Holographic thermalization at strong and intermediate coupling

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Describing a heavy ion collision

Nontrivial observation: Hydrodynamic description of fireball evolution extremely successful with few ‘UV inputs’

1. Relatively easy: Equation of state and freeze-out criterion
2. Hard: Transport coefficients of the plasma ($\eta, \zeta, \ldots$)
3. Very hard: Initial conditions & onset time $\tau_{\text{hydro}}$

Surprise from RHIC/LHC: Extremely fast equilibration into almost ‘ideal fluid’ behavior — hard to explain via weakly coupled quasiparticle picture
Major challenge for theorists: Understand the fast dynamics that take the system from complicated, far-from-equilibrium initial state to near-thermal ‘hydrodynamized’ plasma

Characteristic energy scales and nature of the plasma evolve fast (running coupling) ⇒ Need to efficiently combine both perturbative and nonperturbative machinery
Motivation

Thermalization at weak coupling

Early perturbative dynamics

When describing early (initially perturbative) dynamics of a heavy ion collision, need to take into account

- Longitudinal expansion of the system
- Elastic and inelastic scatterings
- Plasma instabilities

Traditional field theory tools available:

1. Classical (bosonic) lattice simulations — work as long as occupation numbers large\(^1\) (quantum time evolution not feasible)

2. Weak coupling expansions; disagreement related to the role of plasma instabilities, affecting \(\alpha_s\) scaling of \(\tau_{\text{therm}}\)^2

3. Effective kinetic theory — works at smaller occupancies, but breaks down in the description of IR physics\(^3\)

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\(^1\) Berges et al., 1303.5650, 1311.3005

\(^2\) Baier et al., hep-ph/0009237; Kurkela, Moore, 1107.5050; Blaizot et al., 1107.5296

\(^3\) Abraao York, Kurkela, Lu, Moore, 1401.3751
Thermalization in a weakly coupled plasma

Inelastic scatterings drive bottom-up thermalization
- Soft modes quickly create a thermal bath
- Hard splittings lead to $q \sim Q_s$ particles being eaten by the bath

Numerical evolution of expanding SU(2) YM plasma seen to always lead to Baier-Mueller-Schiff-Son type scaling at late times (Berges et al., 1303.5650, 1311.3005)

Ongoing debate about the role of instabilities in hard interactions, argued to lead to slightly faster thermalization: $\tau_{KM} \sim \alpha_s^{-5/2}$ vs. $\tau_{BMSS} \sim \alpha_s^{-13/5}$
**Thermalization beyond weak coupling**

Remarkable progress for the early weak-coupling dynamics of a high energy collision. However, extension of the results to realistic heavy ion collision problematic:

- System clearly not asymptotically weakly coupled ⇒ Direct use of perturbative results requires bold extrapolation
- Dynamics classical only in an overoccupied system — works only for the early dynamics of the system
- Kinetic theory description misses important physics, e.g. instabilities

In absence of nonperturbative first principles techniques, clearly room for alternative approaches

- Needed in particular: Tool to address dynamical problems in strongly coupled field theory — interesting problem in itself!
The holographic way

All approaches to (thermal) QCD are some types of \textit{systematically improvable} approximations: pQCD, lattice QCD, effective theories, ...

Why not consider a different expansion point: SU($N_c$) gauge theory with

- $N_c$ taken to infinity
- Large 't Hooft coupling $\lambda = g^2 N_c$
- Additional adjoint fermions and scalars to make the theory $\mathcal{N} = 4$ supersymmetric and conformal

AdS/CFT conjecture (Maldacena, 1997):

- IIB string theory in $\text{AdS}_5 \times S_5$ exactly dual to $\mathcal{N} = 4$ Super Yang-Mills (SYM) theory living on the 4d Minkowskian boundary of the AdS space
- Strongly coupled, $N_c \to \infty$ SYM $\leftrightarrow$ Classical supergravity
Strong coupling thermalization

Due to conformality, SYM theory very different from QCD at $T = 0$. However:

- At finite temperature, systems much more similar
  - Both describe deconfined plasmas with Debye screening, finite static correlation length,...
  - Conformality and SUSY broken due to introduction of $T$
- Most of the above limits systematically improvable
- *Very* nontrivial field theory problems mapped to classical gravity
Motivation
Thermalization at strong(er) coupling

Important lessons from gauge/gravity calculations at infinite coupling:

- Thermalization always of top-down type (causal argument)
- Thermalization time naturally short, $\sim 1/T$
- Hydrodynamization $\neq$ Thermalization, isotropization
Bridging the gap

Obviously, it would be valuable to bring the two limiting cases closer to each other — and to a realistic setting. Is it possible to:

