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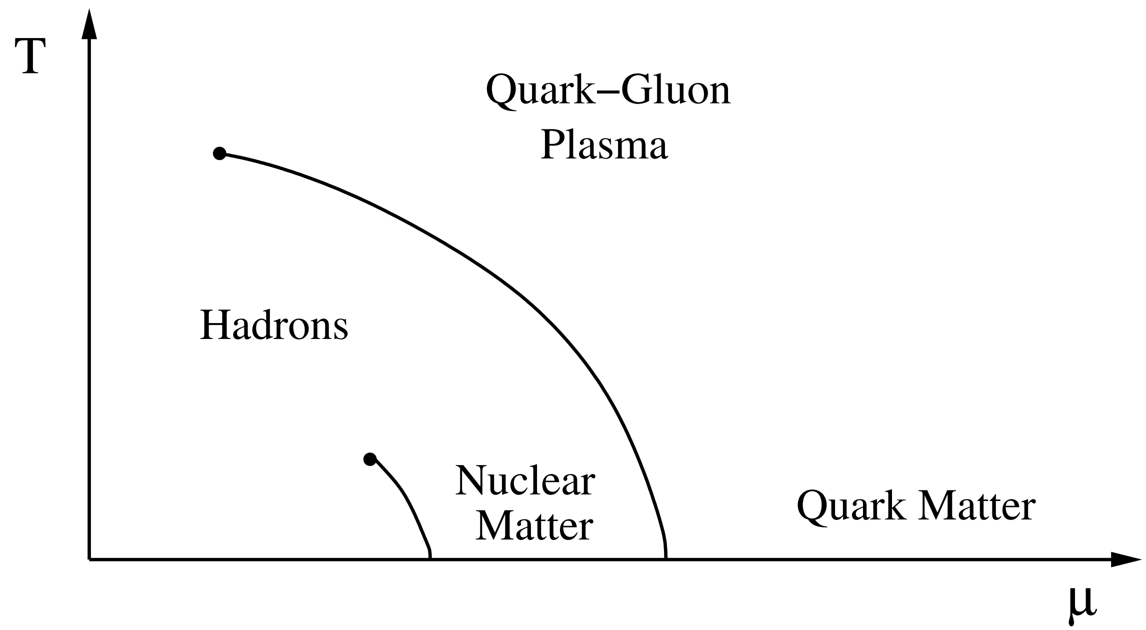
From holography towards real-world nuclear matter

Si-wen Li, Andreas Schmitt, Qun Wang, in preparation

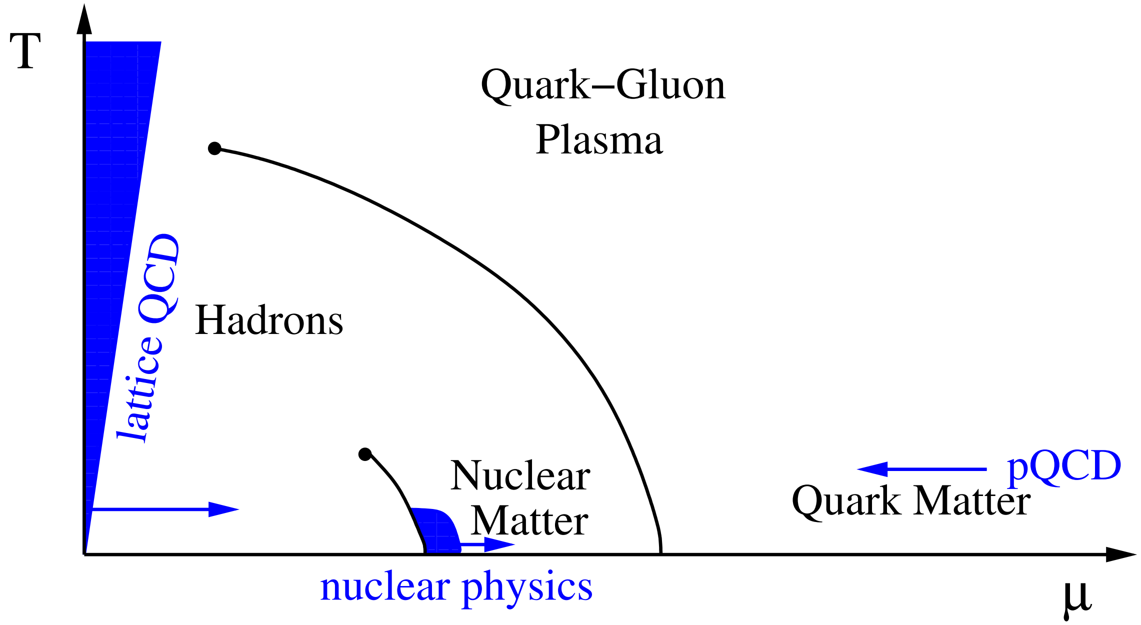
- dense QCD matter:
  relevance for compact stars and theoretical challenges
- the Sakai-Sugimoto model:
  holography as close to QCD as currently possible
- realistic nuclear matter in the Sakai-Sugimoto model?

- **Dense QCD matter: what we know**



- first-order onset of nuclear matter at $\mu = 308\,\text{MeV}$
- weakly interacting quark matter at asymptotically large μ
- as a consequence: must be chiral/deconfinement transition in between (presumably in strongly coupled regime)

- **Dense QCD matter: rigorous methods**



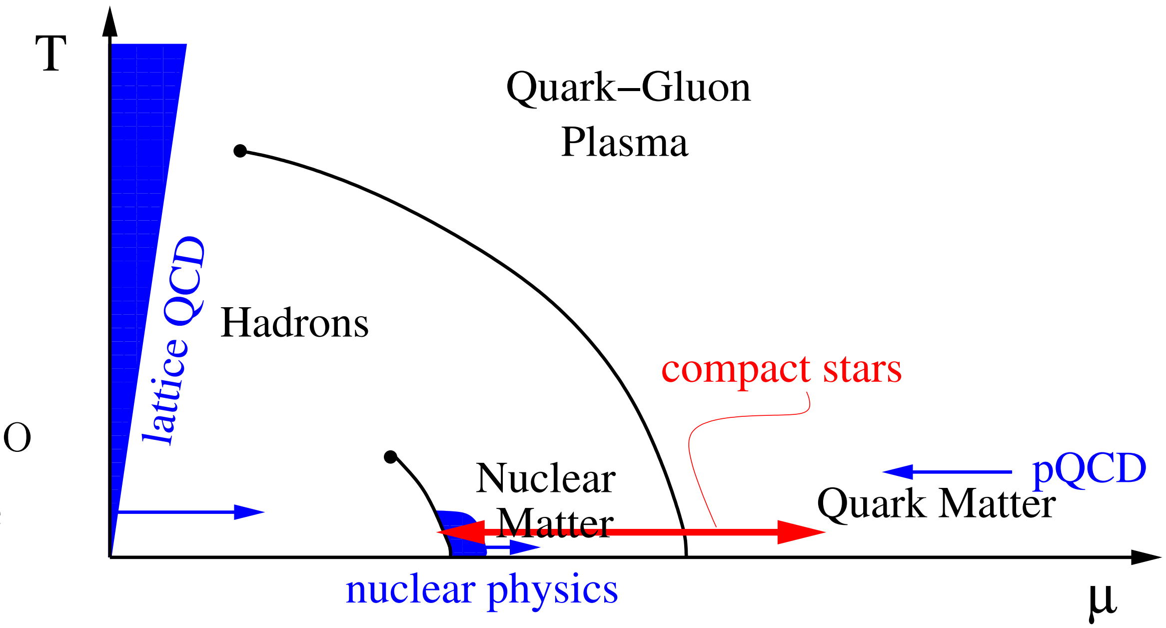
- QCD on the lattice: sign problem at nonzero μ, but recent progress
- perturbative QCD: restricted to ultra-high densities
- "traditional" nuclear physics: input from experiment, restricted to nuclear saturation density

