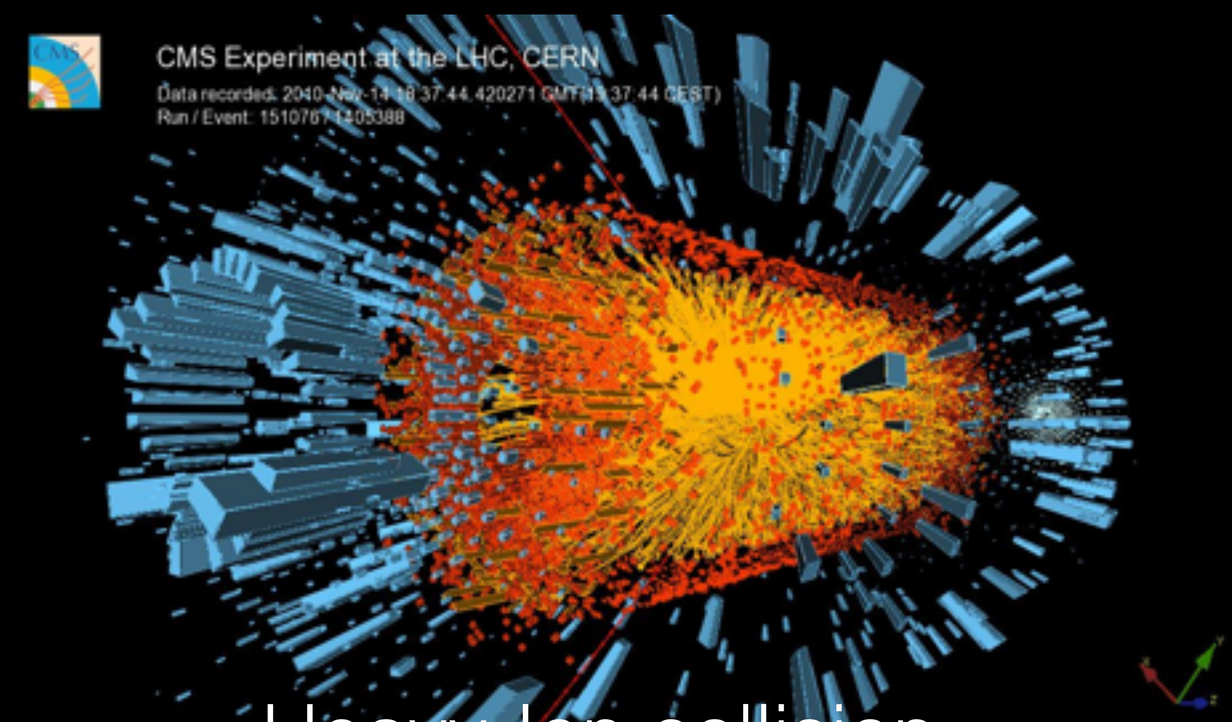


Some examples

- Question about the thermalization mechanism appears in various different systems across all energy scales



Early universe



Heavy-Ion collision

-> Basic mechanism underlying the thermalization process turns out to be very similar for far-from equilibrium systems at vastly different energy scales

Outline

Thermalization process in the early universe (scalar field theory)

- *Basics of thermalization dynamics*
- *Bose condensation far from equilibrium*

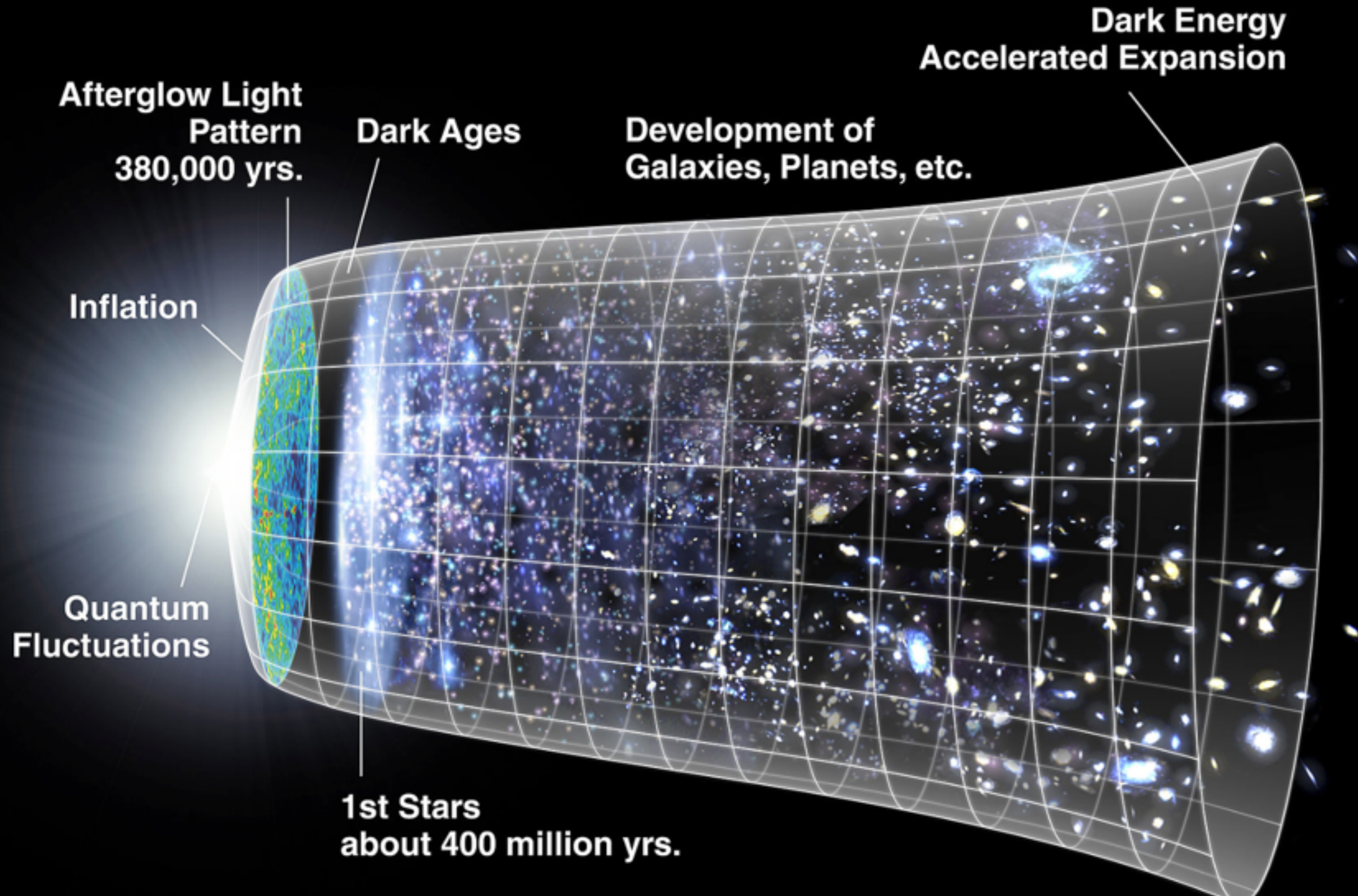
(Micha, Tkachev PRD 70 (2004) 043538; Berges,Boguslavski,SS,Venugopalan JHEP 1405 (2014) 054)

Thermalization process in heavy-ion collisions (Yang-Mills theory)

- *Thermalization process at weak coupling*
- *Universality far from equilibrium*

(Berges,Boguslavski,SS,Venugopalan PRD 89 (2014) 074011 & 114007; PRL114 (2015) 061601)

Summary & Conclusions



Thermalization process in the early universe

Model of reheating after inflation

- Variety of different models for the thermalization of the early universe have been studied
- Consider here only the simplest model of a massless scalar field theory

$$S[\varphi] = \int d^4x \sqrt{-g(x)} \left(\frac{1}{2}(\partial_\mu \varphi_a) g^{\mu\nu} (\partial_\nu \varphi_a) - \frac{\lambda}{24N} (\varphi^2)^2 \right)$$

which can be mapped onto a non-expanding geometry by a conformal transformation *(see e.g. Micha, Tkachev PRD 70 (2004) 043538)*

Coupling constant typically very small

$$\lambda \sim 10^{-8}$$

$$\phi \sim 1/\sqrt{\lambda}$$

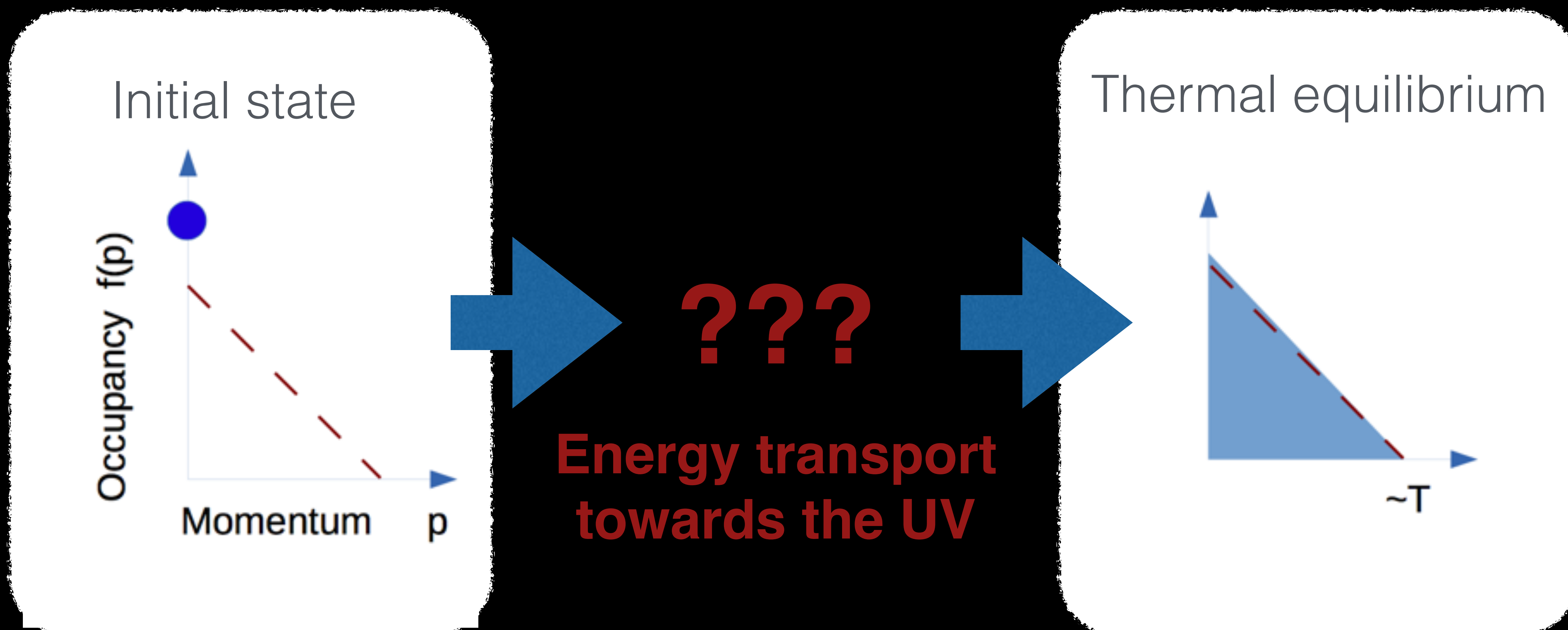
Non-perturbative large field amplitude

-> Even though the coupling constant λ << 1 is small the system is strongly correlated because of large field amplitudes

-> Non-perturbatively large field amplitude allows for an effective description in terms of classical fields

What it takes to thermalize

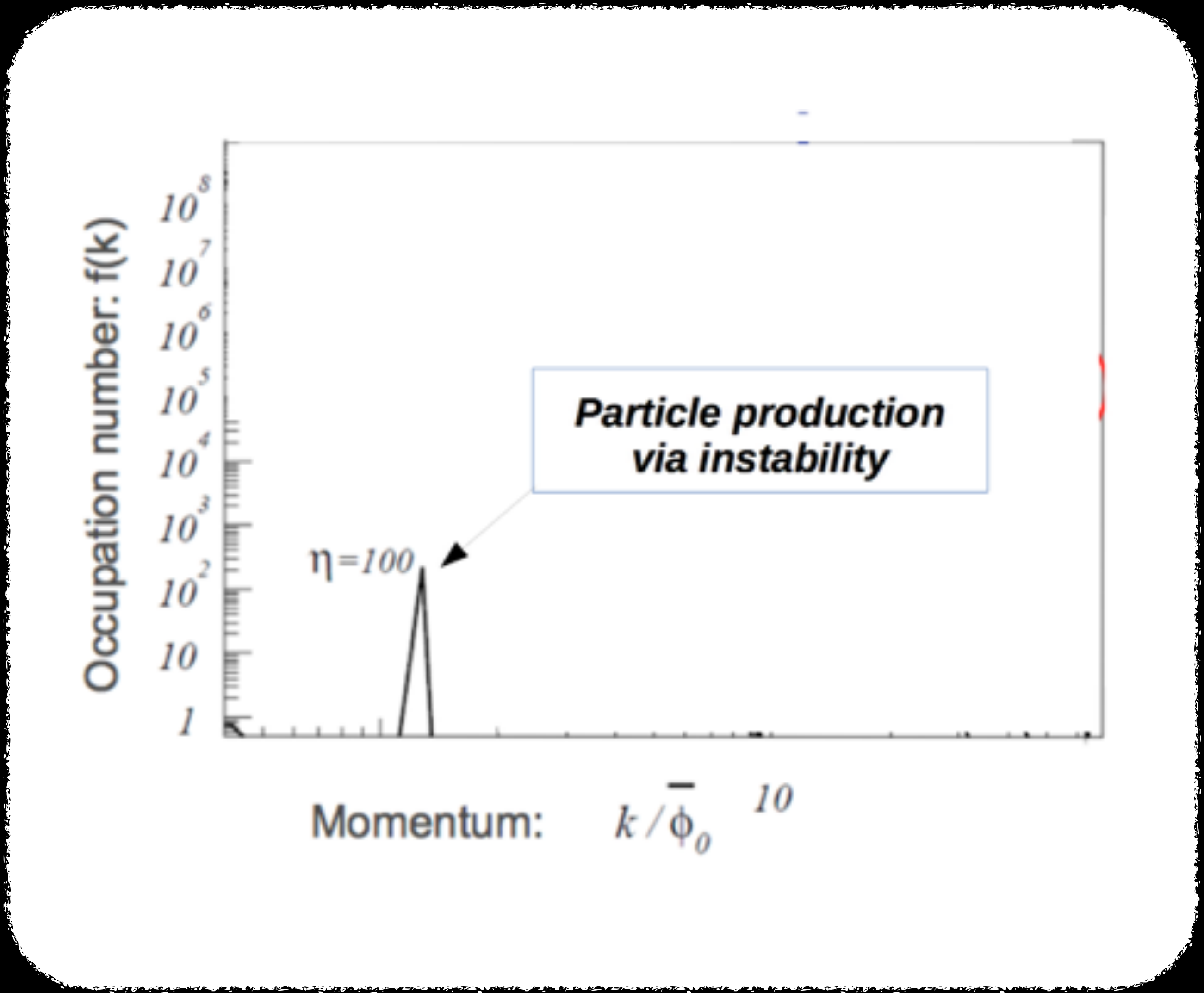
- While initially energy is carried by low momentum excitations, in thermal equilibrium energy is dominated by modes with p~T



-> Thermalization process describes transport of energy towards the ultra-violet

Early time dynamics (Pre-heating)

