

# Phase diagram of the strongly interacting matter with linear sigma model

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# Overview

## 1 Introduction

- Motivation

## 2 The model

- Axial(vector) meson extended linear  $\sigma$ -model
- Parametrization at  $T = 0$
- Polyakov loop

## 3 eLSM at finite $T/\mu_B$

- Extremum equations for  $\phi_{N/S}$  and  $\Phi, \bar{\Phi}$

## 4 Results

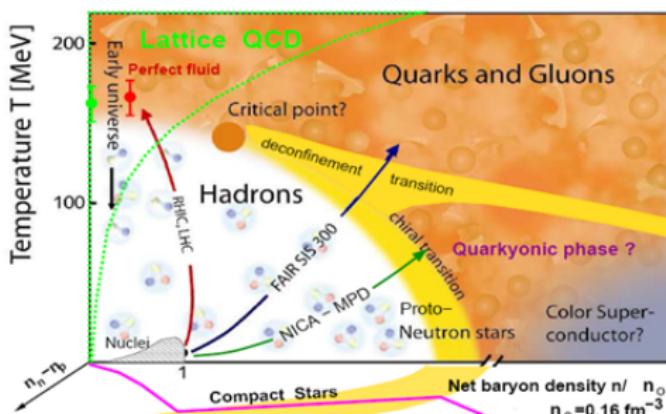
- $T$  dependence of the order parameters
- Critical endpoint
- $T$  dependence of the (pseudo)scalar masses

## 5 Summary

## Motivation

# QCD phase diagram

## Phase diagram in the $T - \mu_B - \mu_J$ space



- At  $\mu_B = 0$   $T_c = 151$  MeV  
Y. Aoki, et al., PLB **643**, 46 (2006)
  - Is there a CP?  
( $T_{CP} = 162$  MeV,  $\mu_{CP} = 360$  MeV, Fodor-Katz)
  - At  $T = 0$  in  $\mu_B$  where is the phase boundary?
  - Behaviour as a function of  $\mu_1/\mu_S$ ?

Details of the phase diagram are heavily studied theoretically (Lattice, EFT), and experimentally (RHIC, LHC, FAIR, NICA)

## Addressed problems

- Which scalars are the chiral partners of the pseudoscalar nonet?
  - Which parameterizations give phenomenologically good description of the phase transition?
  - Which of them predict the existence of the CP?
  - What is the order of phase transition on the  $T=0$  line?
  - How the order parameters behave at finite temperature/chemical potential?
  - How the masses change in medium?

# Efective models

Since QCD is very hard to solve → low energy effective models were set up → reflecting the global symmetries of QCD

- Nambu-Jona-Lasinio model (+Kobayashi-Maskawa)
- Chiral perturbation theory
- Linear and nonlinear (it does not contain degrees of freedom relevant at high T) sigma model

# QCD Lagrangian

$$\begin{aligned}
 \mathcal{L}_{\text{QCD}} &\equiv -\frac{1}{4} G_a^{\mu\nu} G_a^a + \sum_f \bar{q}_f (i\gamma^\mu D_\mu - m_f) q_f \\
 &= -\frac{1}{4} (\partial^\mu G_a^\nu - \partial^\nu G_a^\mu) (\partial_\mu G_a^\alpha - \partial_\nu G_a^\alpha) \\
 &\quad + \sum_f \bar{q}_f^\alpha (i\gamma^\mu \partial_\mu - m_f) q_f^\alpha \\
 &\quad + g_s G_a^\mu \sum_f \bar{q}_f^\alpha \gamma_\mu \left( \frac{\lambda^a}{2} \right)_{\alpha\beta} q_f^\beta \\
 &\quad - \frac{g_s}{2} f^{abc} (\partial^\mu G_a^\nu - \partial^\nu G_a^\mu) G_b^\mu G_c^\nu \\
 &\quad - \frac{g_s^2}{4} f^{abc} f_{ade} G_b^\mu G_c^\nu G_d^\rho G_e^\sigma
 \end{aligned}$$

1-2nd lines: kinetic terms, 3rd: interaction between quarks& gluons

4-5th lines: the cubic and quartic gluon self-interactions;