- Extend weak coupling picture to lower energies, with $\alpha_s(Q) \sim 1$?
- Marry weak coupling description of the early dynamics with strong coupling evolution?
- Bring field theory used in gauge/gravity calculations closer to real QCD?
  - Finite coupling & $N_c$, dynamical breaking of conformal invariance,...
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Rest of the talk: Attempt to relax the $\lambda = \infty$ (and conformality) approximation in studies of holographic thermalization
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AdS/CFT duality: $T = 0$

- Original conjecture: $SU(N_c) \mathcal{N} = 4$ SYM in $\mathbb{R}^{1,3} \leftrightarrow$ IIB ST in $AdS_5 \times S_5$

  "center" of AdS \hspace{5cm} boundary

  \begin{align*}
  & r = 0 \\
  & r = \infty
  \end{align*}

- Pure AdS metric corresponds to vacuum state of the CFT

  \[ ds^2 = L^2 \left( -r^2 dt^2 + \frac{dr^2}{r^2} + r^2 d\mathbf{x}^2 \right) \]

- Dictionary: CFT operators $\leftrightarrow$ bulk fields, with identification

  \[(L/l_s)^4 = \lambda, \quad g_s = \lambda/(4\pi N_c)\]

  $\Rightarrow$ Strongly coupled, large-$N_c$ QFT $\leftrightarrow$ Classical sugra
AdS/CFT duality: $T \neq 0$

- Strongly coupled large-$N_c$ SYM plasma in thermal equilibrium $\leftrightarrow$ Classical gravity in AdS black hole background
  - Metric now features event horizon at $r = r_h$ ($L \equiv 1$ from now on)

$$ds^2 = -r^2(1 - r_h^4/r^4) dt^2 + \frac{dr^2}{r^2(1 - r_h^4/r^4)} + r^2 d\mathbf{x}^2$$

- Identification of field theory temperature with Hawking temperature of the black hole $\Rightarrow T = r_h/\pi$
AdS/CFT duality: Thermalizing system

- Simplest way to take system out of equilibrium: Radial gravitational collapse of a thin planar shell (Danielsson, Keski-Vakkuri, Kruczenski)

  \[ r = 0 \quad r = r_h \quad r = r_s \quad r = \infty \]

- Metric defined in a piecewise manner:

  \[
  ds^2 = -r^2 f(r) dt^2 + \frac{dr^2}{r^2 f(r)} + r^2 dx^2, \quad f(r) = \begin{cases} 
  f_-(r) & \equiv 1, \quad \text{for } r < r_s \\
  f_+(r) & \equiv 1 - \frac{r_h^4}{r^4}, \quad \text{for } r > r_s
  \end{cases}
  \]

- Shell fills entire three-space $\Rightarrow$ Translational and rotational invariance

- Field theory side: Rapid, spatially homogenous injection of energy at all scales
Holographic description of thermalization

Basics of the duality

Shell can be realized by briefly turning on a spatially homogenous scalar source in the CFT, coupled to

- A marginal composite operator in the CFT
- The bulk metric through Einstein equations involving the corresponding bulk field

\[
ds^2 = \frac{1}{u^2} \left( - f(u, t) e^{-2\delta(u,t)} dt^2 + \frac{1}{f(u, t)} du^2 + dx^2 \right), \quad u = r_h^2/r^2
\]
Alternatively can send off the shell from rest at finite radius $r_0$

- For shell EoS $p = c\varepsilon$ radical slowing down of collapse as $c \to 1/3$, assuming mass of final black hole fixed
- $r_0$ only hard scale in the problem $\Rightarrow$ Tempting to speculate about relation to the saturation momentum
Holographic Green’s functions

In- and off-equilibrium correlators offer useful tool for studying thermalization:

- Poles of retarded thermal Green’s functions give dispersion relation of field excitations: Quasiparticle / quasinormal mode spectrum
- Time dependent off-equilibrium Green’s functions probe how fast different energy (length) scales equilibrate
- Related to measurable quantities, e.g. particle production rates
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Example 1: EM current correlator \( \langle J_{\mu}^{\text{EM}} J_{\nu}^{\text{EM}} \rangle \) — photon production

- Obtain by adding to the SYM theory a U(1) vector field coupled to a conserved current corresponding to a subgroup of SU(4)_R
- Excellent phenomenological probe of thermalization because of photons’ weak coupling to plasma constituents
Holographic Green’s functions

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Weak coupling to plasma constituents
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Example 2: Energy momentum tensor correlators $\langle T_{\mu\nu} T_{\alpha\beta} \rangle$ related to e.g. shear and bulk viscosities and dual to metric fluctuations $h_{\mu\nu}$

- Scalar channel: $h_{xy}$
- Shear channel: $h_{tx}, h_{zx}$
- Sound channel: $h_{tt}, h_{tz}, h_{zz}, h_{ii}$
Recipe for the retarded correlator