Classical-statistical field simulations of scalar field theory



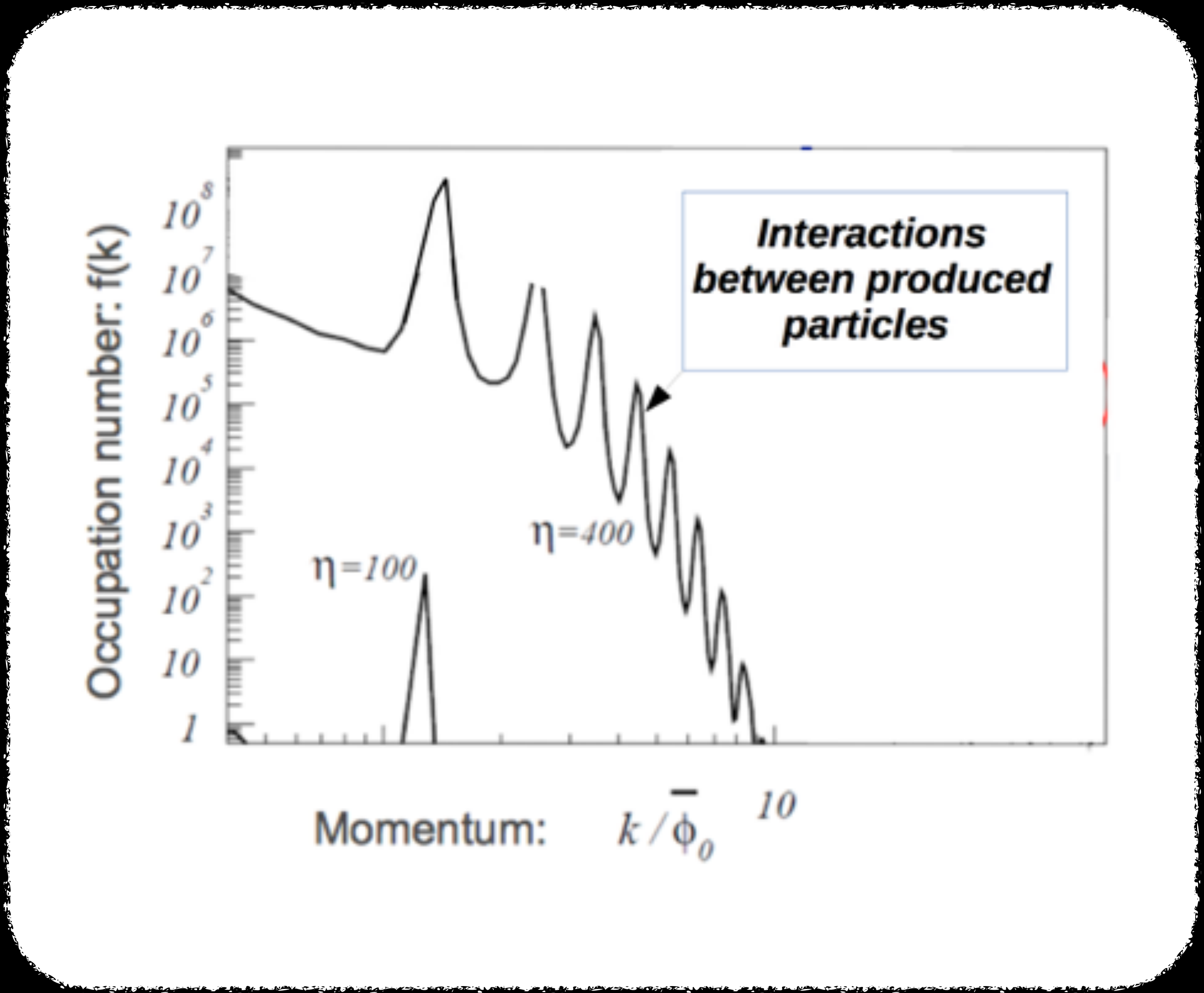
- Oscillation of the background field triggers a parametric resonance instability

-> Particle production in a small momentum region

(Micha, Tkachev PRD 70 (2004) 043538)

Early time dynamics (Pre-heating)

Classical-statistical field simulations of scalar field theory



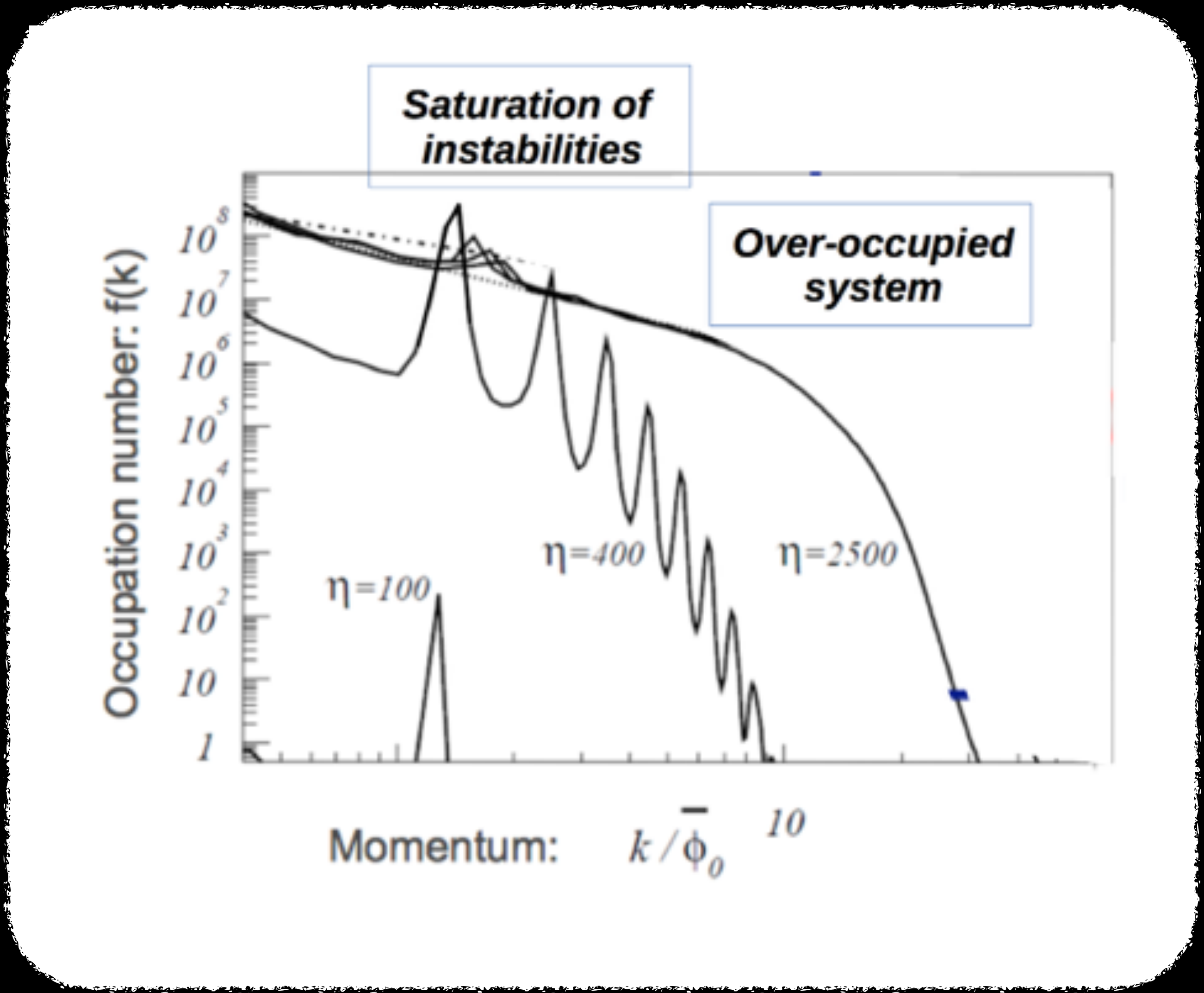
- Once enough particles have been produced, interactions between particles become important

-> Secondary instabilities at larger momenta

(Micha, Tkachev PRD 70 (2004) 043538)

Early time dynamics (Pre-heating)

Classical-statistical field simulations of scalar field theory



- Growth of instabilities saturates, when the occupancy becomes on the order of the inverse self-coupling

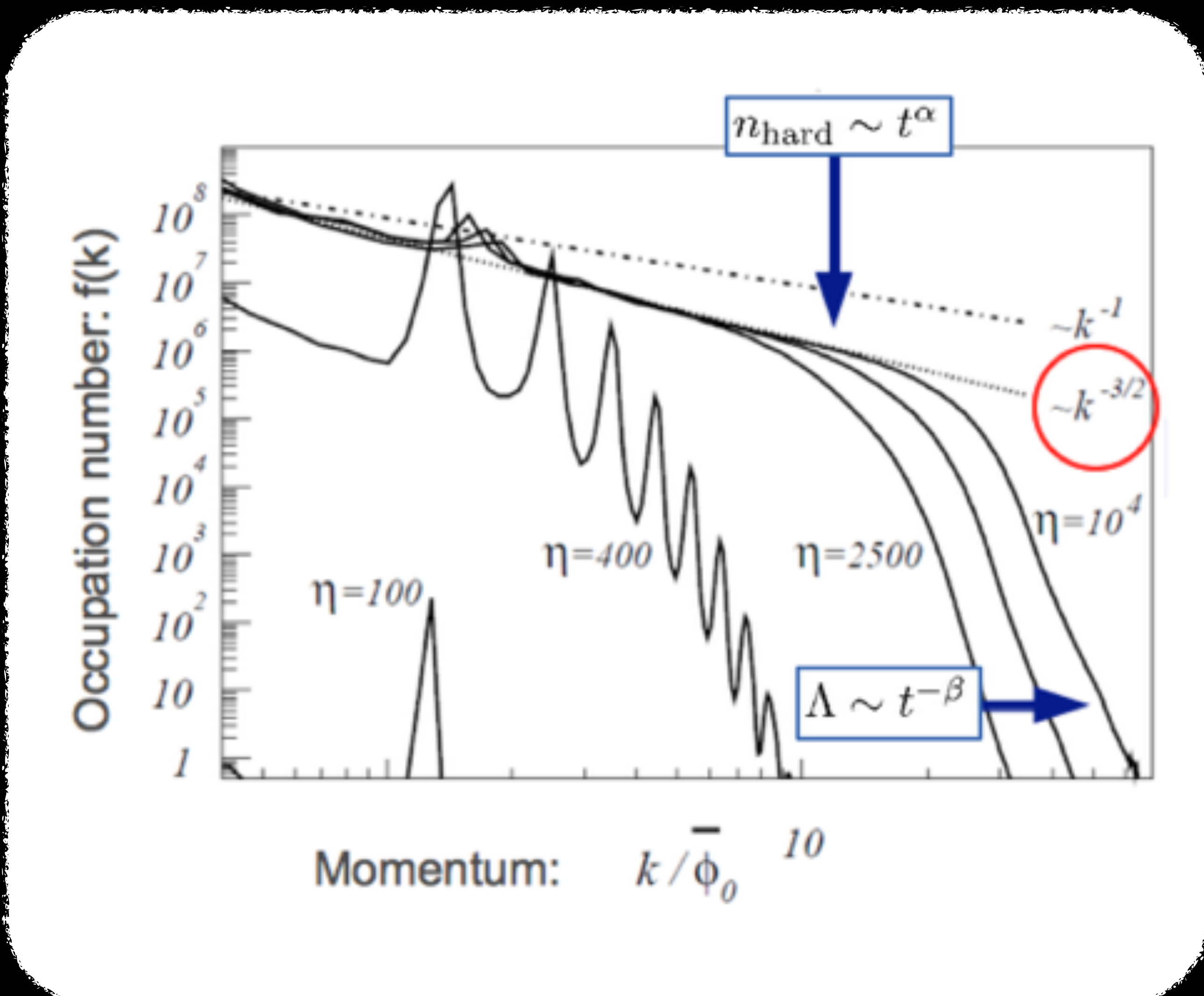
-> Highly-occupied system

Still far from equilibrium

(Micha, Tkachev PRD 70 (2004) 043538)

Thermalization process (Re-heating)

Classical-statistical field simulations of scalar field theory



(Micha, Tkachev PRD 70 (2004) 043538)

- Dynamics becomes self-similar

$$f(p,t) = t^{\alpha}\ f_S(t^{\beta} p)$$

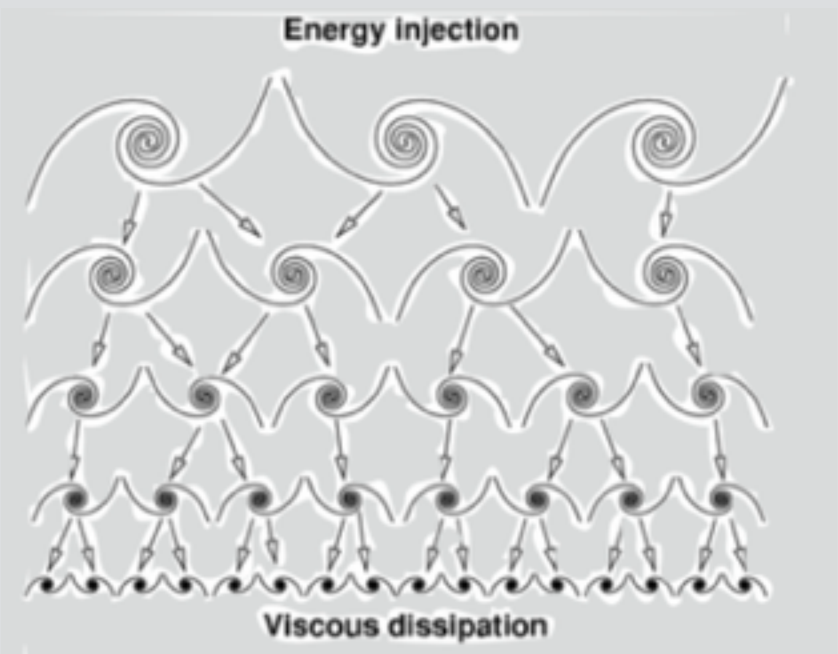
- Evolution of the characteristic momentum and occupancy scales follows simple power law behavior
- Dynamics can be entirely described in terms of scaling exponents *α=-4/5, β=-1/5* and scaling function f_S extracted from simulations

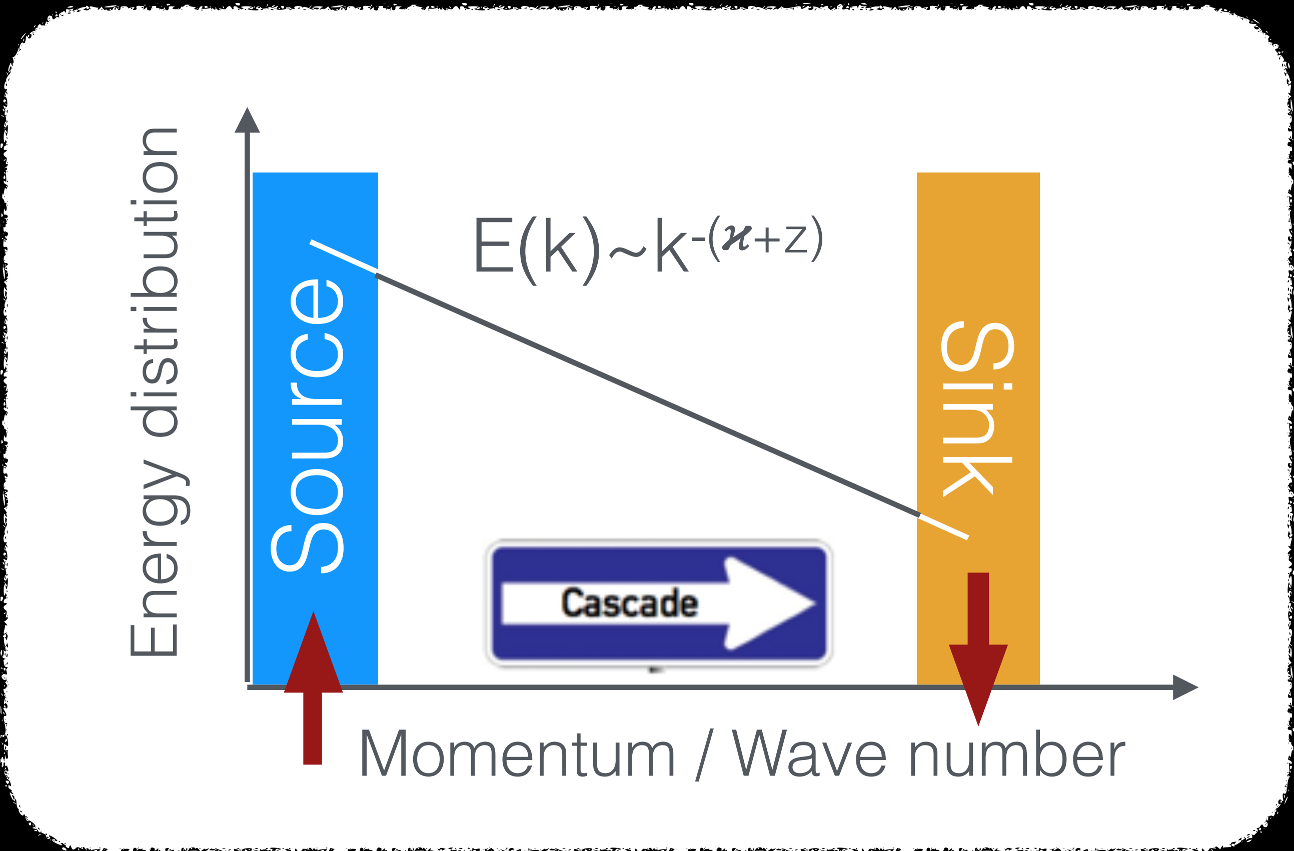
Wave turbulence

- In general turbulent phenomena can be associated with the transport of conserved quantities across a large separation of scales