# Chiral symmetry

If the quark masses are zero (chiral limit)  $\Rightarrow$  QCD invariant under the following global transformation (**chiral symmetry**):

$q_L = (1 - \gamma_5)/2q, \quad q_R = (1 + \gamma_5)/2q$  only the mass term mixes  
 $U(3)_V q = \exp(-i\alpha t)q \quad U(3)_A q = \exp(-i\beta \gamma_5 t)q$

$$U(3)_L \times U(3)_R \simeq U(3)_V \times U(3)_A = \\ SU(3)_V \times SU(3)_A \times U(1)_V \times U(1)_A$$

$U(1)_V$  term  $\longrightarrow$  baryon number conservation

$U(1)_A$  term  $\longrightarrow$  broken through axial anomaly

$SU(3)_A$  term  $\longrightarrow$  broken down by any quark mass

$SU(3)_V$  term  $\longrightarrow$  broken down to  $SU(2)_V$  if  $m_u = m_d \neq m_s$   
 $\longrightarrow$  totally broken if  $m_u \neq m_d \neq m_s$  (**in nature**)

# Meson fields - pseudoscalar and scalar meson nonets

$$\Phi_{PS} = \sum_{i=0}^8 \pi_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\eta_N + \pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N - \pi^0}{\sqrt{2}} & K^0 \\ K^- & K^0 & \eta_S \end{pmatrix} (\sim \bar{q}_i \gamma_5 q_j)$$

$$\Phi_S = \sum_{i=0}^8 \sigma_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\sigma_N + a_0^0}{\sqrt{2}} & a_0^+ & K_S^+ \\ a_0^- & \frac{\sigma_N - a_0^0}{\sqrt{2}} & K_S^0 \\ K_S^- & K_S^0 & \sigma_S \end{pmatrix} (\sim \bar{q}_i q_j)$$

**Particle content:**

Pseudoscalars:  $\pi(138), K(495), \eta(548), \eta'(958)$

Scalars:  $a_0(980 \text{ or } 1450), K_0^*(800 \text{ or } 1430),$

$(\sigma_N, \sigma_S) : 2 \text{ of } f_0(500, 980, 1370, 1500, 1710)$

# Structure of scalar mesons

	Mass (MeV)	width (MeV)	decays
$A_0(980)$	$980 \pm 20$	$50 - 100$	$\pi\pi$ dominant
$A_0(1450)$	$1474 \pm 19$	$265 \pm 13$	$\pi\eta, \pi\eta', K\bar{K}$
$K_s(800) = \kappa$	$682 \pm 29$	$547 \pm 24$	$K\pi$
$K_s(1430)$	$1425 \pm 50$	$270 \pm 80$	$K\pi$ dominant
$f_0(500) = \sigma$	400–550	400 – 700	$\pi\pi$ dominant
$f_0(980)$	$980 \pm 20$	$40 - 100$	$\pi\pi$ dominant
$f_0(1370)$	1200–1500	200 – 500	$\pi\pi \approx 250, K\bar{K} \approx 150$
$f_0(1500)$	$1505 \pm 6$	$109 \pm 7$	$\pi\pi \approx 38, K\bar{K} \approx 9.4$
$f_0(1710)$	$1722 \pm 6$	$135 \pm 7$	$\pi\pi \approx 30, K\bar{K} \approx 71$

Possible scalar states:  $\bar{q}q, \bar{q}q\bar{q}q$ , meson-meson molecules, glueballs  
 pseudoscalar nonet:  $\pi, K, \eta, \eta'$ , scalar nonet:  $A_0, K_0, 2 f_0$   
 multiquark states:  $f_0(980), A_0(980), f_0(600), K_0(800)$  ???  
 meson-meson bound state ( $K\bar{K}$ ):  $f_0(980)$  ???  
 glueballs:  $f_0(1500)$  (weak coupling to  $\gamma\gamma$ ),  $f_0(1710)$  ???