Retarded Green’s functions obtainable within the *quasistatic approximation* with small modifications to the original Son-Starinets recipe:

1. Solve classical EoM for the relevant bulk field inside and outside the shell
2. Match solutions at the shell using Israel junction conditions
   - Quasistatic limit: Ignore time derivatives
   - With $r_s > r_h$, the outside solution has also an outgoing component
3. Obtain the Green’s function from the behavior of the outside solution near the boundary
4. Repeat steps 1-3 for different values of $r_s/r_h$; if desired, combine this information with shell’s trajectory to obtain time dependence
   - Conformal EoS $\Rightarrow$ Parametrically slower evolution
Recipe for the retarded correlator

1. Solve classical EoM for the relevant bulk field inside and outside the shell.
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4. Repeat steps 1-3 for different values of $r_s/r_h$; if desired, combine this information with the shell's trajectory to obtain time dependence.

Conformal EoS $\Rightarrow$ Parametrically slower evolution.
Beyond infinite coupling: $\alpha'$ corrections

Recall key relation from AdS/CFT dictionary: $(L/l_s)^4 = L^4/\alpha'^2 = \lambda$, with $\alpha'$ the inverse string tension

- To go beyond $\lambda = \infty$ limit, need to add $\alpha'$ terms to the sugra action, i.e. determine the first non-trivial terms in a small-curvature expansion
- Leading order corrections $O(\alpha'^3) = O(\lambda^{-3/2})$

End up dealing with $O(\alpha'^3)$ improved type IIB sugra

$$S_{IIB} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G} \left( R_{10} - \frac{1}{2} (\partial \phi)^2 - \frac{F_5^2}{4 \cdot 5!} + \gamma e^{-\frac{3}{2}\phi} (C + T)^4 \right),$$

$$T_{abcdef} \equiv i\nabla_a F^+_{bcdef} + \frac{1}{16} \left( F^+_{abcmn} F^+_{def}^{\ mn} - 3 F^+_{abfmn} F^+_{dec}^{\ mn} \right),$$

$$F^+ \equiv \frac{1}{2} (1 + \star) F_5, \quad \gamma \equiv \frac{1}{8} \zeta(3) \lambda^{-3/2}$$

$\Rightarrow \gamma$-corrected metric and EoMs for different fields
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Quasinormal mode spectra at finite coupling

Analytic structure of retarded thermal Green’s functions $\Rightarrow$ Dispersion relation of field excitations

$$\omega_n(k) = E_n(k) + i\Gamma_n(k)$$

Striking difference between weakly and strongly coupled systems:

- At weak coupling (depending on operator) either long-lived quasiparticles with $\Gamma_n \ll E_n$ or branch cuts
- At strong coupling quasinormal mode spectrum

$$\hat{\omega}_n|_{k=0} = \frac{\omega_n|_{k=0}}{2\pi T} = n(\pm1 - i)$$
QNMs at infinite coupling: Photons

Pole structure of EM current correlator displays usual quasinormal mode spectrum at $\lambda = \infty$. How about at finite coupling?
**QNMs at finite coupling: Photons**

Effect of decreasing $\lambda$: Widths of the excitations consistently decrease $\Rightarrow$ Modes become longer-lived

NB: Convergence of strong coupling expansion not guaranteed, when $\hat{\omega}_n|_{k=0} = n(\pm 1 - i) + \xi_n/\lambda^{3/2}$ shifted from $\lambda = \infty$ value by $O(1)$ amount
Results
Quasinormal modes at finite coupling

QNMs at finite coupling: Photons

Zoom-in to the two lowest modes, \( n = 1 \) and 2: Sensitivity to \( \gamma \)-corrections grows rapidly with \( n \). Understandable from the higher derivative nature of the \( \mathcal{O}(\gamma) \) operators.
QNMs at finite coupling: Photons

Similar shift at nonzero three-momentum: \( k = 2\pi T \)
QNMs at finite coupling: $T_{\mu \nu}$ correlators

Same effect also in the shear (left) and sound (right) channels of energy-momentum tensor correlators (here $k = 0$)
Outside the $\lambda = \infty$ limit, the response of a strongly coupled plasma to infinitesimal perturbations appears to change, with the QNM spectrum moving towards the real axis, eventually forming a branch cut(?)