-> Prominent example of surface waves in a fluid

Richardson cascade



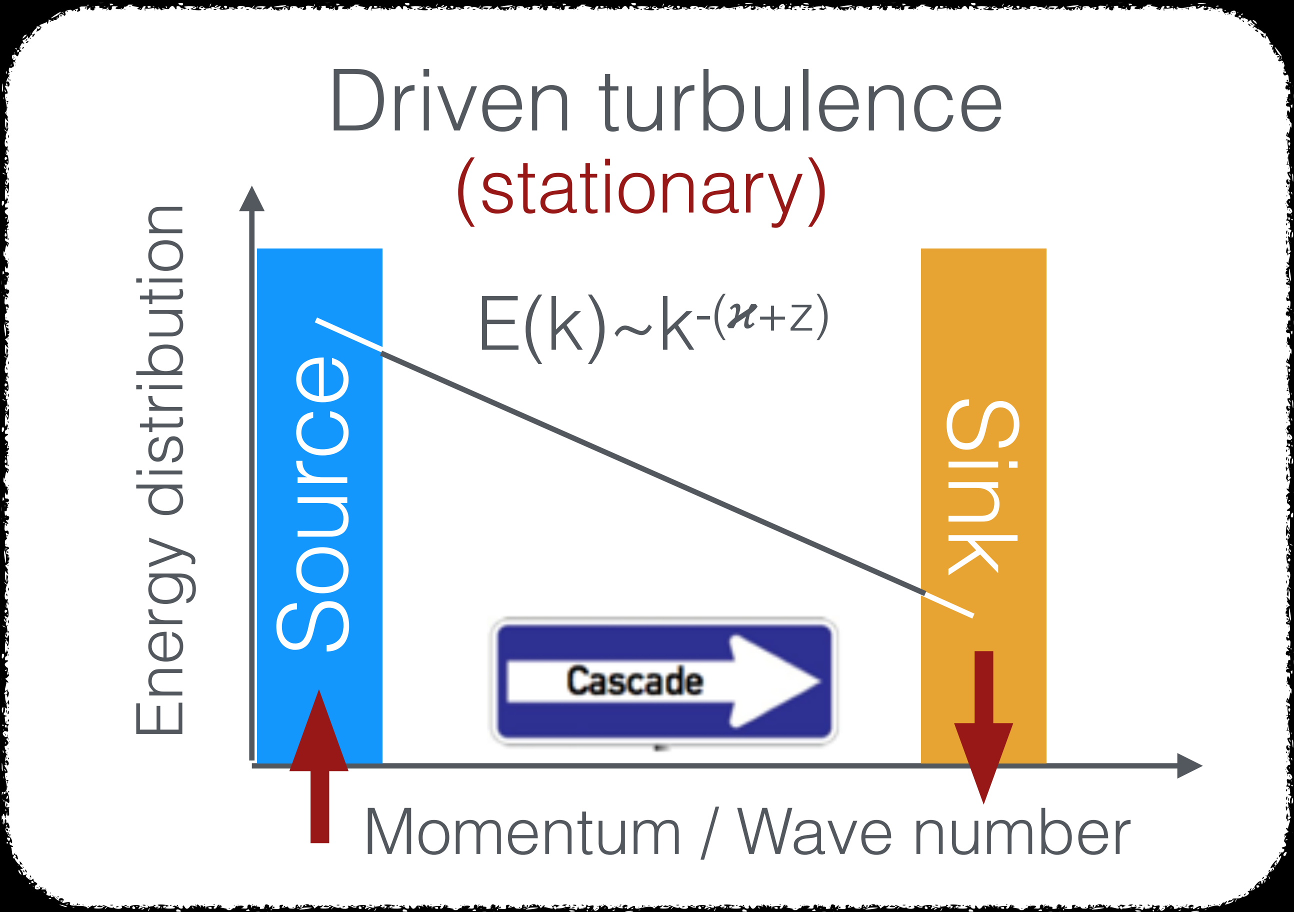


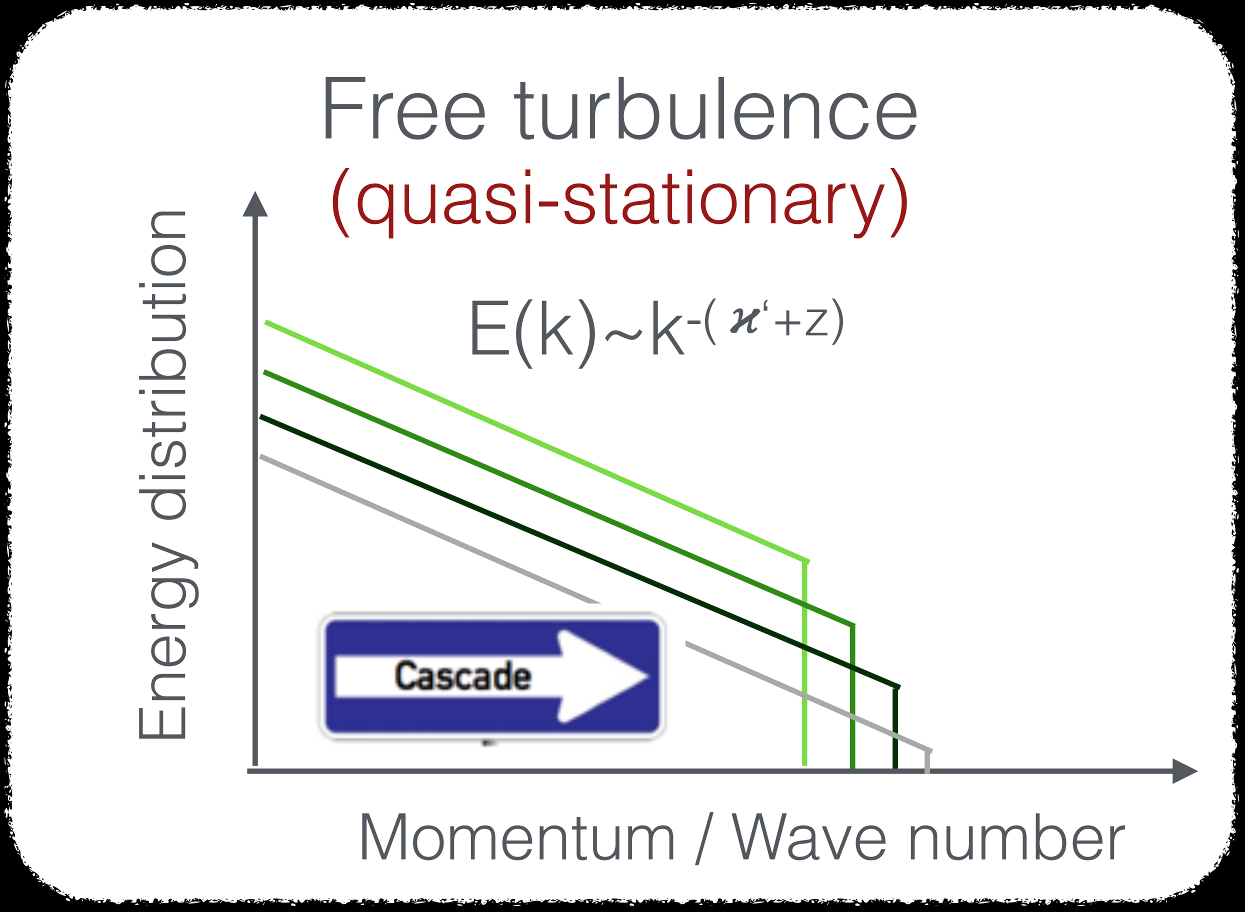
-> When energy is injected at a constant rate ***stationary solutions*** associated with a ***scale independent energy flux*** exist

Characterized by Kolmogoroz-Zhakarov spectra within inertial range of momenta

Stationary turbulence vs. turbulent thermalization

- When considering the thermalization process in a closed system energy is conserved *-> No Source & Sink*





-> Instead of stationary non-equilibrium solution system exhibits quasi-stationary/self-similar solution associated with energy transport

Empirical evidence from simulations suggest that spectral exponents $\varkappa$ and $\varkappa'$ are the same for driven and free turbulence.

Kinetic description

- When the characteristic occupancies become f<1/λ in addition to the classical field description, the dynamics of hard excitations can also be described by kinetic theory

$$\frac{\partial f(t,\mathbf{p})}{\partial t} = C[f](t,\mathbf{p})$$

Search for self-similar scaling solution

$$f(p,t) = t^{\alpha}\ f_S(t^{\beta} p)$$

Scaling behavior of the collision integral

$$\xrightarrow[(f \gg 1)]{scale\ \ invariance} C[f](p,t) = t^{\mu} C[f_S](t^{\beta} p)$$

-> Boltzmann equation can be decomposed into

$$[\alpha + \beta\, \mathbf{p} \cdot \nabla_{\mathbf{p}}]\, f_S(\mathbf{p}) = C[f_S](1,\mathbf{p}),$$

$$\alpha - 1 = \mu(\alpha, \beta)$$

time independent fixed-point condition

scaling relation

(Micha, Tkachev PRD 70 (2004) 043538)

Kinetic description

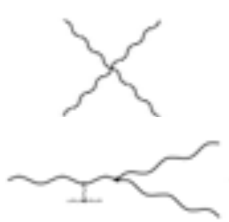
- Dynamical scaling exponents *α, β* are uniquely determined by

Scaling of the collision integral + **Conservation laws (energy)**

$$\alpha - 1 = \mu(\alpha, \beta)$$

$$\alpha = \beta(d + z)$$

- Independence of microscopic parameters (e.g. coupling constant, number of field components,…) allows for a universal classification scheme

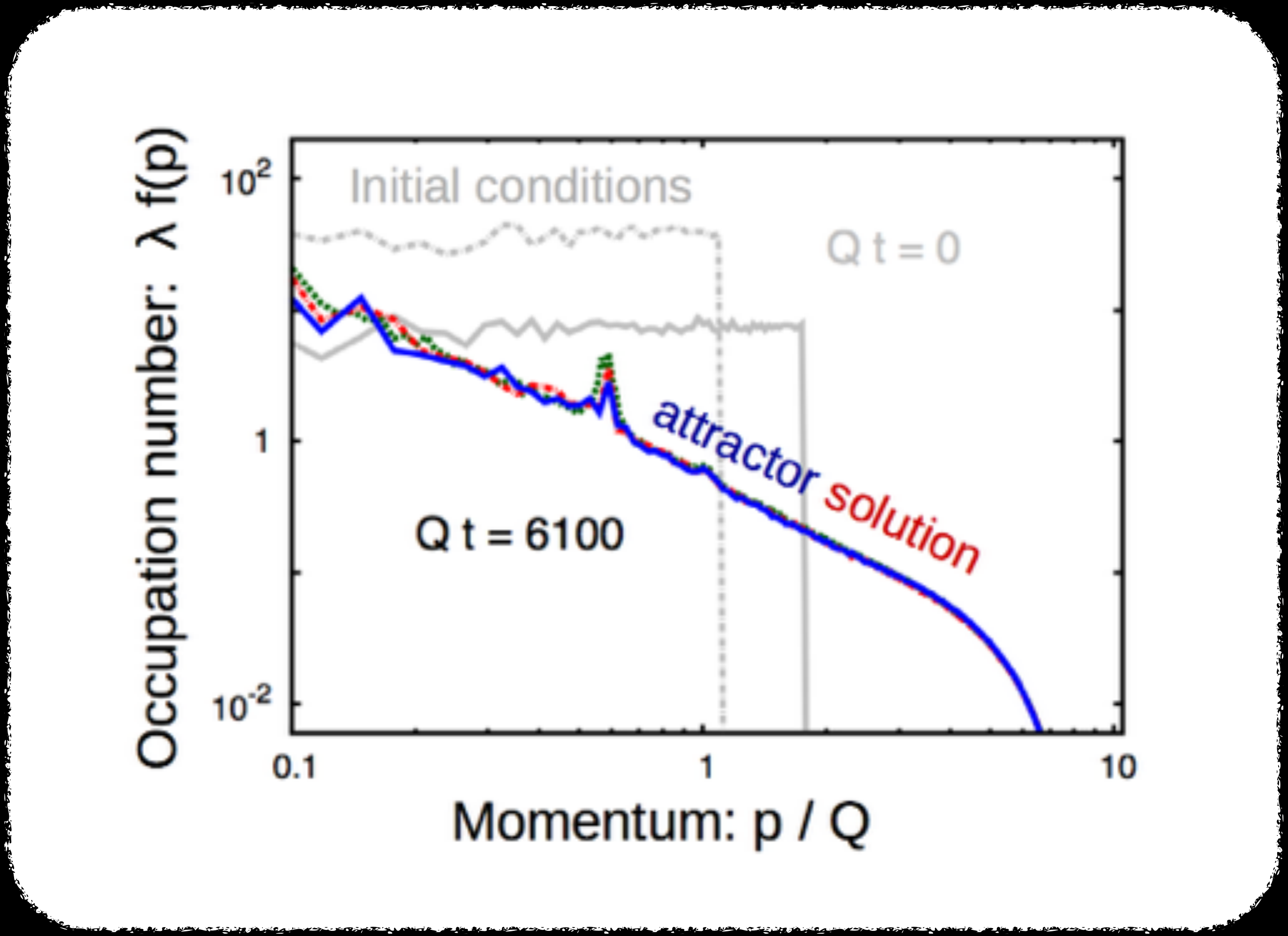
	Interaction		Hard scale evolution (Exponent α)	Occupancy evolution (Exponent β)
scalar theory		**2<->1+soft**	**-1/5**	**-4/5**
gauge theory		**2<->2 & eff. 2<->1**	**-1/7**	**-4/7**

(relativistic theories (z=1) in d=3 dimensions)

(Micha, Tkachev, Kurkela, Moore, Berges, SS, Venugopalan, …)

Independence of initial conditions

- Consider a different set of initial conditions where instead of a large background field the energy is carried by a large number of low momentum excitations