# Included fields - vector meson nonets

$$V^\mu = \sum_{i=0}^8 \rho_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N + \rho^0}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & \frac{\omega_N - \rho^0}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \omega_S \end{pmatrix}^\mu$$

$$A^\mu = \sum_{i=0}^8 b_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{f_{1N} + a_1^0}{\sqrt{2}} & a_1^+ & K_1^+ \\ a_1^- & \frac{f_{1N} - a_1^0}{\sqrt{2}} & K_1^0 \\ K_1^- & \bar{K}_1^0 & f_{1S} \end{pmatrix}^\mu$$

## Particle content:

Vector mesons:  $\rho(770)$ ,  $K^*(894)$ ,  $\omega_N = \omega(782)$ ,  $\omega_S = \phi(1020)$

Axial vectors:  $a_1(1230)$ ,  $K_1(1270)$ ,  $f_{1N}(1280)$ ,  $f_{1S}(1426)$

# Lagrangian (2/1)

$$\begin{aligned}
 \mathcal{L}_{\text{Tot}} = & \text{Tr}[(D_\mu \Phi)^\dagger (D_\mu \Phi)] - m_0^2 \text{Tr}(\Phi^\dagger \Phi) - \lambda_1 [\text{Tr}(\Phi^\dagger \Phi)]^2 - \lambda_2 \text{Tr}(\Phi^\dagger \Phi)^2 \\
 & - \frac{1}{4} \text{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) + \text{Tr} \left[ \left( \frac{m_1^2}{2} + \Delta \right) (L_\mu^2 + R_\mu^2) \right] + \text{Tr}[H(\Phi + \Phi^\dagger)] \\
 & + \textcolor{red}{c_1 (\det \Phi + \det \Phi^\dagger)} + i \frac{g_2}{2} (\text{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \text{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) \\
 & + \frac{h_1}{2} \text{Tr}(\Phi^\dagger \Phi) \text{Tr}(L_\mu^2 + R_\mu^2) + h_2 \text{Tr}[(L_\mu \Phi)^2 + (\Phi R_\mu)^2] + 2h_3 \text{Tr}(L_\mu \Phi R^\mu \Phi^\dagger). \\
 & + g_3 [\text{Tr}(L_\mu L_\nu L^\mu L^\nu) + \text{Tr}(R_\mu R_\nu R^\mu R^\nu)] + g_4 [\text{Tr}(L_\mu L^\mu L_\nu L^\nu) \\
 & + \text{Tr}(R_\mu R^\mu R_\nu R^\nu)] + g_5 \text{Tr}(L_\mu L^\mu) \text{Tr}(R_\nu R^\nu) + g_6 [\text{Tr}(L_\mu L^\mu) \text{Tr}(L_\nu L^\nu) \\
 & + \text{Tr}(R_\mu R^\mu) \text{Tr}(R_\nu R^\nu)]
 \end{aligned}$$

$$+ \bar{\Psi} i \not{\partial} \Psi - g_F \bar{\Psi} (\Phi_S + i \gamma_5 \Phi_{PS}) \Psi + g_V \bar{\Psi} \gamma^\mu \left( V_\mu + \frac{g_A}{g_V} \gamma_5 A_\mu \right) \Psi$$

+ Polyakov loops

D. Parganlija, P. Kovacs, Gy. Wolf, F. Giacosa, D.H. Rischke,  
 Phys. Rev. D87 (2013) 014011

Axial(vector) meson extended linear  $\sigma$ -model

## Lagrangian (2/2)

where

$$D^\mu \Phi = \partial^\mu \Phi - ig_1(L^\mu \Phi - \Phi R^\mu) - ie A_e^\mu [T_3, \Phi]$$

$$\Phi = \sum_{i=0}^8 (\sigma_i + i\pi_i) T_i, \quad H = \sum_{i=0}^8 h_i T_i \quad T_i : U(3) \text{ generators}$$

$$R^\mu = \sum_{i=0}^8 (\rho_i^\mu - b_i^\mu) T_i, \quad L^\mu = \sum_{i=0}^8 (\rho_i^\mu + b_i^\mu) T_i$$

$$L^{\mu\nu} = \partial^\mu L^\nu - ieA_e^\mu[T_3, L^\nu] - \{\partial^\nu L^\mu - ieA_e^\nu[T_3, L^\mu]\}$$

$$R^{\mu\nu} = \partial^\mu R^\nu - ieA_e^\mu [T_3, R^\nu] - \{\partial^\nu R^\mu - ieA_e^\nu [T_3, R^\mu]\}$$

$$\bar{\Psi} = (\bar{u}, \bar{d}, \bar{s})$$

non strange – strange base:

$$\varphi_N = \sqrt{2/3}\varphi_0 + \sqrt{1/3}\varphi_8,$$

$$\varphi_S = \sqrt{1/3}\varphi_0 - \sqrt{2/3}\varphi_8, \quad \varphi \in (\sigma_i, \pi_i, \rho_i^\mu, b_i^\mu, h_i)$$

broken symmetry: non-zero condensates  $\langle \sigma_N \rangle, \langle \sigma_S \rangle \longleftrightarrow \phi_N, \phi_S$