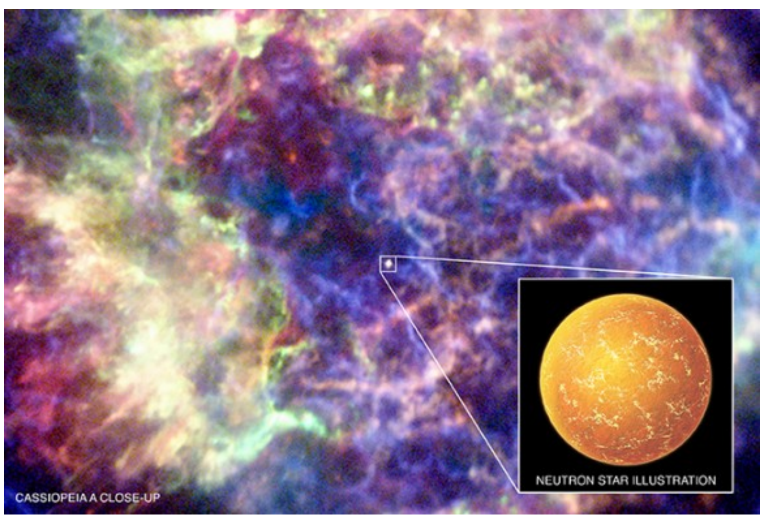
• Dense QCD matter in compact stars

- density *profile* in a compact star

$$n_B \sim (1-10)\, n_0$$

- phase transition to quark matter possible

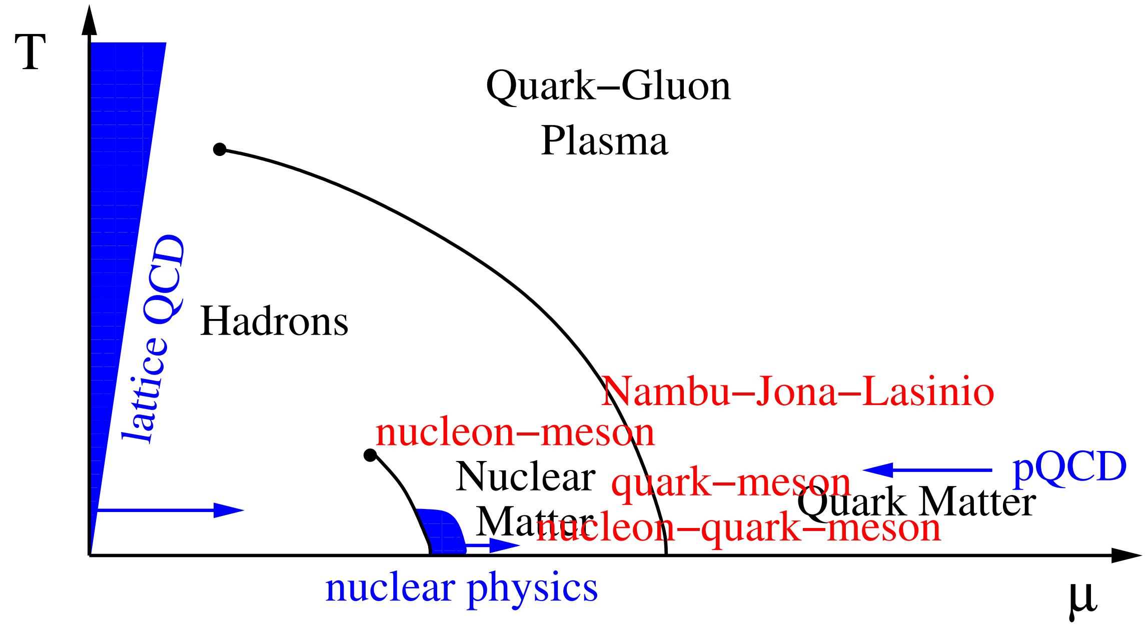




- equation of state + gravity → mass/radius of the star

> equation of state over wide density regime highly desired!

• Dense QCD matter: models



- Nambu–Jona-Lasinio (usually no nuclear matter)
- quark-meson (no nucleons), nucleon-meson (no quarks)
- nucleon-quark-meson (patched together, many parameters)
- extrapolations from nuclear to weakly interacting quark matter

→ even without rigor: models for compact stars hard to construct!

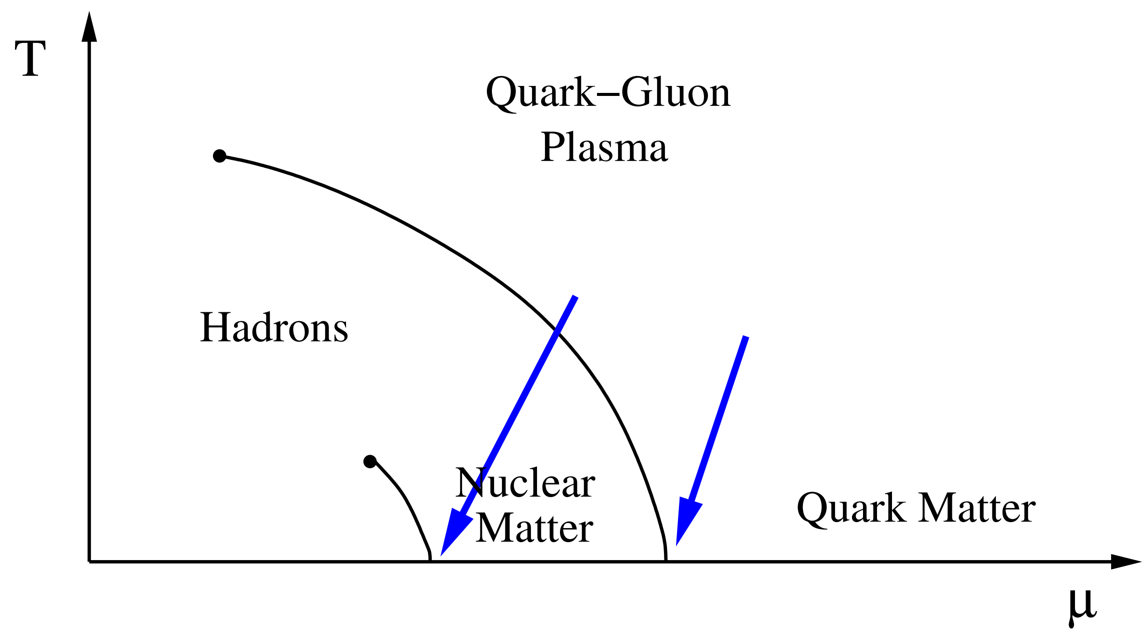
• Can holography help?