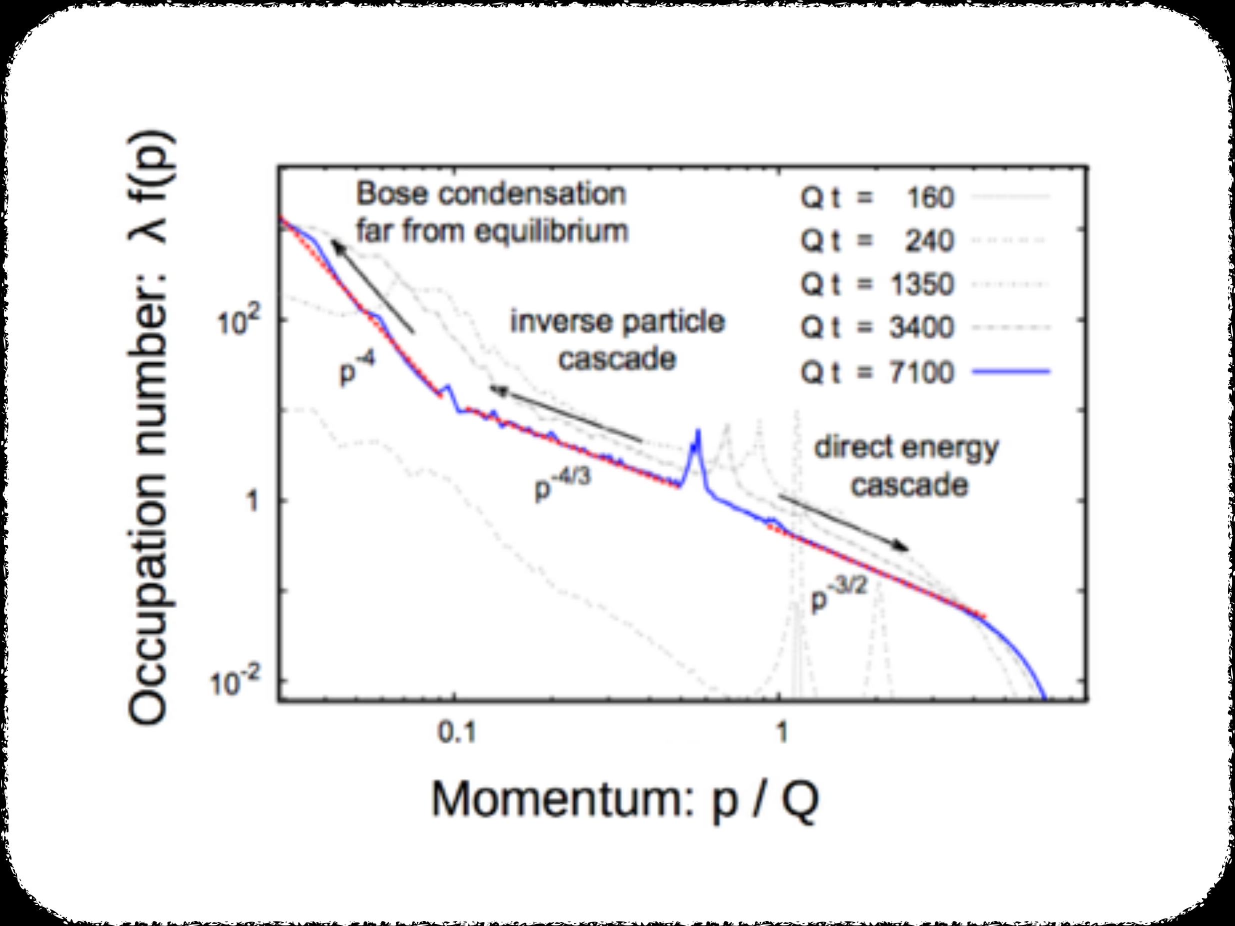
- Effective memory loss occurs already at early times
- The turbulent scaling behavior is independent of the initial conditions

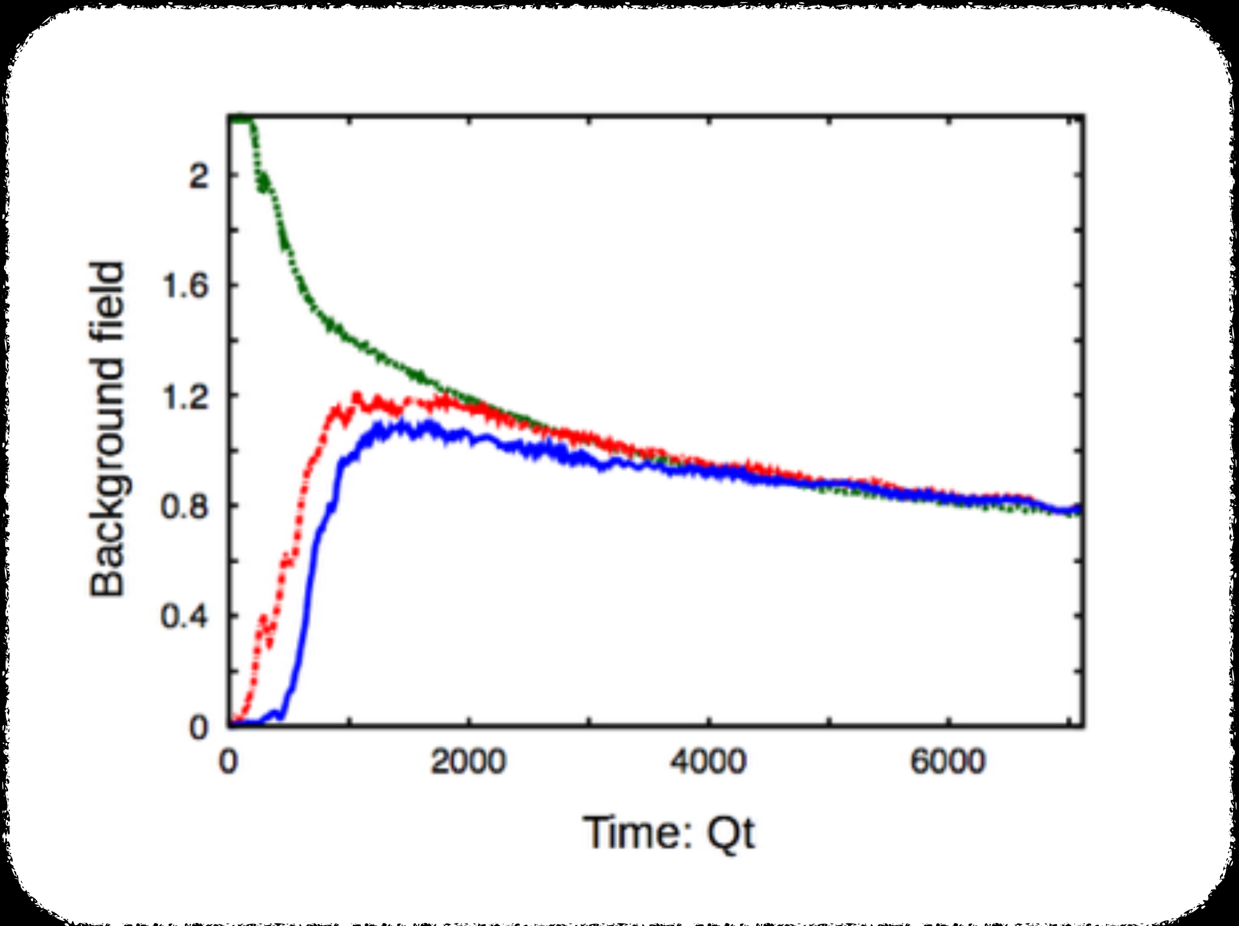
-> Generic property of the thermalization process

(Berges,Boguslavski,SS,Venugopalan JHEP 1405 (2014) 054)

Infrared dynamics & Bose condensation

- Since particle number changing processes are highly suppressed in the scalar theory, an infrared particle cascade develops and leads to the formation of a Bose condensate

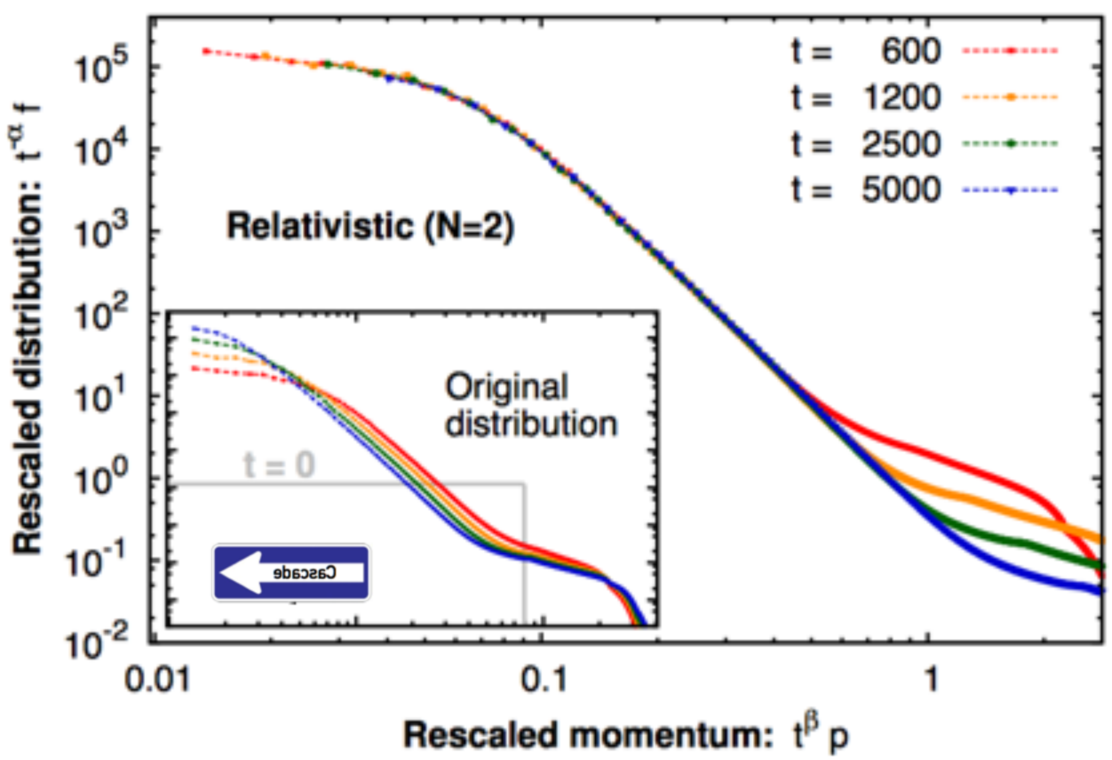




(Berges,Boguslavski,SS,Venugopalan JHEP 1405 (2014) 054)

Infrared dynamics & Bose condensation

O(2) scalar



- Infrared dynamics described by an inverse particle number cascade with dynamical scaling exponents

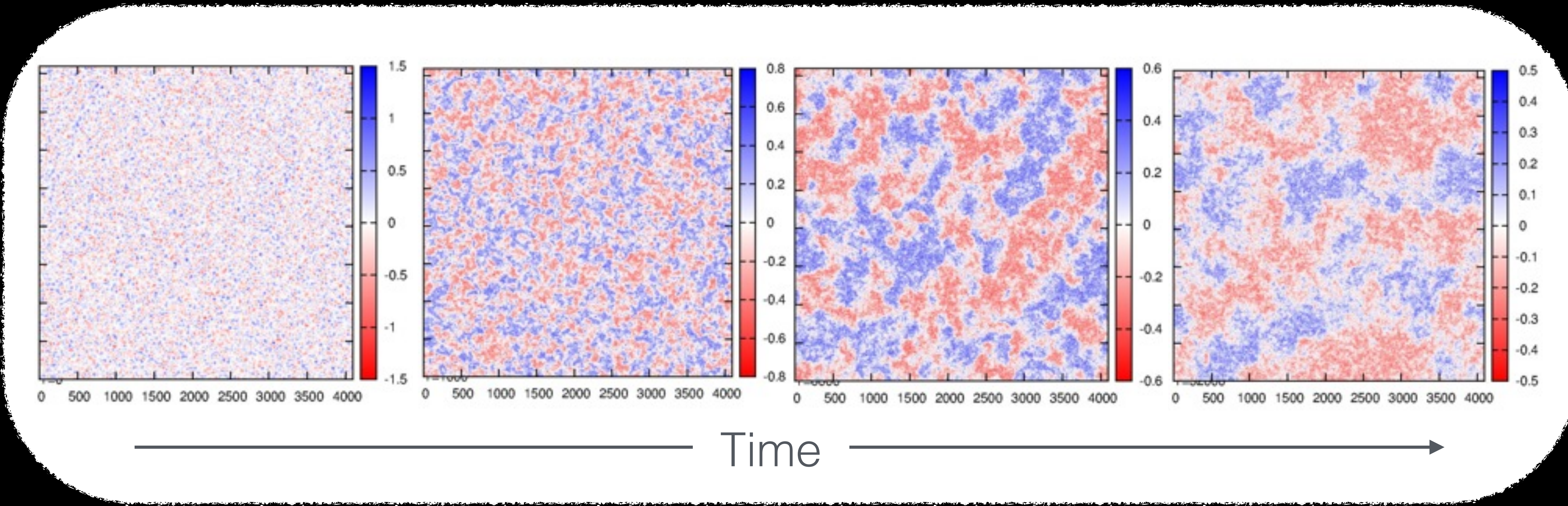
$$\alpha = 1.51 \pm 0.13\,, \qquad \beta = 0.51 \pm 0.04\,,$$

- Can be understood in terms of eff. kinetic description with vertex corrections (2PI 1/N)

(Berges,Boguslavski,Pinero Orioli arXiv:1503.02498)

Coordinate space picture

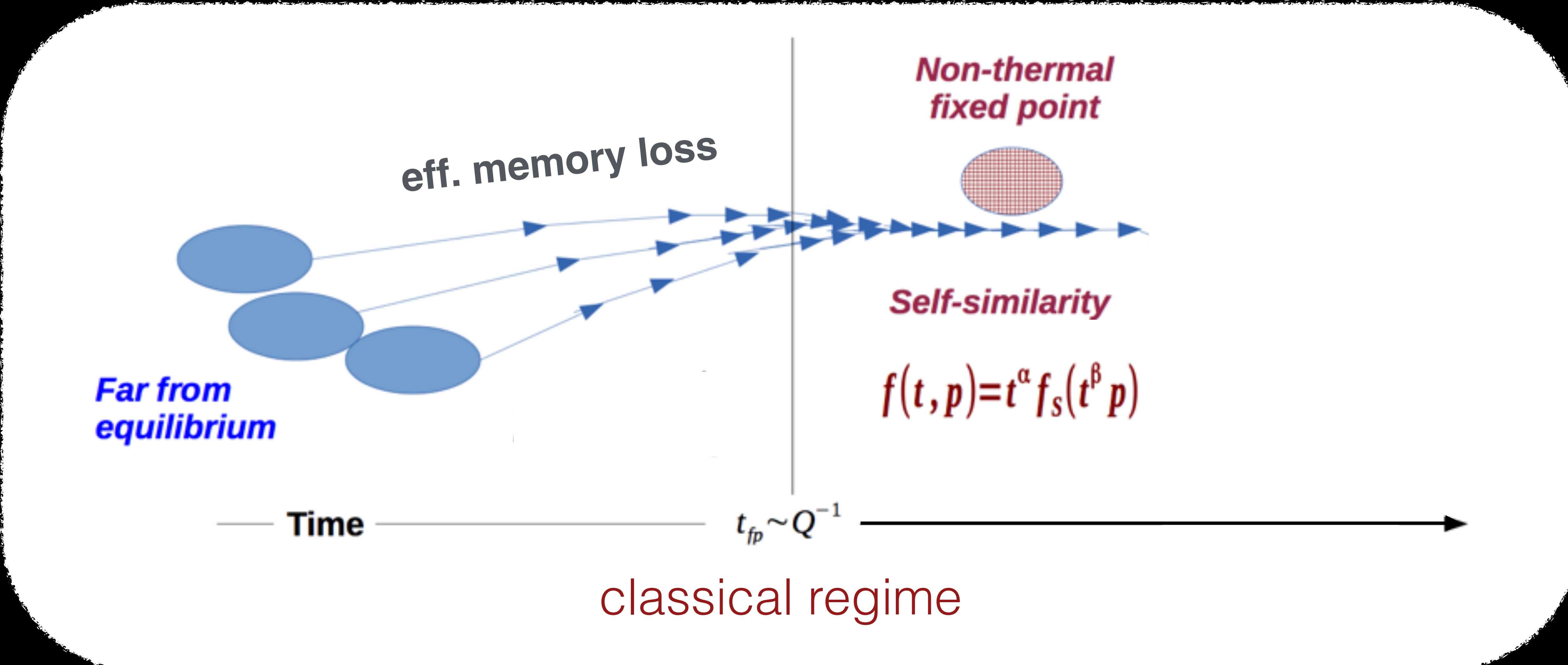
- Visualization of infrared dynamics in 2+1 D massless scalar theory



(SS, Yin work in progress)

-> Emergence of macroscopic domains of correlated Φ fields and domain growth as a function of time