What happens if we take the system further away from equilibrium?
Off-equilibrium Green’s functions: Definitions

Natural quantities to study: Spectral density $\chi(\omega, k) \equiv \text{Im} \Pi_R(\omega, k)$ and related particle production rate (here photons)

$$k^0 \frac{d\Gamma_\gamma}{d^3k} = \frac{1}{4\pi k} \frac{d\Gamma_\gamma}{dk_0} = \frac{\alpha_{\text{EM}}}{4\pi^2} \eta^{\mu\nu} \Pi_{\mu\nu}(k_0 \equiv \omega, k) = \frac{\alpha_{\text{EM}}}{4\pi^2} \eta^{\mu\nu} n_B(\omega) \chi_\mu^{\mu}(\omega, k)$$

Useful measure of ‘out-of-equilibriumness’: Relative deviation of spectral density from the thermal limit

$$R(\omega, k) \equiv \frac{\chi(\omega, k) - \chi_{\text{therm}}(\omega, k)}{\chi_{\text{therm}}(\omega, k)}$$

Important consistency check: $R \to 0$, as $r_s \to r_h$
Production rates: Real (on-shell) photons

Left: Photon production rate for $\lambda = \infty$ and $r_s/r_h = 1.1, 1.01, 1.001, 1$

Right: Photon production rate for $r_s/r_h = 1.01$ and $\lambda = \infty, 120, 80, 40$

Note the much weaker dependence on $\lambda$ than in the QNM spectrum
Spectral density and $R$ at $\lambda = \infty$: Photons

Left: Photon spectral functions for different virtualities ($c = k/\omega$) in thermal equilibrium and $r_s/r_h = 1.1$

Right: Relative deviation $R \equiv (\chi - \chi_{th})/\chi_{th}$ for dileptons ($c = 0$) with $r_s/r_h = 1.1$ and 1.01 together with analytic WKB results, valid at large $\omega$

Note: Clear top-down thermalization pattern (as always at $\lambda = \infty$)
Relative deviation at finite $\lambda$: Photons

Relative deviation $R \equiv (\chi - \chi_{th})/\chi_{th}$ for on-shell photons with $r_s/r_h = 1.01$ and $\lambda = \infty, 500, 300$ (left) and $150, 100, 75$ (right)

NB: Change of pattern with decreasing $\lambda$: UV modes no longer first to thermalize.
Relative deviation at finite $\lambda$: $T_{\mu\nu}$ correlators

Relative deviation $R \equiv (\chi - \chi_{th})/\chi_{th}$ in the shear and sound channels for $r_s/r_h = 1.2$, $\lambda = 100$, and $k/\omega = 0$ (black), 6/9 (blue) and 8/9 (red)
Reliability of results

So what to make of all this? Indications of the holographic plasma starting to behave like a system of weakly coupled quasiparticles, or simply

- ... due to the breakdown of some approximation?
  - Quasistatic limit OK as long as $\omega / T \gg 1$
  - Strong coupling expansion applied with care: $(\text{NLO-LO})/\text{LO} \lesssim \mathcal{O}(1/10)$

- ... a peculiarity of the channels considered?
  - EM current and $T_{\mu\nu}$ correlators probe system in different ways
  - Recent results for purely geometric probes display different behavior\(^4\)

- ... a sign of the unphysical nature of the collapsing shell model?
  - Difficult to rule out. However, at least QNM results universal.

∴ Clearly, more work needed to generalize results — in particular to more realistic and dynamical models of thermalization

\(^4\)Galante, Schvellinger, 1205.1548
Implications for holography

For a given quantity,

\[ X(\lambda) = X(\lambda = \infty) \times \left( 1 + \frac{X_1}{\lambda^{3/2}} + O(1/\lambda^2) \right) \]

define critical coupling \( \lambda_c \) such that \( |\frac{X_1}{\lambda_c^{3/2}}| = 1 \). Then:

<table>
<thead>
<tr>
<th>Quantity</th>
<th>( \lambda_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure</td>
<td>0.9</td>
</tr>
<tr>
<td>Transport/hydro coeffs. (( \eta/S, \tau_H, \kappa ))</td>
<td>7 \pm 1</td>
</tr>
<tr>
<td>Spectral densities in equilibrium</td>
<td>( \lambda_c(\omega = 0) = 40 ),  ( \lambda_c(\omega \to \infty) = 0.8, ... )</td>
</tr>
<tr>
<td>Quasinormal mode ( n ) for photons / ( T_{\mu\nu} )</td>
<td>( \lambda_c(n = 1) = 200 ), ( \lambda_c(n = 2) = 500 ), ( \lambda_c(n = 3) = 1000, ... )</td>
</tr>
</tbody>
</table>

Lesson: What is weak/strong coupling strongly depends on the quantity. Thermalization appears particularly sensitive to strong coupling corrections.
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**Take home messages**

1. Holographic (thermalization) calculations can — and should — be taken away from the $\lambda = \infty$ limit

2. QNM spectrum and thermalization related properties particularly sensitive to strong coupling corrections: $\lambda \sim 10$ far from the strong coupling regime

3. Tentative indications that a holographic system obtains weakly coupled characteristics within the realm of a strong coupling expansion
   - QNM poles flow in the direction of a quasiparticle spectrum / branch cut
   - Top-down thermalization pattern weakens and shifts towards bottom-up