Axial(vector) meson extended linear  $\sigma$ -model

## Symmetry properties of the model

## Global $U(3)_L \times U(3)_R$ transformation:

$$\begin{aligned}\Phi &\rightarrow U_L \Phi U_R^\dagger \\ L^\mu &\rightarrow U_L L^\mu U_L^\dagger \\ R^\mu &\rightarrow U_R R^\mu U_R^\dagger\end{aligned}$$

Consequences (using the unitarity of U's)

$$\begin{aligned} D^\mu \Phi &\rightarrow U_L D^\mu \Phi U_R^\dagger \\ L^{\mu\nu} &\rightarrow U_L L^{\mu\nu} U_L^\dagger \\ R^{\mu\nu} &\rightarrow U_R R^{\mu\nu} U_R^\dagger \end{aligned}$$

$$(\text{Tr}(\Phi^\dagger \Phi))' = \text{Tr}(U_R \Phi^\dagger U_L^\dagger U_L \Phi U_R^\dagger) = \text{Tr}(U_R \Phi^\dagger \Phi U_R^\dagger) = \text{Tr}(\Phi^\dagger \Phi U_R^\dagger U_R) = \text{Tr}(\Phi^\dagger \Phi)$$

All terms are invariant except the determinant and the explicit symmetry breaking term.

## Determinant term

$$U_L \equiv e^{-i\omega_L^a T^a} \quad U_R \equiv e^{-i\omega_R^a T^a}$$

$$\omega_V^a = 0.5(\omega_I^a + \omega_R^a) \quad \omega_A^a = 0.5(\omega_I^a - \omega_R^a)$$

By  $SU(3)_L \times SU(3)_R$  transformation (if  $\omega_I^0 = \omega_R^0 = 0 = \omega_V^0 = \omega_A^0$ )

$$(\det \Phi)' = \det(U_L \Phi U_R^\dagger) = \det U_L \det \Phi \det U_R^\dagger = \det \Phi$$

Similarly  $\det \Phi^\dagger$  is also invariant.

If  $\omega_V^0 \neq 0$  and all the other  $\omega$ 's are 0 ( $[T^a, T^0] = 0$ )

$$(\det \Phi)' = \det(e^{-i\omega_V^0 T^0} \Phi e^{i\omega_V^0 T^0}) = \det(e^{-i\omega_V^0 T^0} e^{i\omega_V^0 T^0} \Phi) = \det \Phi$$

On the other hand, if  $\omega_A^0 \neq 0$  and all the other  $\omega$ 's are 0

$$(\det \Phi)' = \det(e^{-i\omega_A^0 T^0} \Phi e^{-i\omega_A^0 T^0}) = \det(e^{-i\omega_A^0 T^0} e^{-i\omega_A^0 T^0} \Phi) = e^{-i2\omega_A^0} \det \Phi \text{Tr } T^0$$

So the determinant term is invariant under  $U(3)_V \times SU(3)_A$  transformation and breaks explicitly the  $U(1)_A$  symmetry.

# Explicit breaking term: $\text{Tr}[\hat{\epsilon}(\Phi + \Phi^\dagger)]$

$$\hat{\epsilon} = \sum_{i=0}^8 \epsilon_i T_i = \begin{pmatrix} \frac{\epsilon_N}{2} & 0 & 0 \\ 0 & \frac{\epsilon_N}{2} & 0 \\ 0 & 0 & \frac{\epsilon_S}{\sqrt{2}} \end{pmatrix} \quad \text{only } \epsilon^0, \epsilon^8 \neq 0$$

- axial transformation: if at least  $\epsilon^0 \neq 0$   $U(3)_A$  is broken:

$$(\text{Tr}[\hat{\epsilon}(\Phi)])' = \text{Tr}(e^{-i2\omega_A^a T^a} \hat{\epsilon} \Phi)$$

- vector transformation

$$(\text{Tr}[\hat{\epsilon}(\Phi)])' = \text{Tr}(e^{-i\omega_V^a T^a} \hat{\epsilon} e^{i\omega_V^a T^a} \Phi)$$

Since  $[\hat{\epsilon}, T^0] = 0$ ,  $U(1)_V$  symmetry is preserved.