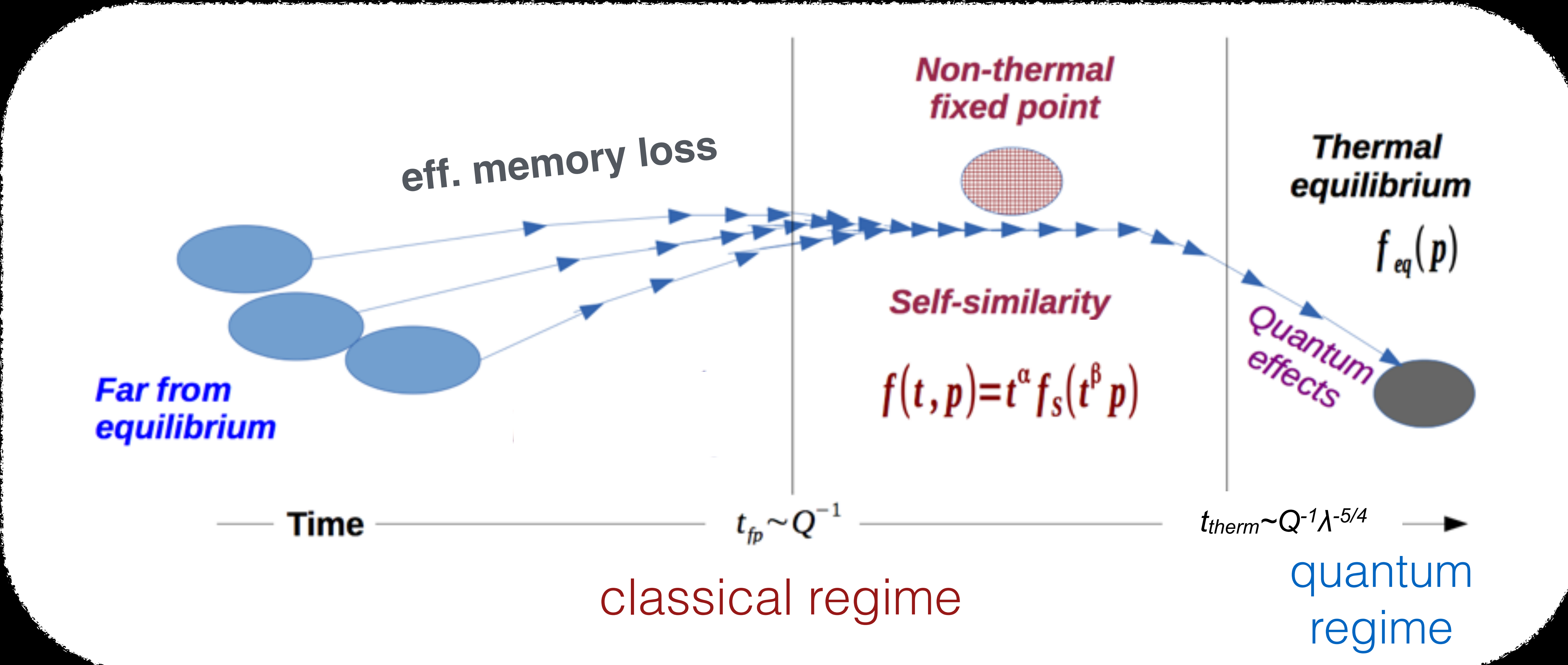
Qualitative picture



-> Effective memory loss occurs on a time scale of a few interactions and different initial conditions lead to the same attractor solution

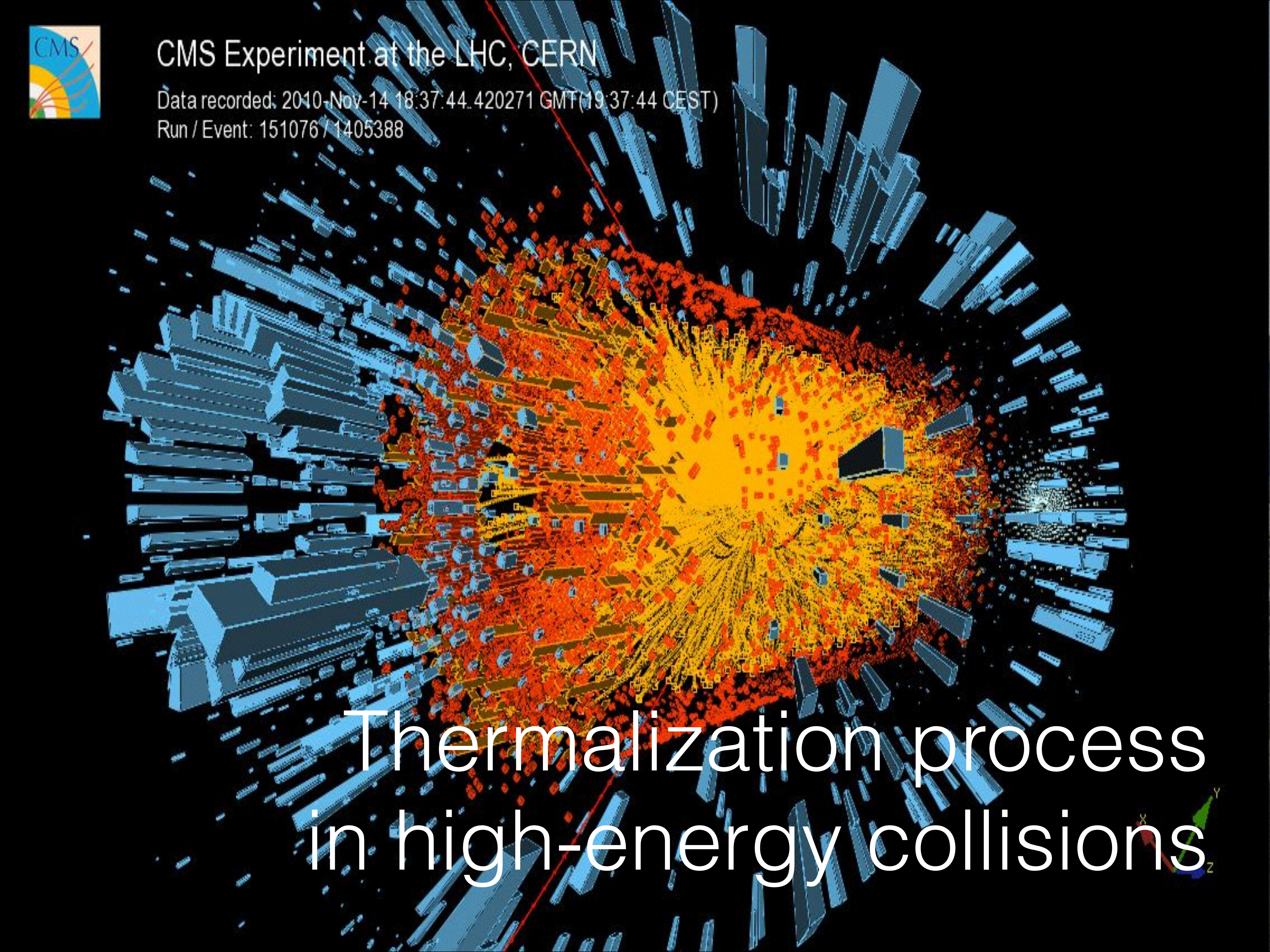
-> Thermalization process characterized by a self-similar evolution in time associated with energy transport towards the UV

Qualitative picture



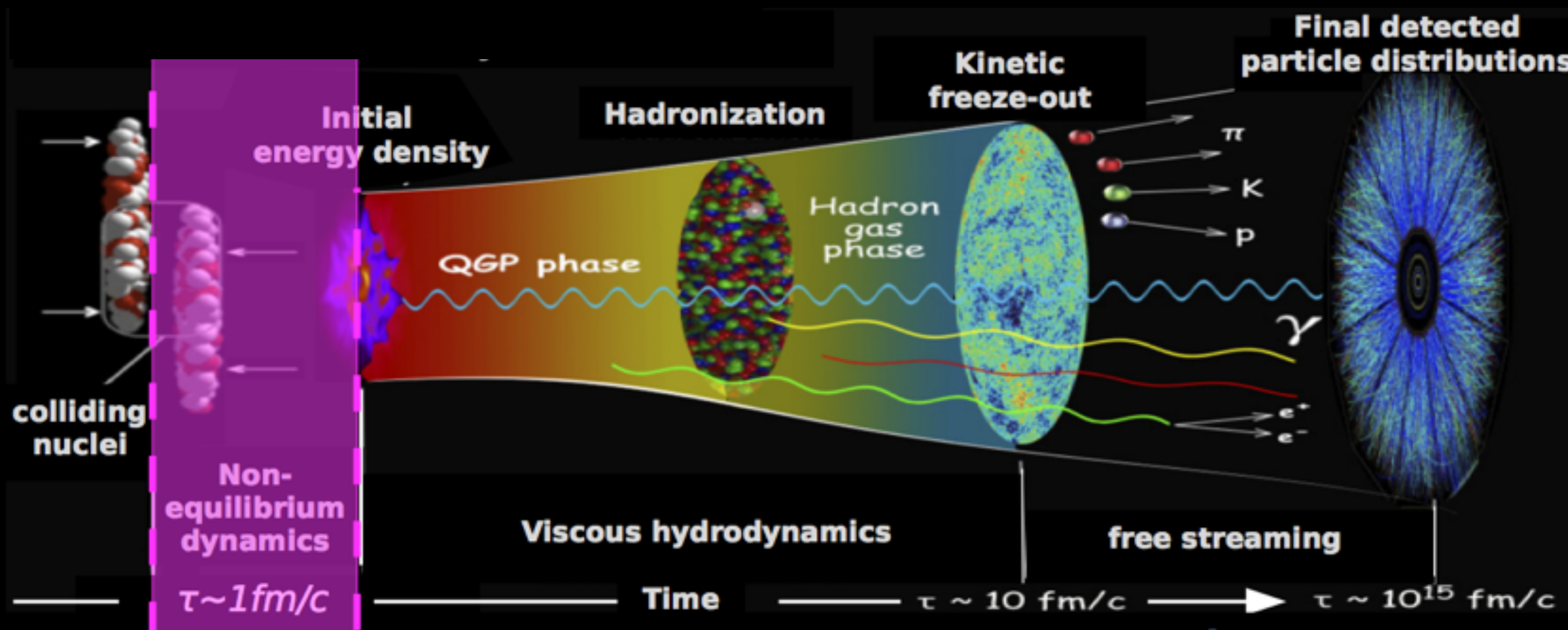
-> Energy transport towards UV is accomplished on the same time sale when the typical occupancies become small f~1

-> Even though the classical description is no longer applicable beyond this time scale the system is essentially thermal at this point and quantum effects will ultimately drive it towards equilibrium.



Thermalization process in high-energy collisions

Standard model of a heavy-ion collision

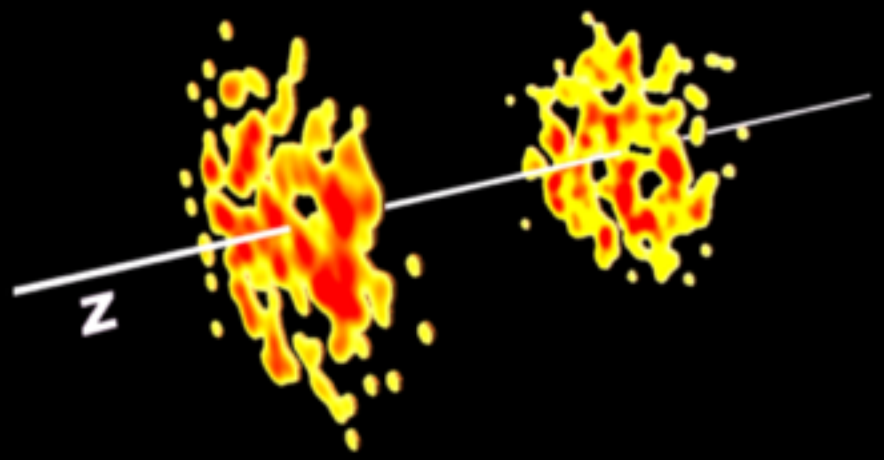


- Well established phenomenology describing the space-time evolution of heavy-ion collisions at later
- Little understanding of how a thermalized quark gluon plasma is formed starting from the collision of heavy nuclei

Weak-coupling picture

Color glass condensate framework

- High-energy nuclei feature a large number of small-x gluons with typical momentum $Q_s \gg \Lambda_{QCD}$



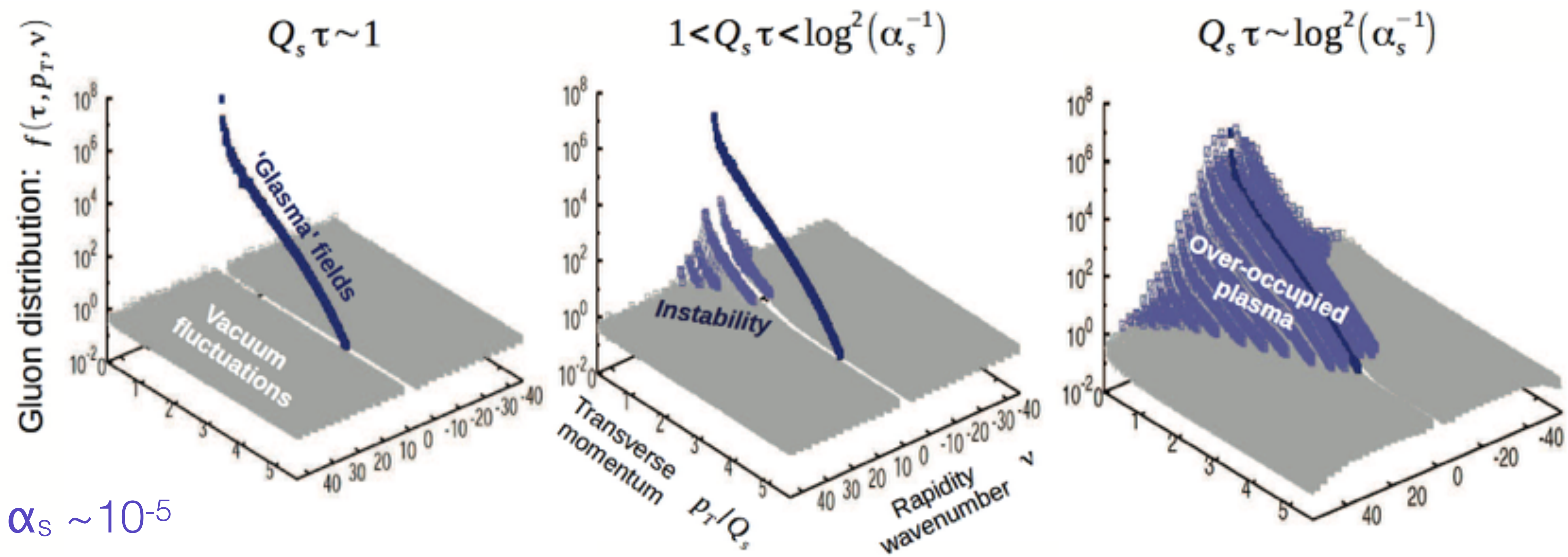
- Collision of high-energy nuclei leads to a far-from equilibrium state 'Glasma' characterized by a large phase space occupancy of gluons

$$f(p \sim Q_s) \sim 1/\alpha_s$$

-> Even though the relevant coupling $\alpha_s(Q_s) \ll 1$ is small the system is strongly correlated because of high gluon density

Early time dynamics

Classical Yang-Mills simulations in CGC framework @ NLO



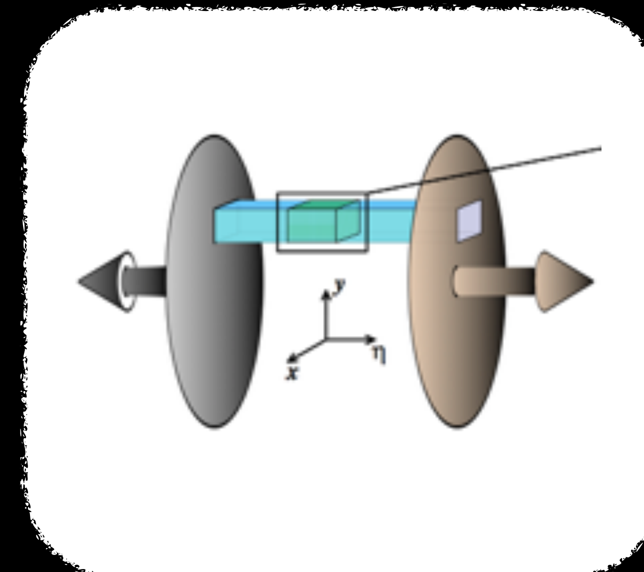
Berges, Schenke, SS, Venugopalan arXiv:1409.1638

- Early time dynamics is described by plasma instabilities

  -> Highly occupied plasma

Qualitative description of thermalization process

- We will neglect the transverse expansion of the system and consider a system which is only expanding in the longitudinal direction