J. M. Maldacena, Int. J. Theor. Phys. 38, 1113 (1999) [Adv. Theor. Math. Phys. 2, 231 (1998)]

- dual of QCD: probably exists, but currently out of reach
- reliable strong-coupling calculation (usually infinite coupling)
- Sakai-Sugimoto model:
  T. Sakai and S. Sugimoto, Prog. Theor. Phys. 113, 843 (2005)
  - top-down approach with only 3 parameters
  - dual to large-N_c QCD, however in inaccessible limit
  - contains all necessary ingredients:
    baryons, quark matter, chiral/deconfinement phase transitions

- **Goal**

Does cold and dense holographic matter show
a first-order baryon onset
and
a chiral phase transition to quark matter?

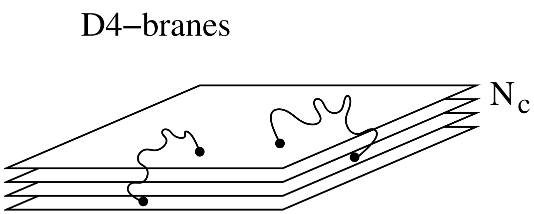


(ignore superfluidity in nuclear matter and color superconductivity)

• Sakai-Sugimoto model: background geometry (p. 1/2)

E. Witten, Adv. Theor. Math. Phys. 2, 505 (1998)

N_c D4-branes compactified on circle $x_4 \equiv x_4 + 2\pi/M_{\mathrm{KK}}$



- 4-4 strings → adjoint scalars & fermions, gauge fields
- periodic x_4 → break SUSY by giving mass $\sim M_{\mathrm{KK}}$ to scalars & fermions

$\Rightarrow SU(N_c)$ gauge theory

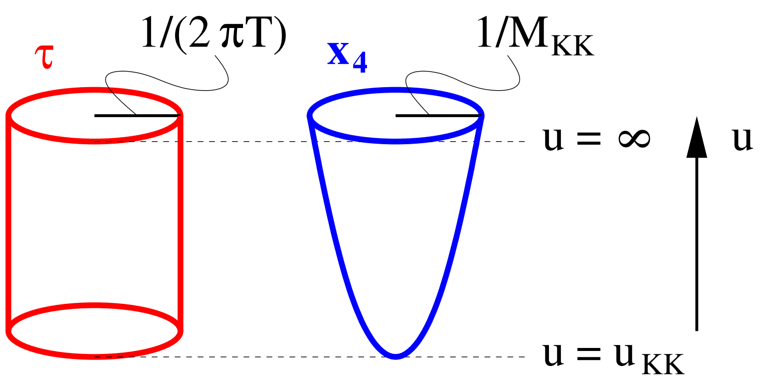
$$\lambda = \underbrace{\frac{g_5^2}{2\pi/M_{\mathrm{KK}}}}_{g_4^2} N_c$$

	$\lambda \ll 1$	$\lambda \gg 1$
gravity approximation	x	$\surd$
dual to large-N_c QCD (at energies $\ll M_{\mathrm{KK}}$)	$\surd$ $\Lambda_{\mathrm{QCD}} \ll M_{\mathrm{KK}}$	x $\Lambda_{\mathrm{QCD}} \sim M_{\mathrm{KK}}$

• Background geometry (page 2/2): two solutions

Confined phase

$$ds^2_{\rm conf} = \left(\frac{u}{R}\right)^{3/2}\left[d\tau^2 + d\mathbf{x}^2 + \tilde{f}(u)dx_4^2\right] + \left(\frac{R}{u}\right)^{3/2}\left[\frac{du^2}{\tilde{f}(u)} + u^2 d\Omega_4^2\right]$$

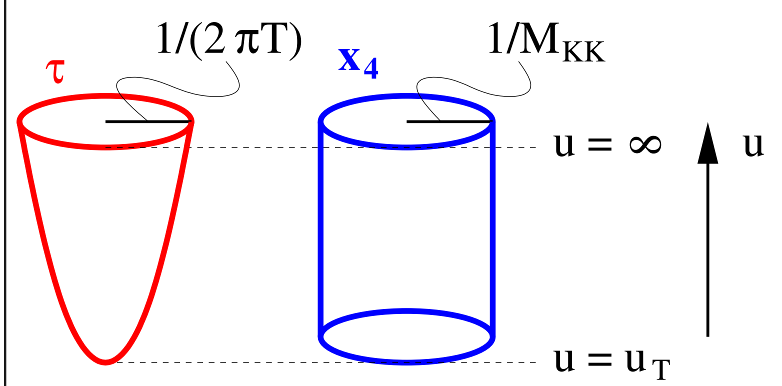


$$M_{\rm KK} = \frac{3}{2}\frac{u_{\rm KK}^{1/2}}{R^{3/2}} \qquad \tilde{f}(u) \equiv 1 - \frac{u_{\rm KK}^3}{u^3}$$

Wick rotated regular geometry

Deconfined phase

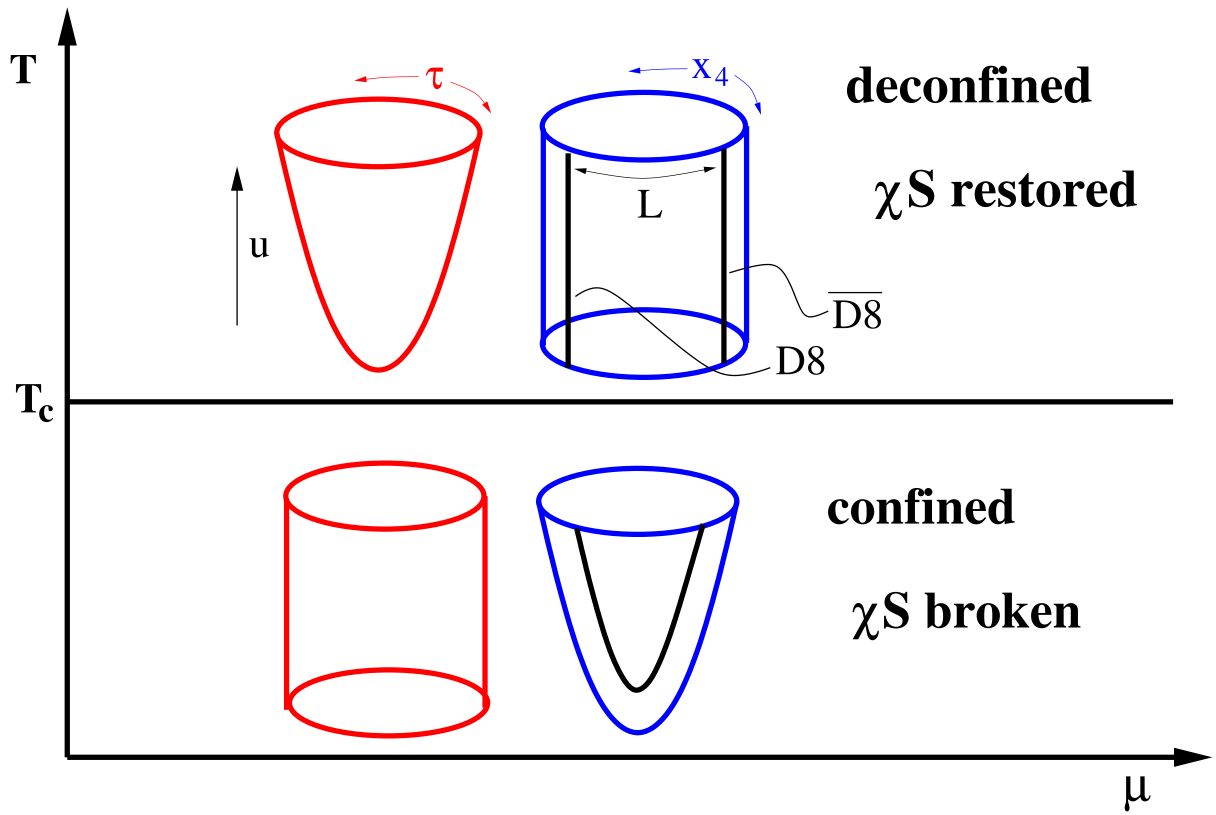
$$ds^2_{\rm deconf} = \left(\frac{u}{R}\right)^{3/2}\left[f(u)d\tau^2 + d\mathbf{x}^2 + dx_4^2\right] + \left(\frac{R}{u}\right)^{3/2}\left[\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right]$$