If all  $\epsilon^a = 0$  except  $\epsilon^0$ ,  $U(3)_V$  is preserved.

If  $\epsilon^8$  also non zero, then since  $[T^K, T^8] = 0$  if  $k = 1, 2, 3$ ,  $U(1)_V \times SU(2)_V$  survives (isospin symmetry)

(If  $\epsilon^3 \neq 0$  too, then the isospin symmetry is broken, only  $U(1)_V$ .)

# Spontaneous symmetry breaking

Interaction is approximately chiral symmetric, spectra not, so SSB:

$$\sigma_{N/S} \rightarrow \sigma_{N/S} + \phi_{N/S} \quad \phi_{N/S} \equiv \langle \sigma_{N/S} \rangle$$

For tree level masses we have to select all terms quadratic in the new fields. Some of the terms include mixings arising from terms like  $\text{Tr}[(D_\mu \Phi)^\dagger (D_\mu \Phi)]$ :

$$\pi_N - a_{1N}^\mu : -g_1 \phi_N a_{1N}^\mu \partial_\mu \pi_N,$$

$$\pi - a_1^\mu : -g_1 \phi_N (a_1^{\mu+} \partial_\mu \pi^- + a_1^{\mu 0} \partial_\mu \pi^0) + \text{h.c.},$$

$$\pi_S - a_{1S}^\mu : -\sqrt{2} g_1 \phi_S a_{1S}^\mu \partial_\mu \pi_S,$$

$$K_S - K_\mu^\star : \frac{ig_1}{2} (\sqrt{2} \phi_S - \phi_N) (\bar{K}_\mu^{\star 0} \partial^\mu K_S^0 + K_\mu^{\star -} \partial^\mu K_S^+) + \text{h.c.},$$

$$K - K_1^\mu : -\frac{g_1}{2} (\phi_N + \sqrt{2} \phi_S) (K_1^{\mu 0} \partial_\mu \bar{K}^0 + K_1^{\mu +} \partial_\mu \bar{K}^-) + \text{h.c.}.$$

Parametrization at  $T = 0$ 

# Determination of the parameters of the Lagrangian

16 unknown parameters ( $m_0, \lambda_1, \lambda_2, c_1, m_1, g_1, g_2, h_1, h_2, h_3, \delta_S, \Phi_N, \Phi_S, g_F, g_V, g_A$ ) → Determined by the min. of  $\chi^2$ :

$$\chi^2(x_1, \dots, x_N) = \sum_{i=1}^M \left[ \frac{Q_i(x_1, \dots, x_N) - Q_i^{\text{exp}}}{\delta Q_i} \right]^2,$$