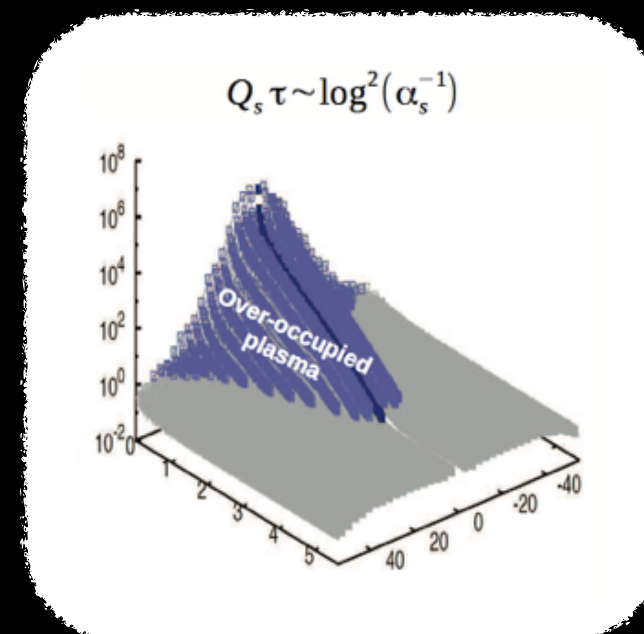


- Characterize the initial state at $\tau_0 \sim 1/Q_s$ in terms of the initial gluon distribution

initial occupancy

$$f(p_T, p_z, \tau_0) = \frac{n_0}{\alpha_s} \Theta\left(Q - \sqrt{p_T^2 + (\xi_0 p_z)^2}\right)$$

initial anisotropy

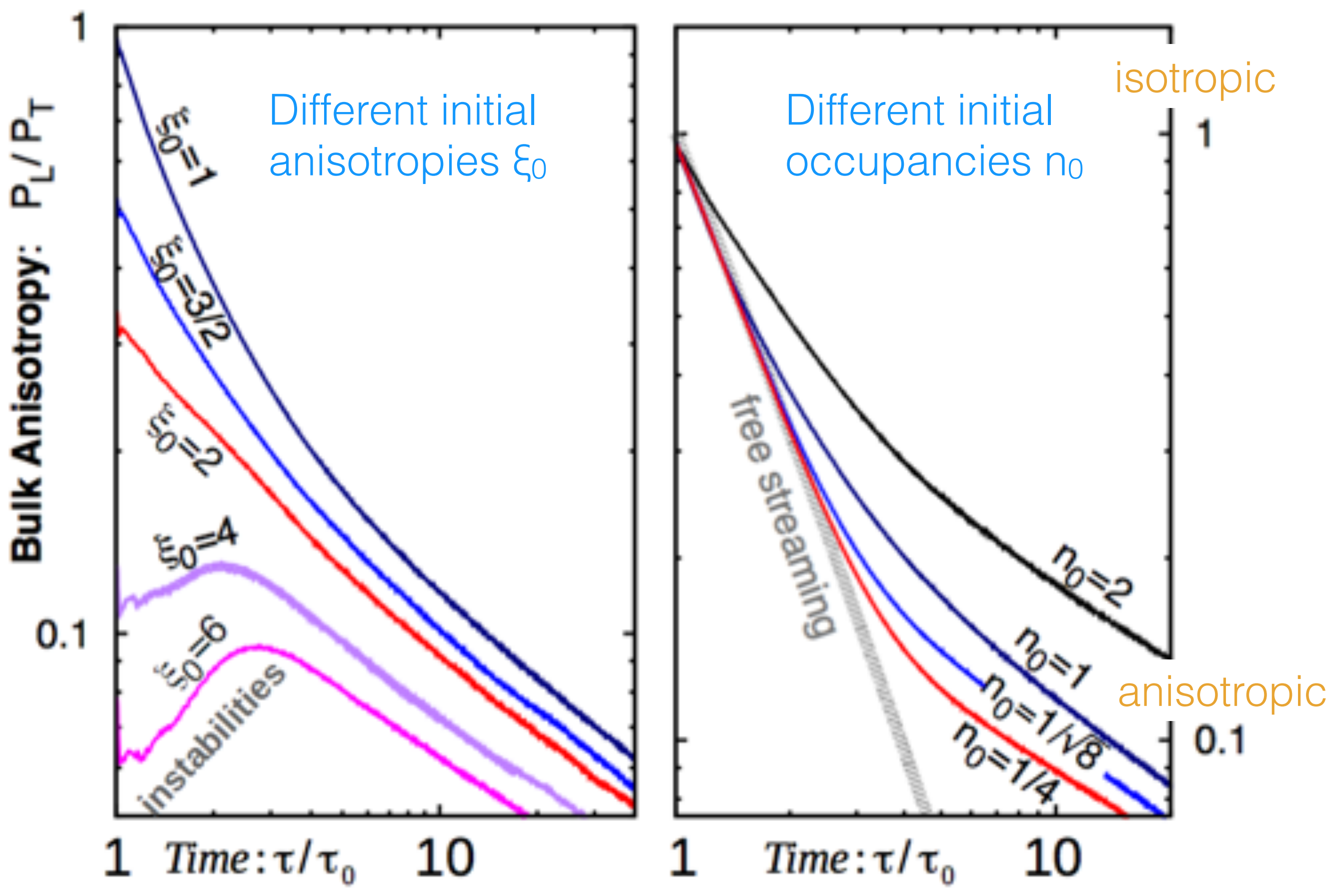


-> High gluon densities allow for an effectively classical description of the initial stages

Hydrodynamic quantities

Classical Yang-Mills simulations for a variety of initial conditions

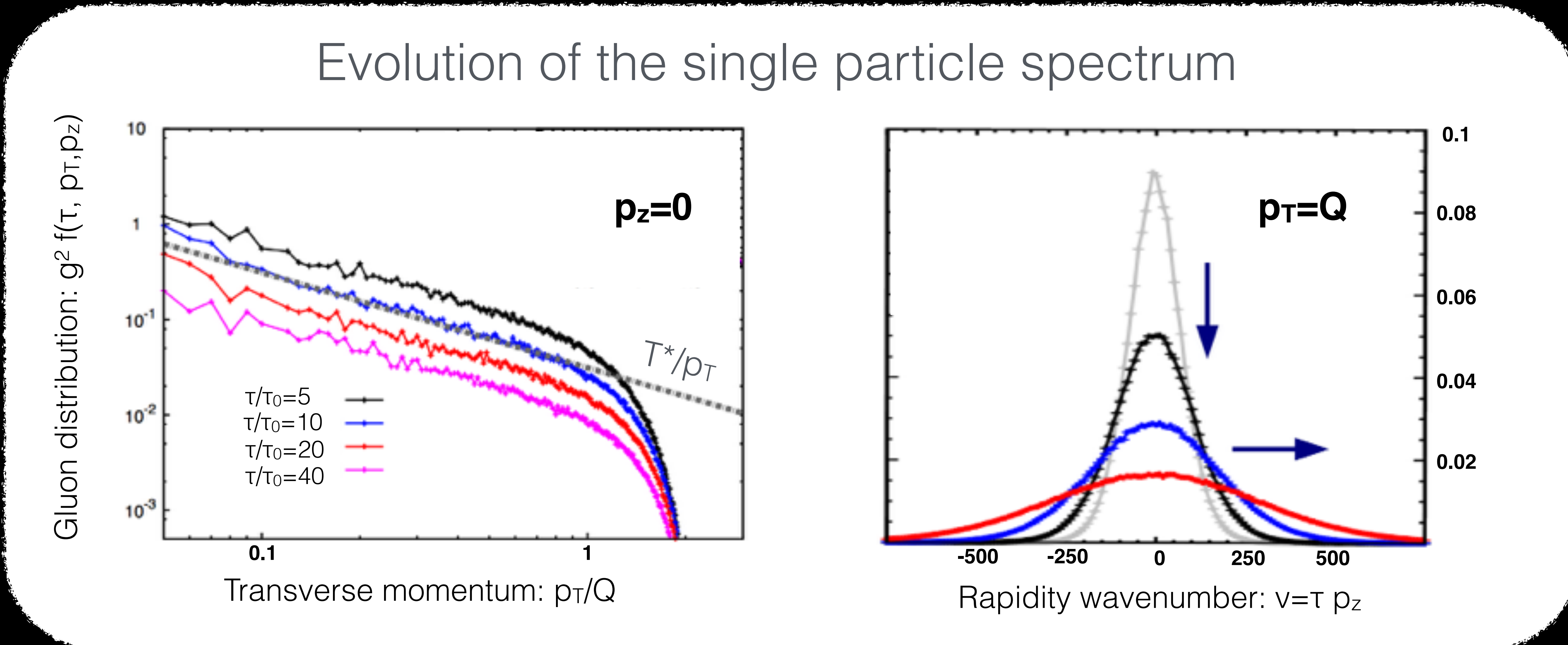
Evolution of the bulk anisotropy



- Small occupancy leads to initial free streaming behavior.
- Large initial anisotropy leads to transient increase of isotropy via plasma instabilities
- The system remains ***strongly interacting*** throughout the entire evolution.
- At late times, the evolution becomes ***insensitive to the details of the initial conditions*** and the anisotropy increases.

(Berges,Boguslavski,SS,Venugopalan PRD 89 (2014) 074011; 89 (2014) 114007)

Microscopic properties

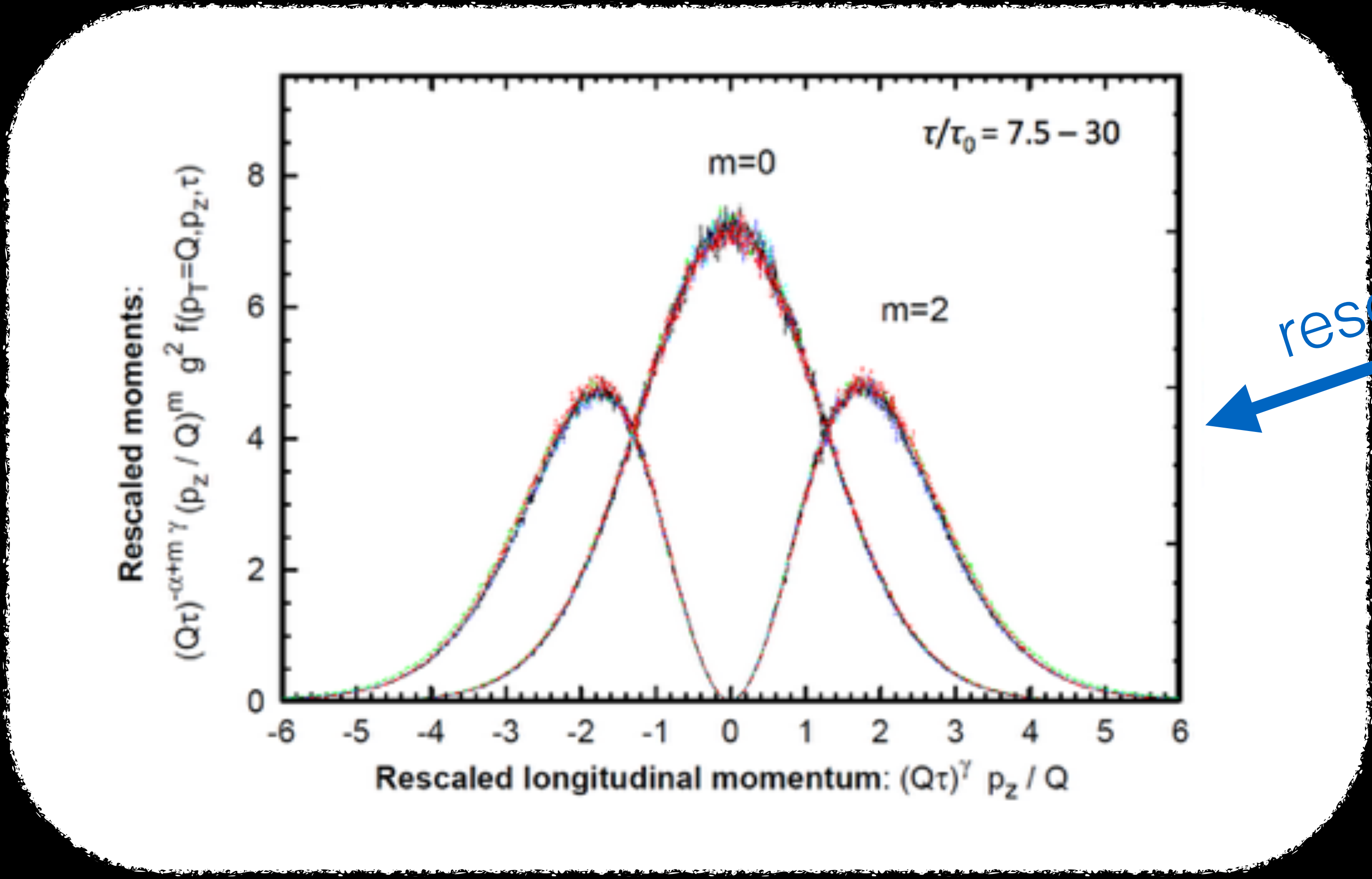


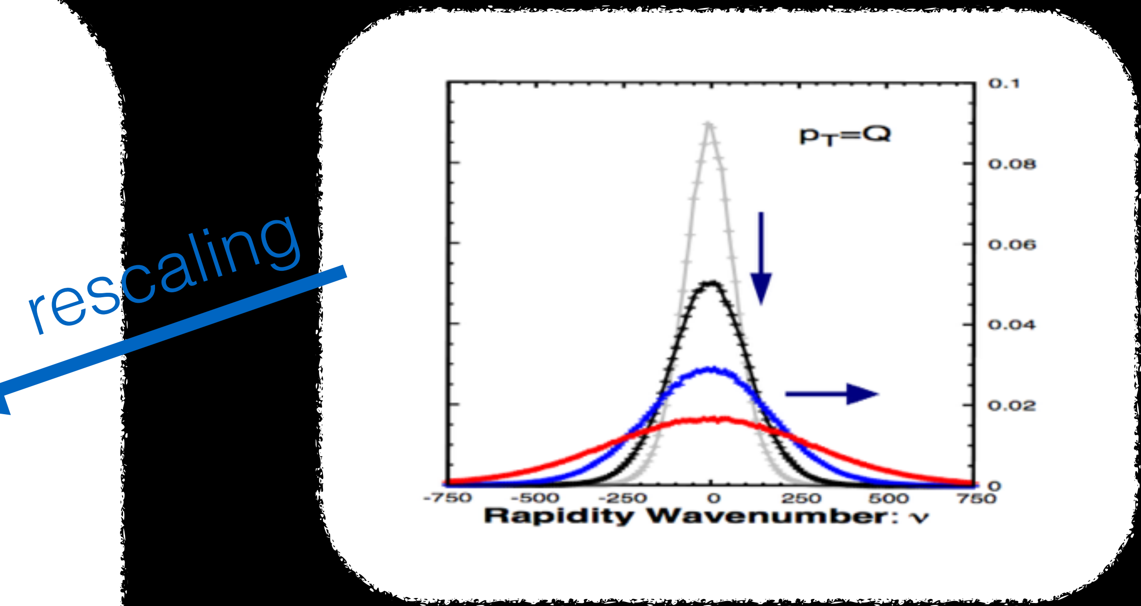
Transverse spectrum shows thermal-like $1/p_T$ behavior up to Q_s.