$$T = \frac{3}{4\pi}\frac{u_T^{1/2}}{R^{3/2}} \qquad f(u) \equiv 1 - \frac{u_T^3}{u^3}$$

Wick rotated black brane

• Chiral transition in the Sakai-Sugimoto model (p. 1/3)

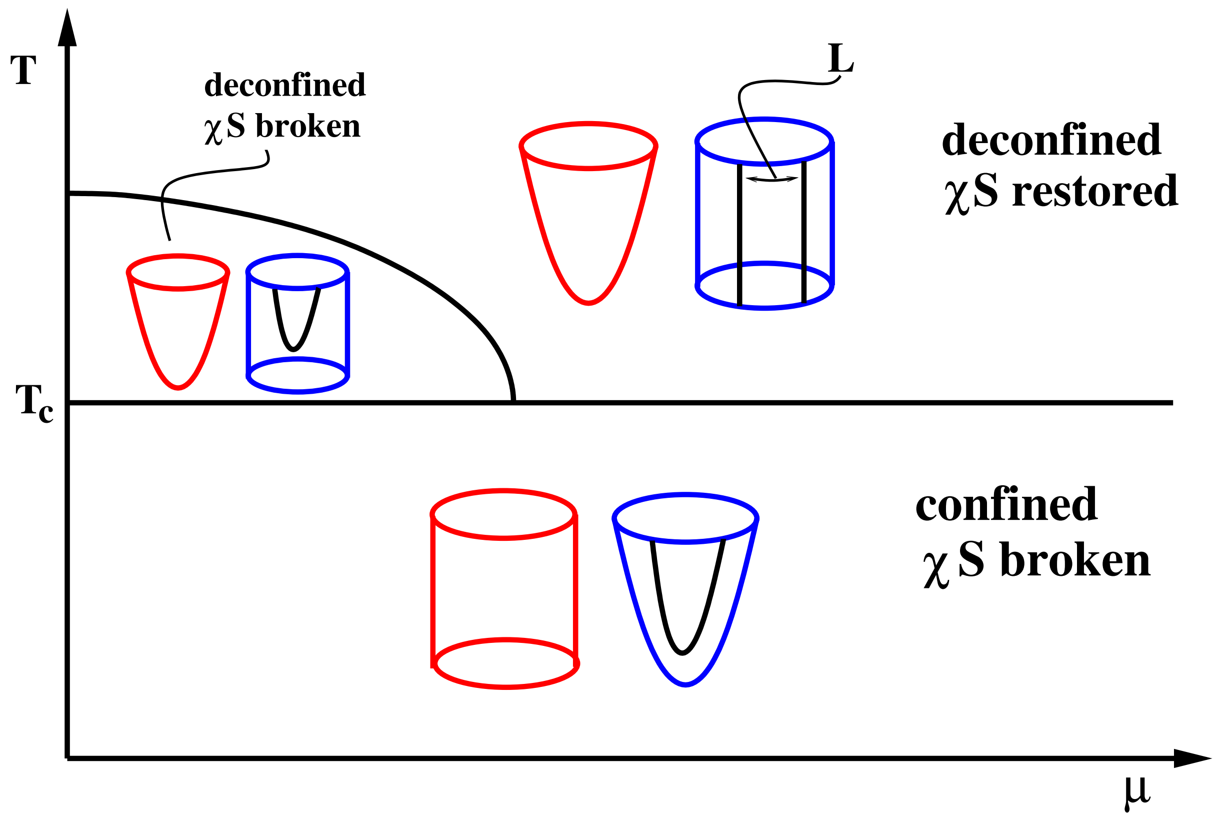


- in probe brane ("quenched") approximation: phase transition unaffected by quantities on flavor branes (μ, B, …)
- not unlike expectation from large-N_c QCD