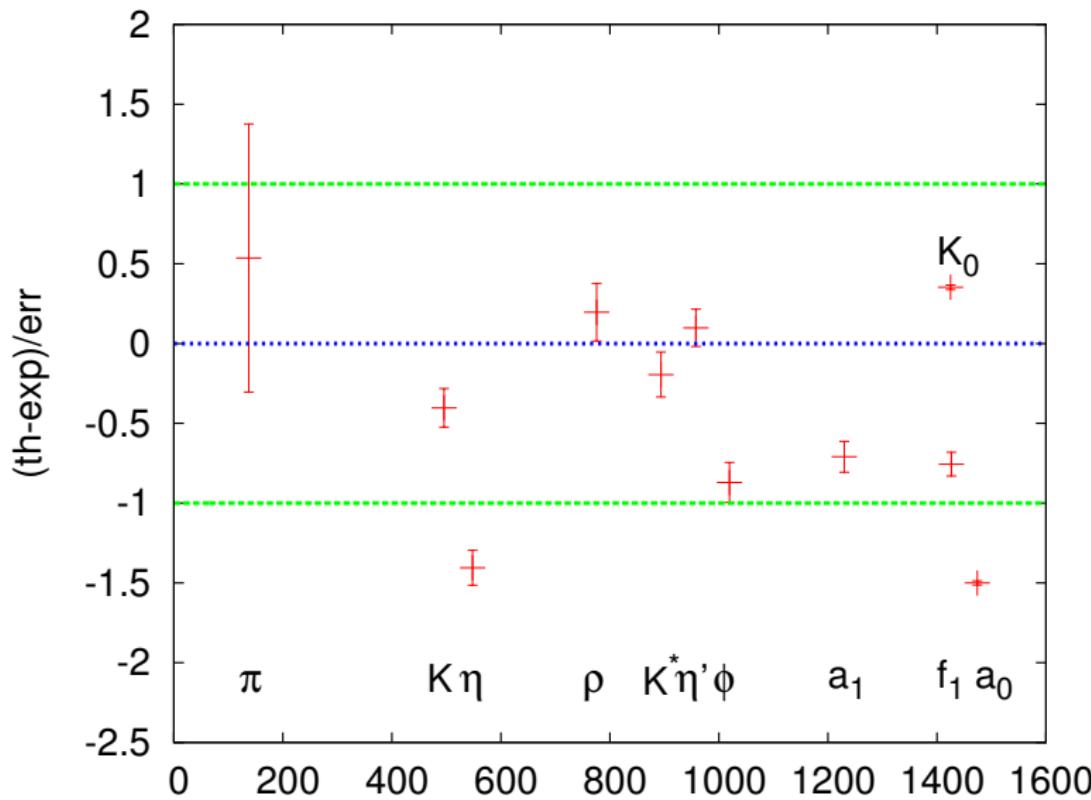
where  $(x_1, \dots, x_N) = (m_0, \lambda_1, \lambda_2, \dots)$ ,  $Q_i(x_1, \dots, x_N)$  calculated from the model, while  $Q_i^{\text{exp}}$  taken from the PDG

multiparametric minimization → MINUIT

- PCAC → 2 physical quantities:  $f_\pi, f_K$
- Tree-level masses → 16 physical quantities:  
 $m_u/d, m_s, m_\pi, m_\eta, m_{\eta'}, m_K, m_\rho, m_\Phi, m_{K^*}, m_{a_1}, m_{f_1^H}, m_{K_1}, m_{a_0}, m_{K_s}, m_{f_0^L}, m_{f_0^H}$
- Decay widths → 12 physical quantities:  
 $\Gamma_{\rho \rightarrow \pi\pi}, \Gamma_{\Phi \rightarrow KK}, \Gamma_{K^* \rightarrow K\pi}, \Gamma_{a_1 \rightarrow \pi\gamma}, \Gamma_{a_1 \rightarrow \rho\pi}, \Gamma_{f_1 \rightarrow KK^*}, \Gamma_{a_0}, \Gamma_{K_S \rightarrow K\pi}, \Gamma_{f_0^L \rightarrow \pi\pi}, \Gamma_{f_0^L \rightarrow KK}, \Gamma_{f_0^H \rightarrow \pi\pi}, \Gamma_{f_0^H \rightarrow KK}$