Dynamics in the scaling regime consists of *longitudinal momentum broadening* — not strong enough to completely compensate for red-shift due to longitudinal expansion

(Berges,Boguslavski,SS,Venugopalan PRD 89 (2014) 074011; 89 (2014) 114007)

Self-similarity





Dynamics can be entirely described in terms of universal scaling exponents *α=-2/3, β=0, γ=1/3* and scaling function f_S extracted from simulations

$$f(\mathrm{p_T}, \mathrm{p_z}, \tau) = (Q\tau)^{\alpha} f_S\Big((Q\tau)^{\beta}\mathrm{p_T}, (Q\tau)^{\gamma}\mathrm{p_z}\Big)$$

(Berges,Boguslavski,SS,Venugopalan PRD 89 (2014) 074011; 89 (2014) 114007)

Kinetic theory interpretation

Consider the Boltzmann equation (c.f. Baier et al. PLB 502 (2001) 51-58)

$$[\partial_\tau - \frac{p_z}{\tau}\partial_{p_z}] f(p_T, p_z, \tau) = C[f](p_T, p_z, \tau)$$

with a self-similar evolution

$$f(p_T, p_z, \tau) = (Q\tau)^\alpha f_S((Q\tau)^\beta p_T, (Q\tau)^\gamma p_z)$$

→ ***Non-thermal fixed point solution*** $(f \gg 1)$

$$[\alpha + \beta p_T \partial_{p_T} + (\gamma - 1) p_z \partial_{p_z}] f_S(p_T, p_z) = Q^{-1} C[f_S](p_T, p_z)$$

→ ***Scaling exponents determined by scaling relations for***

- Small angle elastic scattering $(2\alpha - 2\beta + \gamma = -1)$
- Energy conservation $(\alpha - 3\beta - \gamma = -1)$
- Particle number conservation $(\alpha - 2\beta - \gamma = -1)$

→ $\alpha = -2/3, \beta = 0, \gamma = 1/3$ ***in excellent agreement with lattice data!***

Scaling exponents independent of detailed microscopic features
-> Chance to observe identical behavior in different physical systems

Expanding scalar field theory

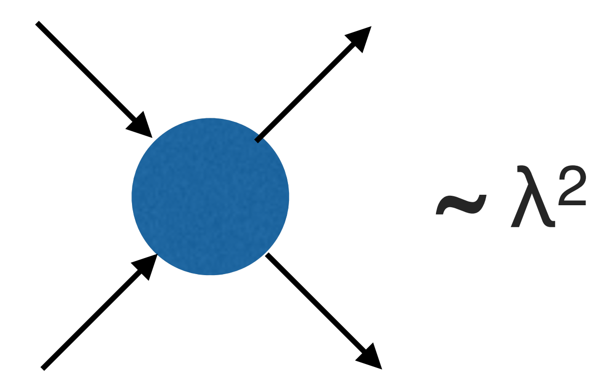
Consider massless N-component scalar field theory with quartic self-interaction

$$S[\varphi] = \int d^4x \sqrt{-g(x)} \left(\frac{1}{2}(\partial_\mu \varphi_a) g^{\mu\nu} (\partial_\nu \varphi_a) - \frac{\lambda}{24N} (\varphi^2)^2 \right)$$

in a longitudinally expanding setup.

Even though perturbatively there is no preference for small angle scattering, one at least expects elastic processes to dominate

-> Energy conservation
& Particle number conservation

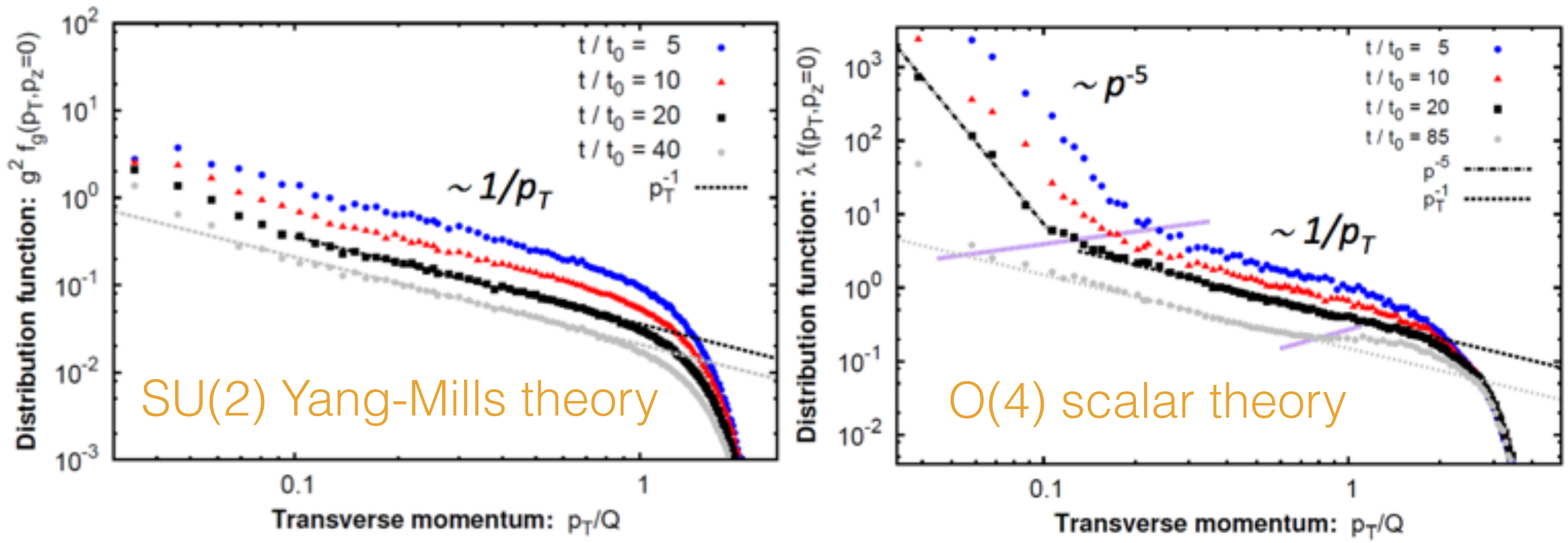


(Berges,Boguslavski,SS,Venugopalan PRL114 (2015) 061601)

Comparison of simulations

Comparison of longitudinally expanding Yang-Mills and scalar field theory in the classical regime of high occupancy

Evolution of the single particle spectrum



Scalar theory shows three distinct scaling regimes at soft ($\sim p^{-5}$), intermediate ($\sim 1/p_T$) and hard momenta (~const)

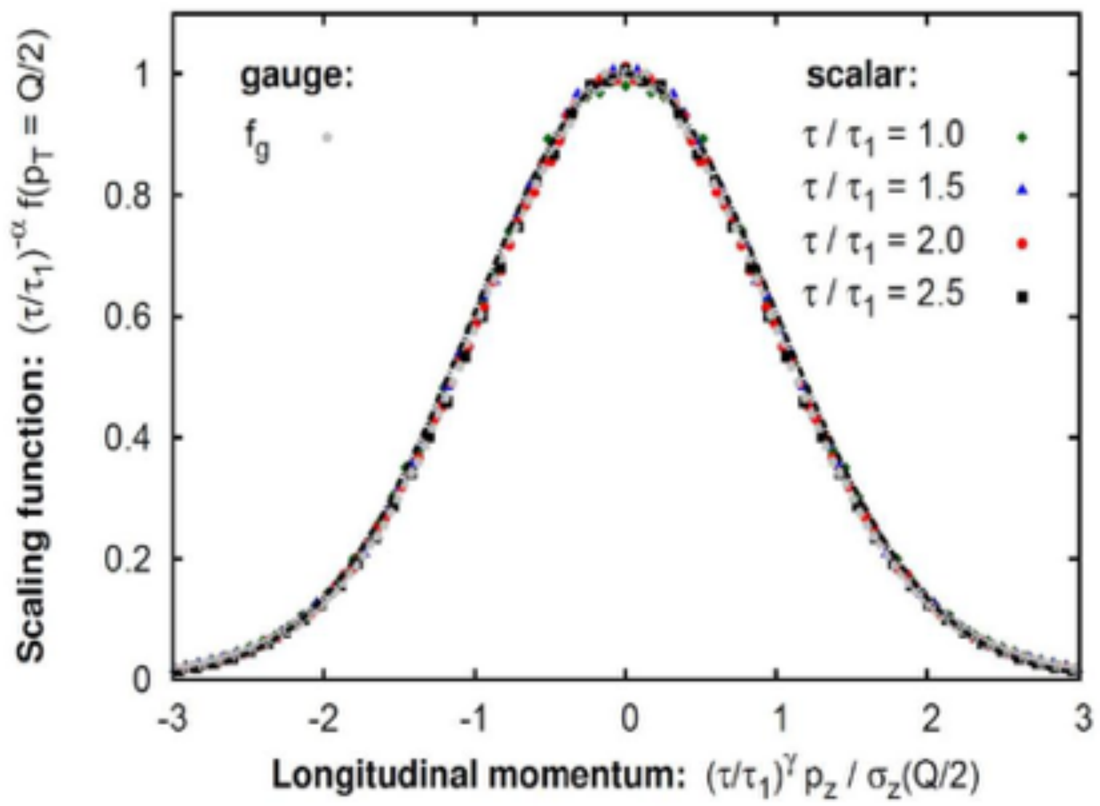
->Common $\sim 1/p_T$ scaling regime

(Berges,Boguslavski,SS,Venugopalan PRL114 (2015) 061601)

Universality far from equilibrium

- Scaling exponents and scaling functions agree in the inertial range of momenta, where both theories show $1/p_T$ behavior

Normalized fixed-point distribution



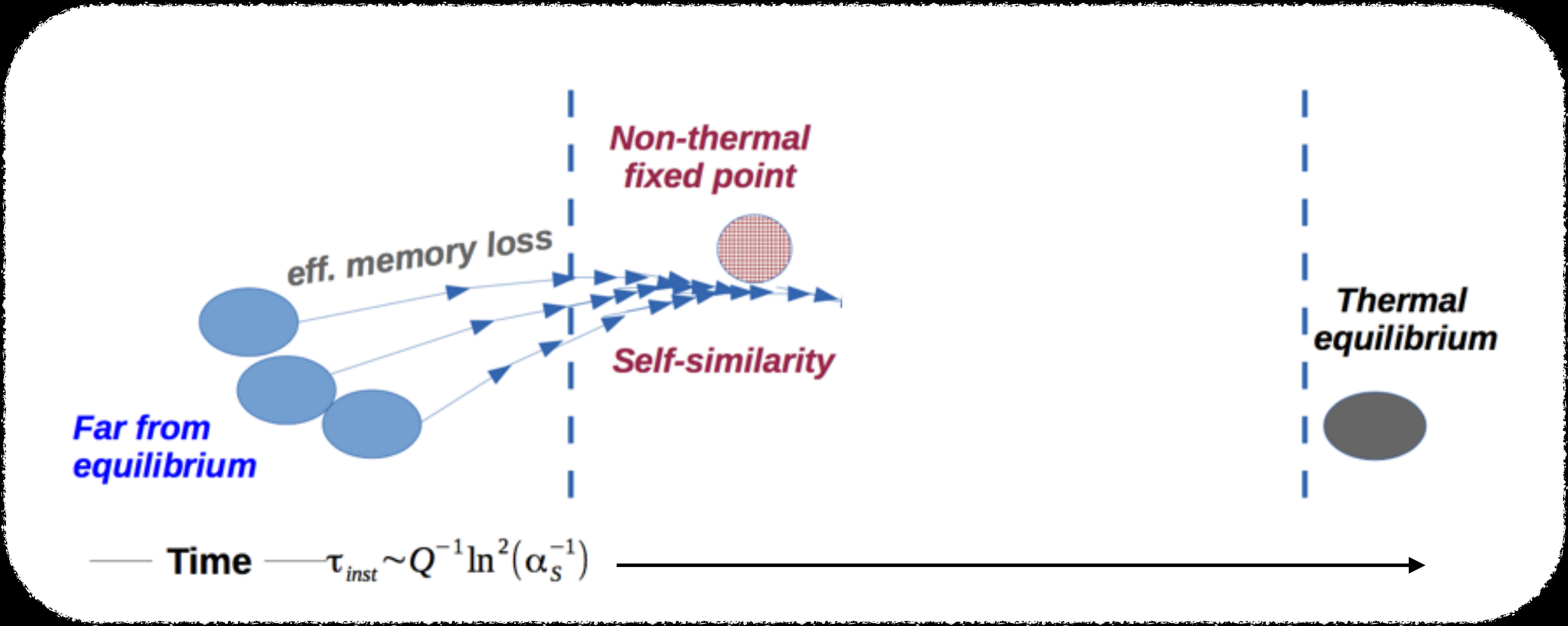
Kinetic interpretation in scalar theory still unclear —

Vertex corrections? More general than small angle scattering?

(Berges,Boguslavski,SS,Venugopalan PRL114 (2015) 061601)

Non-thermal fixed point

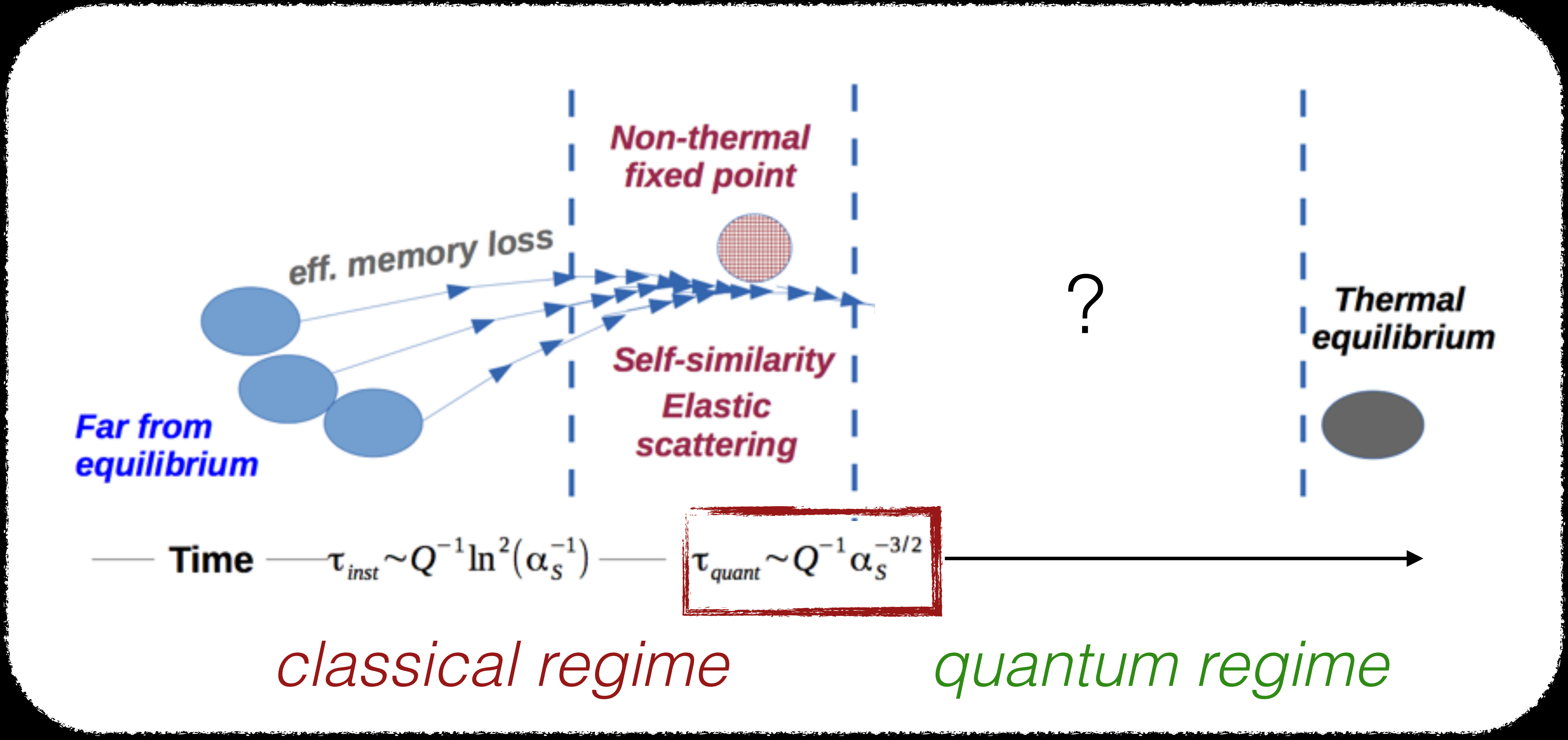
Non-equilibrium evolution leads to a universal attractor solution, where the system exhibits self-similar scaling behavior.