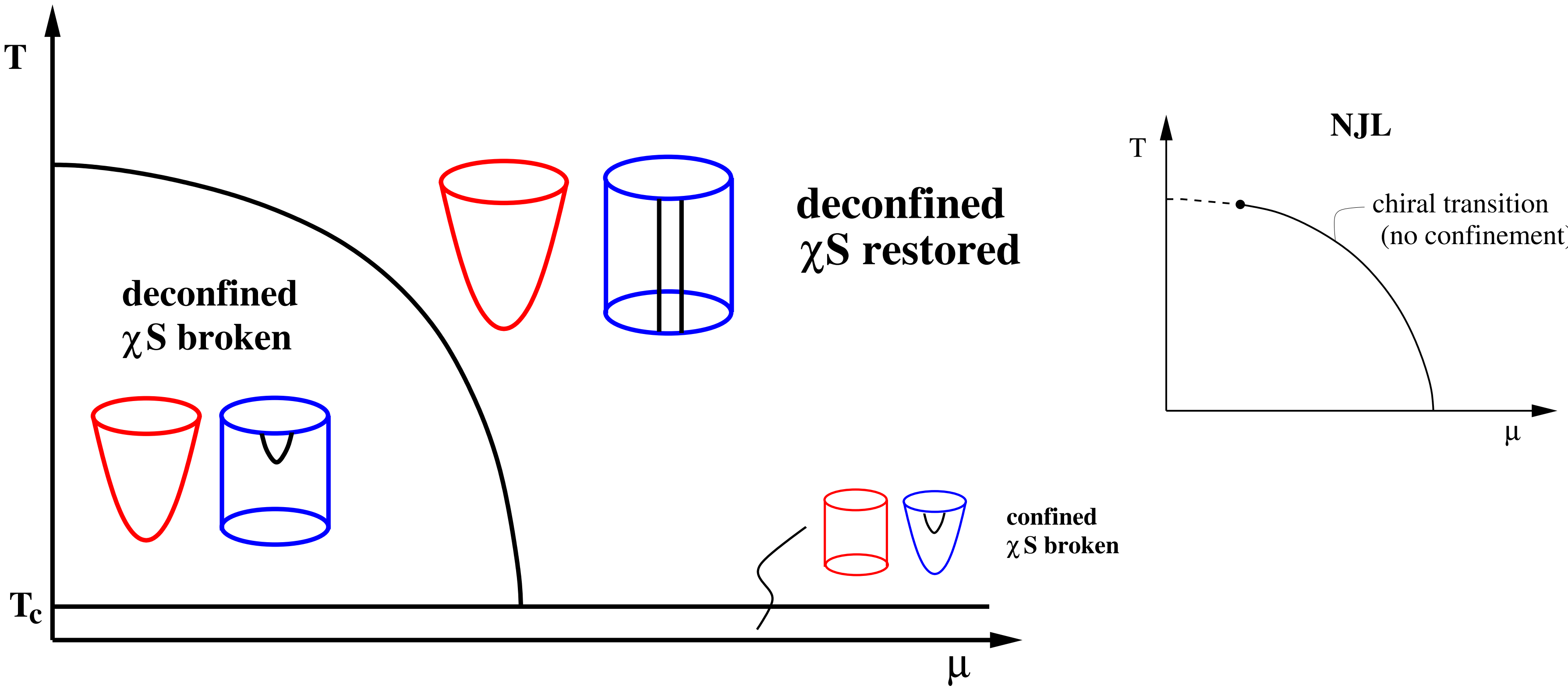
• Chiral transition in the Sakai-Sugimoto model (p. 2/3)

- less "rigid" behavior for smaller L
- deconfined, chirally broken phase for $L < 0.3\,\pi/M_{\rm KK}$

O. Aharony, J. Sonnenschein, S. Yankielowicz, Annals Phys. 322, 1420 (2007)
N. Horigome, Y. Tanii, JHEP 0701, 072 (2007)



• Chiral transition in the Sakai-Sugimoto model (p. 3/3)



- "decompactified" limit → gluon dynamics decouple
- "NJL-like" dual field theory

E. Antonyan, J. A. Harvey, S. Jensen, D. Kutasov, hep-th/0604017

J. L. Davis, M. Gutperle, P. Kraus, I. Sachs, JHEP 0710, 049 (2007)

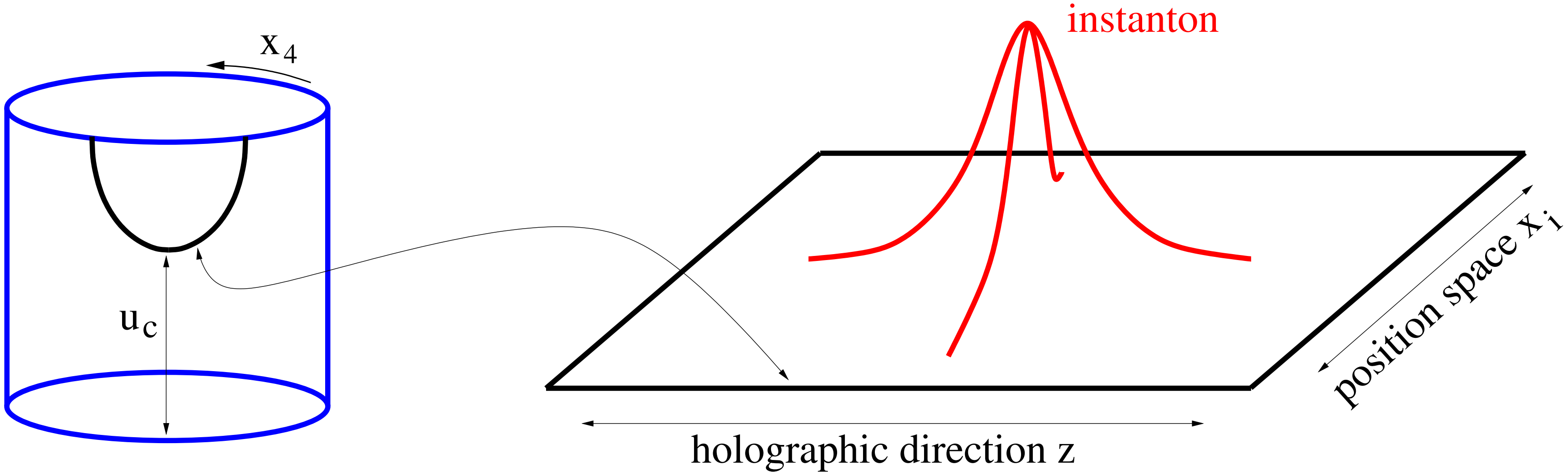
F. Preis, A. Rebhan and A. Schmitt, Lect. Notes Phys. 871, 51 (2013)

• Baryons in Sakai-Sugimoto

- baryons in AdS/CFT: wrapped D-branes with N_c string endpoints
  E. Witten, JHEP 9807, 006 (1998); D. J. Gross, H. Ooguri, PRD 58, 106002 (1998)
- baryons in Sakai-Sugimoto:
  - D4-branes wrapped on S^4
  - equivalently: instantons on D8-branes

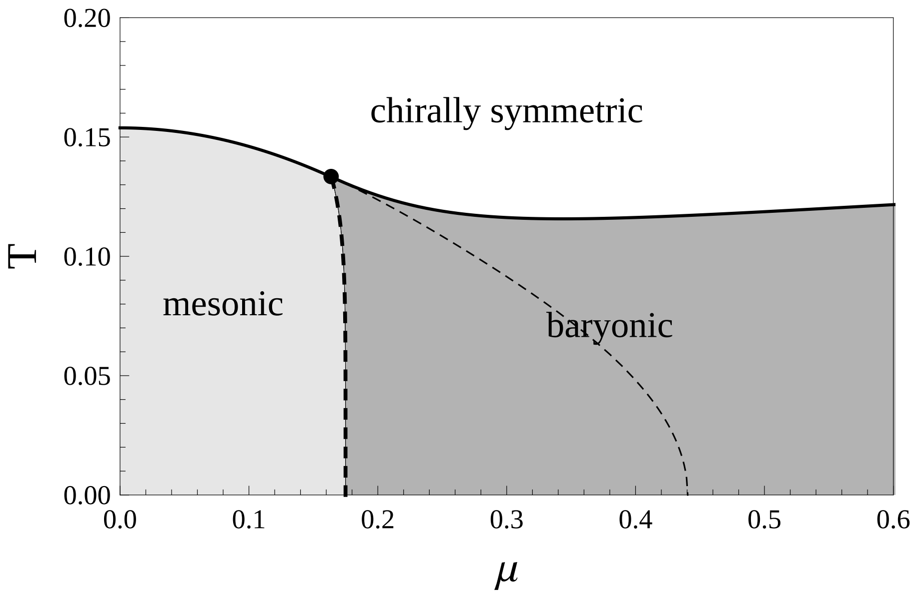
  T. Sakai, S. Sugimoto, Prog. Theor. Phys. 113, 843-882 (2005)

  H. Hata, T. Sakai, S. Sugimoto, S. Yamato, Prog. Theor. Phys. 117, 1157 (2007)



• Baryonic matter: pointlike approximation

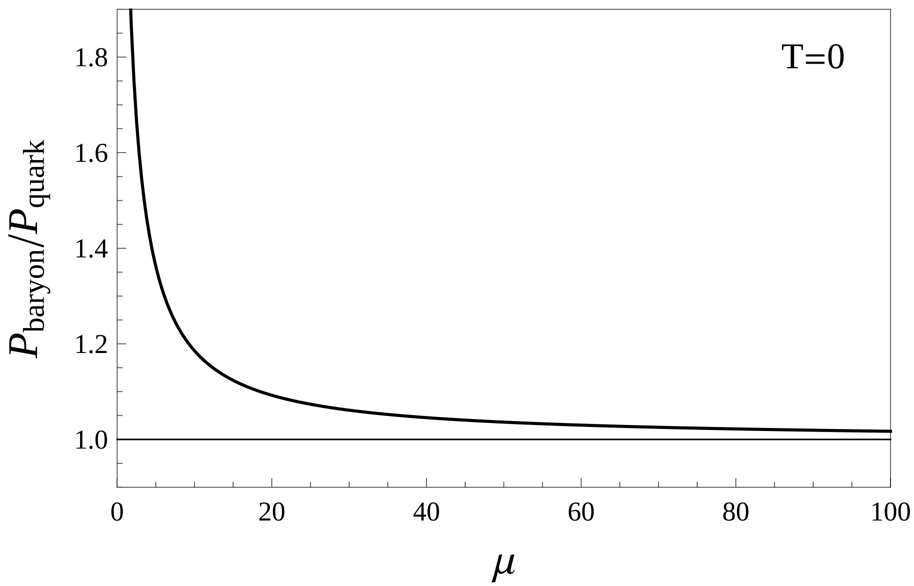
O. Bergman, G. Lifschytz, M. Lippert, JHEP 0711, 056 (2007)



- second-order baryon onset
- no chiral restoration at small T

- baryonic pressure approaches quark matter pressure for $\mu \to \infty$

F. Preis, A. Rebhan, A. Schmitt, JPG 39, 054006 (2012)



• Baryonic matter: beyond the pointlike approximation

Si-wen Li, Andreas Schmitt, Qun Wang, in preparation

1. Instanton gas approximation

   K. Ghoroku, K. Kubo, M. Tachibana, T. Taminato and F. Toyoda, PRD 87, 066006 (2013)

2. "Homogeneous ansatz"

   M. Rozali, H. H. Shieh, M. Van Raamsdonk and J. Wu, JHEP 0801, 053 (2008)

- **Setup**

- D8-brane action

$$S = \underbrace{T_8 V_4 \int_{x^\mu}\int_z e^{-\Phi}\sqrt{\det(g+2\pi\alpha' F)}}_{\text{Dirac-Born-Infeld (DBI)}} + \underbrace{\frac{N_c}{8\pi^2}\int_{x^\mu}\int_z \hat{A}_0 \mathrm{Tr}[F_{ij}F_{kz}]\epsilon_{ijk}}_{\text{Chern-Simons (CS)}}$$

- gauge fields in the bulk ($\to$ global symmetry at the boundary)

$$N_f = 2: \qquad F_{\mu\nu} = \hat{F}_{\mu\nu} + F^a_{\mu\nu}\sigma_a$$

- abelian part $U(1)$: chemical potential $\mu = \hat{A}_0(z = \pm\infty)$

- non-abelian part $SU(2)$: baryon number (instantons)

$$N_B = -\frac{1}{8\pi^2}\int_{\vec{x}}\int_z \mathrm{Tr}[F_{ij}F_{kz}]\epsilon_{ijk}$$

• Instanton gas approximation (page 1/2)

- single instanton from non-abelian gauge fields

$$A_z(\vec{x},z) = -i\phi\,\psi\,\partial_z\psi^{-1}\,, \qquad A_i(\vec{x},z) = -i\phi\,\psi\,\partial_i\psi^{-1}\,,$$

with

$$\phi(\vec{x},z) = \frac{\xi^2}{\xi^2+\rho^2}\,, \quad \psi(\vec{x},z) = \frac{z - i\vec{x}\cdot\vec{\sigma}}{\xi}\,, \qquad \xi^2 \equiv (\vec{x}-\vec{x}_0)^2 + (z-z_0)^2$$

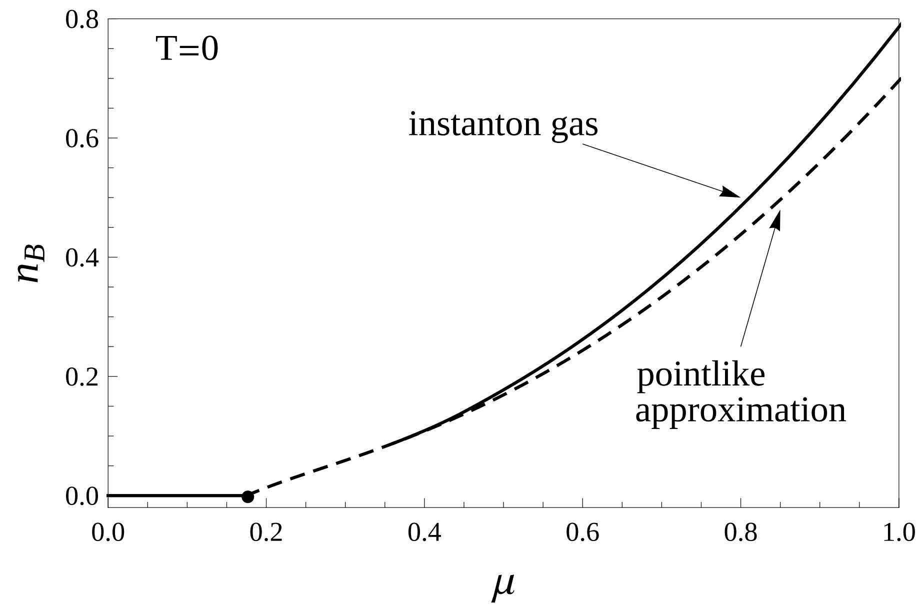
- here: homogeneous gas of instantons, at $z = 0$ in the bulk