Similar picture as previously. However, system is still far from equilibrium at the end of the classical regime

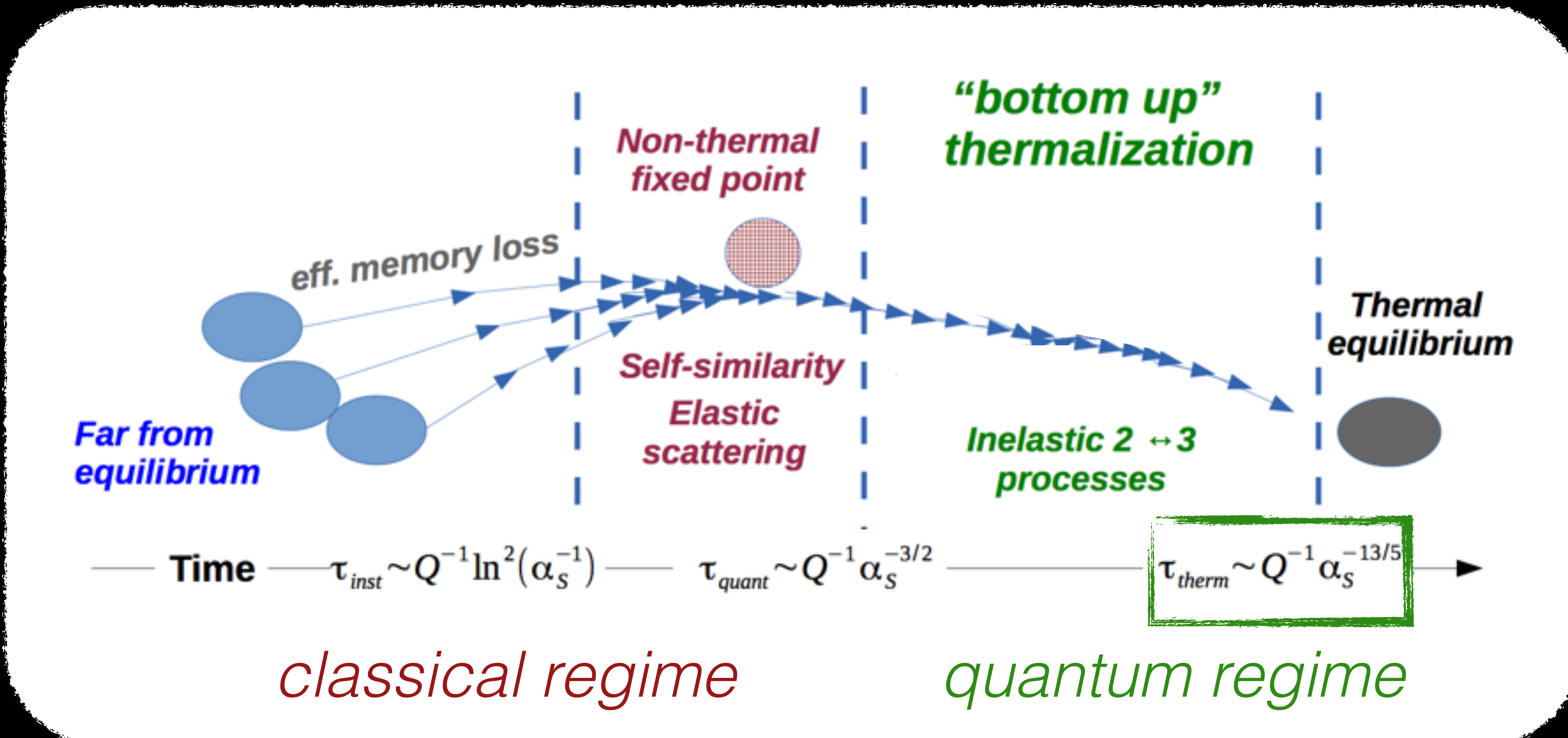
Thermalization process

Classical description breaks down once the typical occupancies become of order one -> *Departure from classical fixed point*



Thermalization process

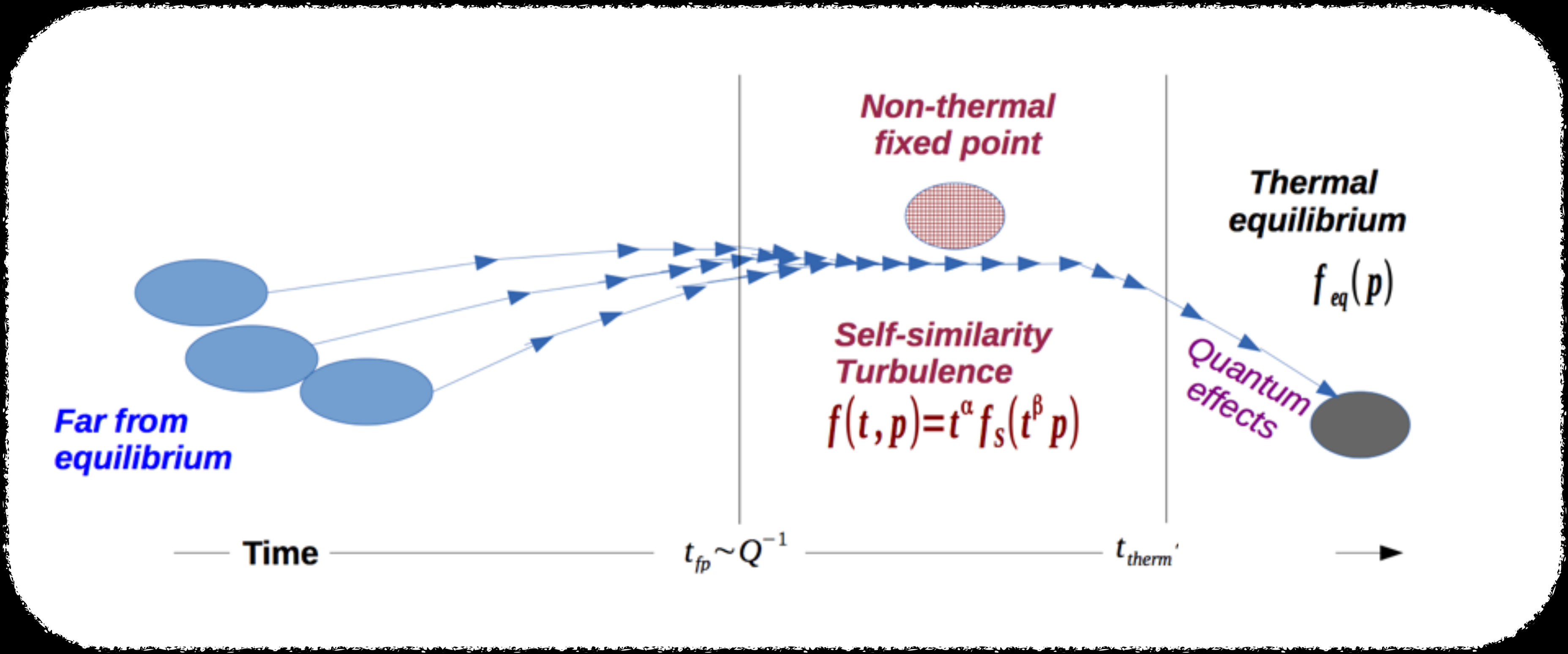
Quantum regime can be studied within effective kinetic description
(Baier, Mueller, Schiff & Son PLB 502 (2001) 51-58)



-> Inelastic processes lead to formation of soft thermal bath
Hard gluons loose energy to the soft bath

Summary & Conclusions

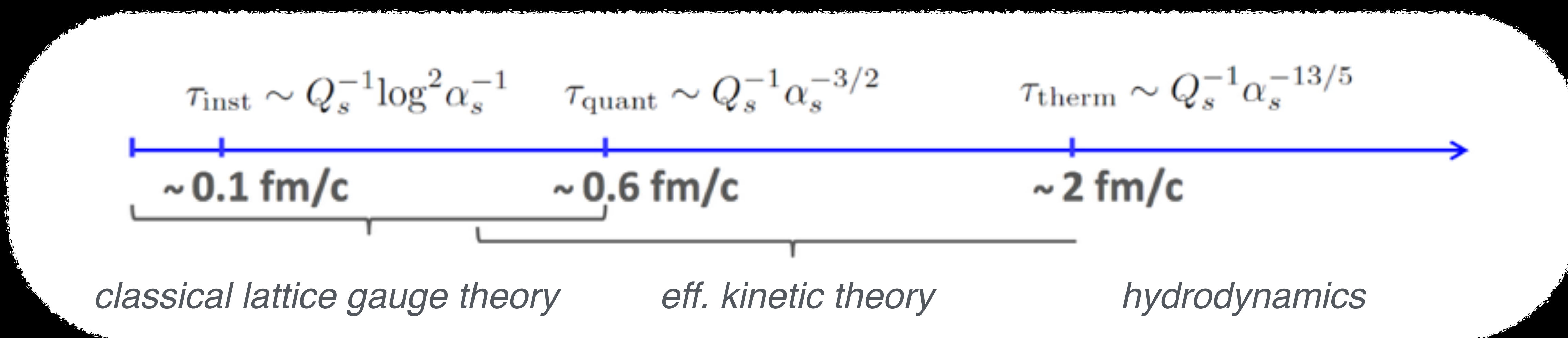
- Thermalization process in far-from equilibrium systems proceeds via self-similar evolution associated to non-thermal fixed points



- Similar mechanism observed across systems at various energy scales *(big bang, little bang, ultra-cold bang,…)*
- Many possible applications — Evolution of sound wave spectrum and production of gravitational waves *(Shuryak, Kalaydzhyan arXiv:1412.5147)*

Summary & Conclusions

- Qualitative understanding of the dynamics of the thermalization process in A+A collisions at weak coupling. Different methods agree within the overlapping range of validity.
- Can now compute entire thermalization process in A+A from an interplay of different weak coupling methods



- Striking universality observed between expanding scalar and gauge theory in the early time classical regime.

-> Even though precise origin is still unclear there is an exciting possibility to learn about the dynamics of thermalization process from different physical systems.

Backup

Quantum regime

Effective kinetic description used to study the dynamics of the quantum regime up to complete equilibration

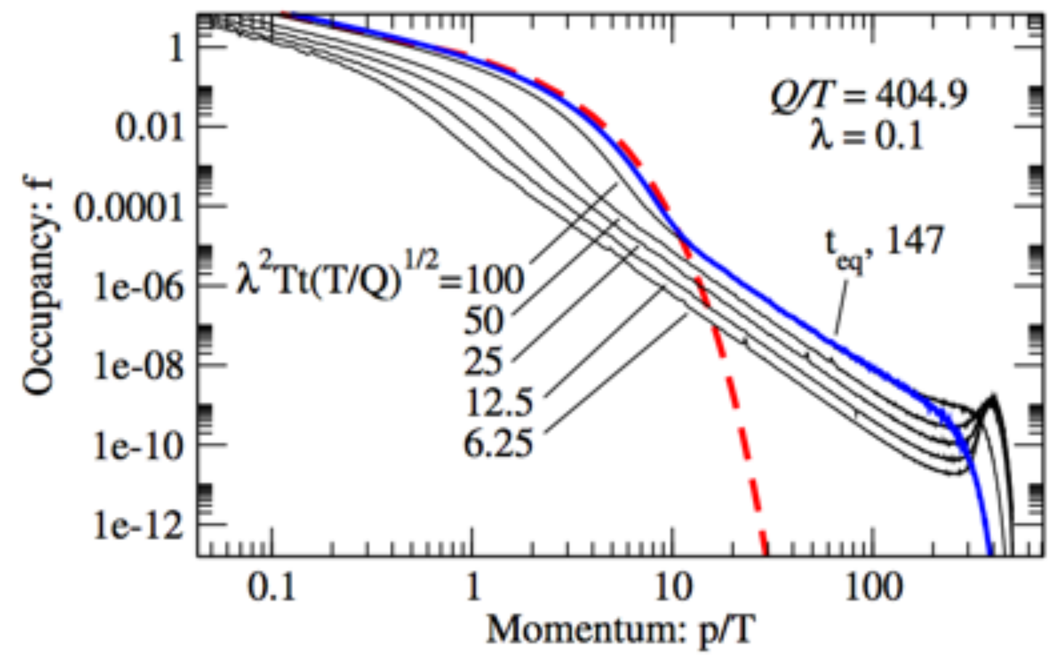
$$\partial_t f(p,t) = -\mathcal{C}_{2\leftrightarrow 2}[f](p) - \mathcal{C}_{1\leftrightarrow 2}[f](p).$$

Quantitative estimate of the thermalization time

$$t_{\rm eq}^{\rm under\ occ.} \approx \frac{34. + 21.\ln(Q/T)}{1+0.037\log\lambda^{-1}} \left(\frac{Q}{T}\right)^{1/2} \frac{1}{\lambda^2 T}$$

-> τ_{eq}~0.2- 2 fm/c

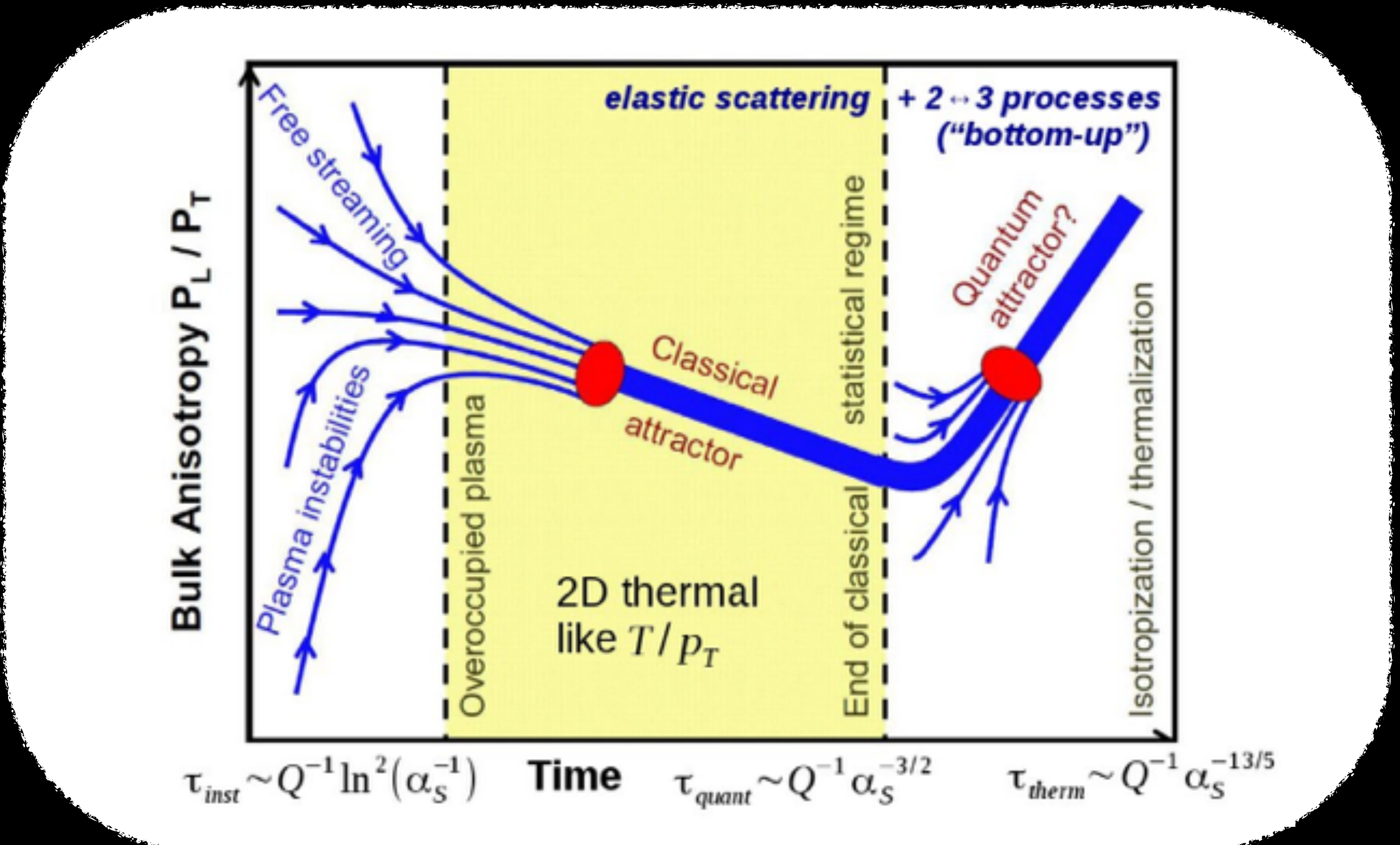
Evolution of the single particle spectrum



(Kurkela,Lu,Moore,York PRD89 (2014) 074036 Kurkela, Lu PRL 113 (2014) 18, 182301)

Thermalization process

Thermalization scenario based on classical-statistical and kinetic theory simulations



(Berges,Boguslavski,SS,Venugopalan PRD 89 (2014) 074011; 89 (2014) 114007)

(c.f. Baier, Mueller, Schiff and Son Phys.Lett. B502 (2001) 51-58)