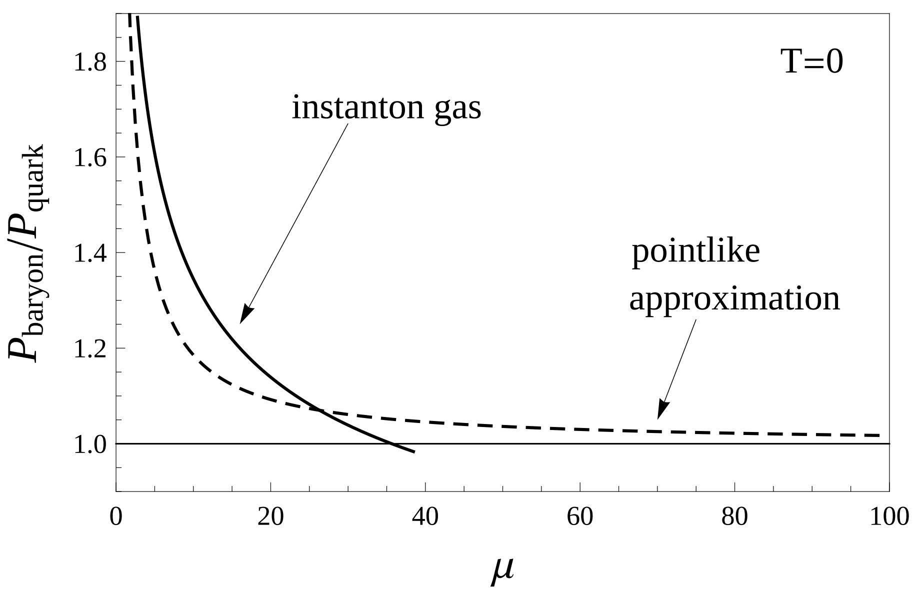
$$\mathrm{Tr}[F^2] \sim \frac{\rho^4}{(\xi^2+\rho^2)^4} \to \frac{1}{V}\sum_{n=1}^{N_I}\int d^3x\,\frac{\rho^4}{[(\vec{x}-\vec{x}_{0n})^2 + z^2 + \rho^2]^4}$$

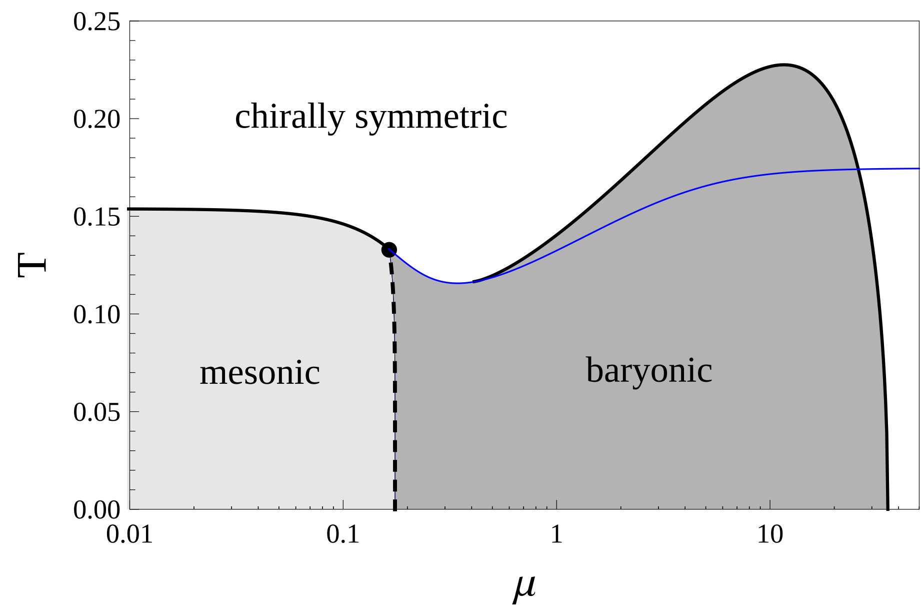
- calculation:
  - solve equations of motion for $\hat{A}_0$, x_4
  - minimize free energy with respect to ρ, N_I, u_c
  - compare free energy with mesonic and quark matter phases

• Instanton gas approximation (page 2/2)

- pointlike approximation recovered for small n_B
- second-order baryon onset
- (very) large μ: chiral restoration







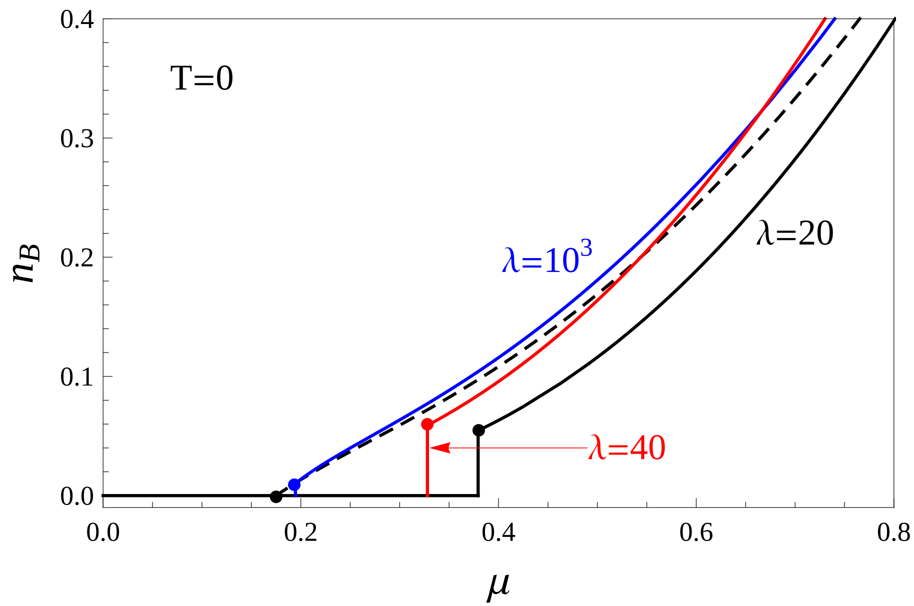
• Homogeneous ansatz (page 1/3)

- non-abelian gauge fields

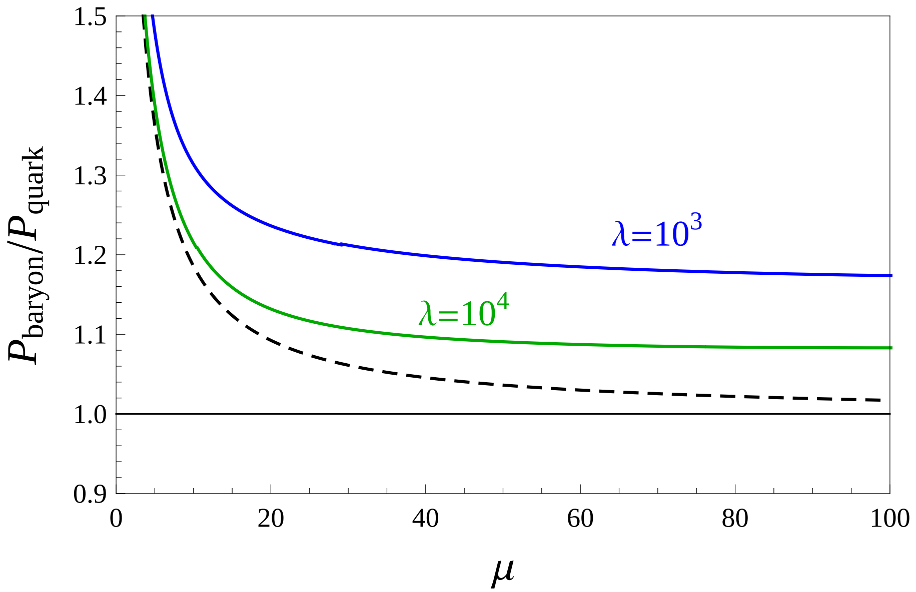
$$A_z = 0\,, \qquad A_i(z) = -\sigma_i \frac{h(z)}{2}$$

- no further approximation: determine $h(z)$ dynamically
- nonzero baryon number requires discontinuity of $h(z)$ at $z = 0$
- ansatz induces explicit dependence on 't Hooft coupling $\lambda = g^2 N_c$
- calculation:
  - solve equations of motion for $\hat{A}_0$, x_4, h
  - minimize free energy with respect to $h(z = 0)$, u_c
  - compare free energy with mesonic and quark matter phases

• Homogeneous ansatz (page 2/3)



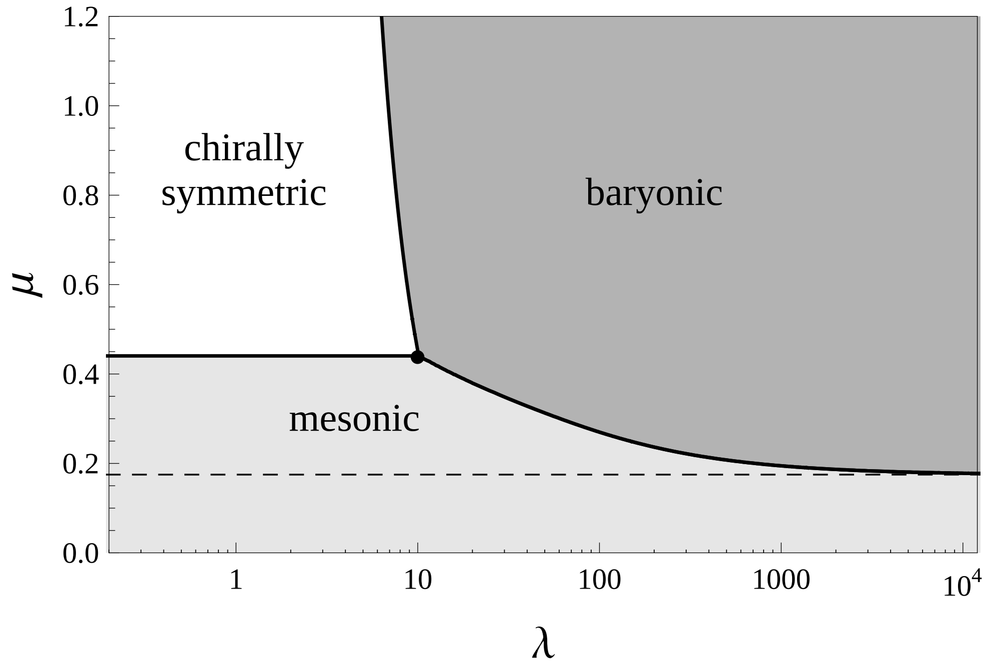
- first-order baryon onset
- pointlike approximation recovered for $\lambda \to \infty$



- no chiral restoration for any μ

• Homogeneous ansatz (page 3/3)

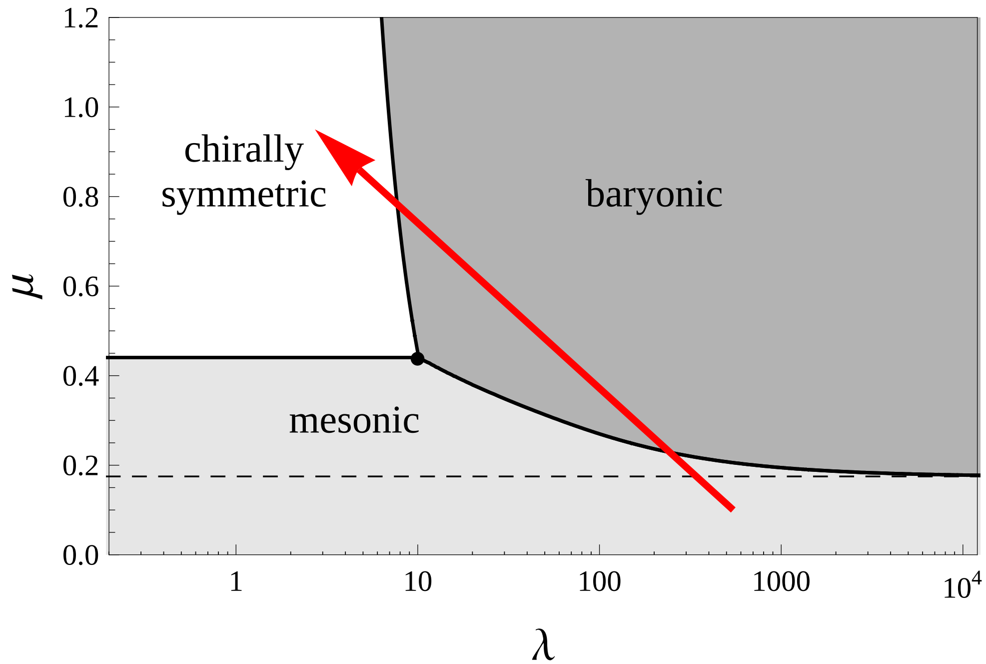
- phases in μ-λ plane at $T = 0$:



- large λ: baryon onset approaches pointlike approximation
- $\lambda \gtrsim 10$: vacuum $\to$ baryons
- $\lambda \lesssim 10$: vacuum $\to$ quark matter ($\to$ baryons)

• Homogeneous ansatz (page 3/3)

- phases in μ-λ plane at $T = 0$:



- large λ: baryon onset approaches pointlike approximation
- $\lambda \gtrsim 10$: vacuum $\rightarrow$ baryons
- $\lambda \lesssim 10$: vacuum $\rightarrow$ quark matter ($\rightarrow$ baryons)
- running coupling: vacuum $\rightarrow$ baryons $\rightarrow$ quark matter

- **Summary**

- compact stars: need to understand nuclear *and* quark matter over fairly wide density regime
- currently no first-principle calculations and very few/crude models that cover *both* phases
- holography: useful because of strong coupling, however (more or less) different from QCD
- nuclear matter in Sakai-Sugimoto ("decompactified" limit):

	first-order baryon onset	chiral restoration
pointlike approximation	x	x
instanton gas	x	√
homogeneous ansatz	√	x

• Outlook

- improve on present results:
  - understand relation between instanton approach and homogeneous ansatz
  - instantons without $SO(4)$ symmetry (λ dependence!)
    vacuum: M. Rozali, J. B. Stang and M. van Raamsdonk, JHEP 1402, 044 (2014)
  - determine distribution of instantons in the bulk dynamically

- use (improved) results for phenomenology:
  - fit parameters to nuclear saturation,
    compute equation of state, speed of sound, ...
  - nuclear matter in a magnetic field
    pointlike: F. Preis, A. Rebhan, A. Schmitt, JPG 39, 054006 (2012)