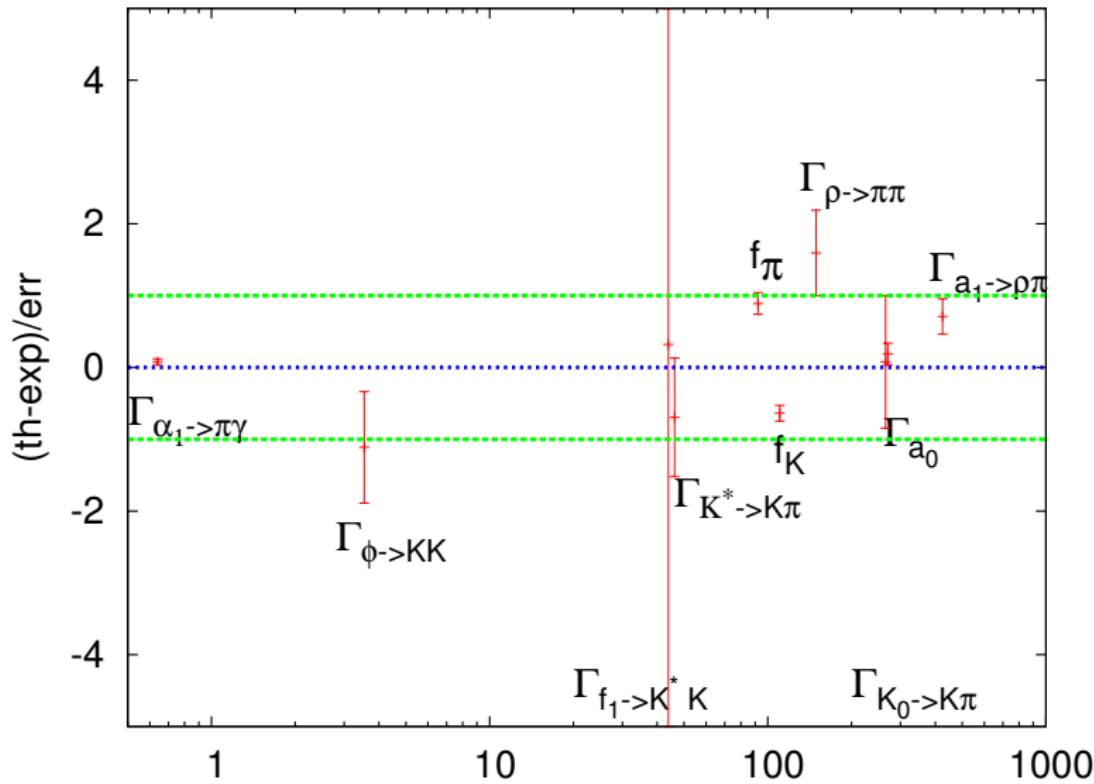
Parametrization at  $T = 0$ 

## Results: Masses



Parametrization at  $T = 0$ 

## Results: widths



Parametrization at  $T = 0$ 

# Results

	Cal(GeV)	Mass		Cal(GeV)	Width
$m_\pi$	0.1383	0.1380	$f_\pi$	0.0923	0.0922
$m_K$	0.5060	0.4956	$f_K$	0.1099	0.1100
$m_\eta$	0.5280	0.5479	$\Gamma_{f_0 L \rightarrow KK}$	0.1437	0.150
$m_{\eta'}$	0.9651	0.9578	$\Gamma_{f_0 H \rightarrow KK}$	0.158	0.0714
$m_\rho$	0.7672	0.7755	$\Gamma_\rho$	0.1694	0.149
$m_\phi$	1.0152	1.0195	$\Gamma_\phi$	0.001535	0.00177
$m_{K^*}$	0.9006	0.8938	$\Gamma_{K^*}$	0.044194	0.0462
$m_{f_1 H}$	1.4083	1.4264	$\Gamma_{f_1 \rightarrow KK}$	0.0438	0.0439
$m_{a_1}$	1.1829	1.2300	$\Gamma_{A_1 \rightarrow \rho \pi}$	0.656	0.425
$m_{K_1}$	1.2999	1.2720	$\Gamma_{A_1 \rightarrow \gamma \pi}$	0.000705	0.000640
$m_{a_0}$	1.4467	1.4740	$\Gamma_{A_0}$	0.254	0.265
$m_{K_s}$	1.5390	1.4250	$\Gamma_{K_s}$	0.363	0.270
$m_{f_0 L}$	1.303	1.3700	$\Gamma_{f_0 L \rightarrow \pi \pi}$	0.172	0.250
$m_{f_0 H}$	1.5865	1.7200	$\Gamma_{f_0 H \rightarrow \pi \pi}$	0.032744	0.0297
$m_{ud}$	0.292	0.314	$m_s$	0.314	0.500

Parametrization at  $T = 0$ 

# Parameters

Parameter	Value
$\phi_N$ [GeV]	0.1622
$\phi_S$ [GeV]	0.1262
$C_1$ [GeV $^2$ ]	-0.7537
$C_2$ [GeV $^2$ ]	0.3953
$\lambda_1$	undetermined
$\lambda_2$	65.3221
$h_1$	undetermined
$h_2$	11.6586
$h_3$	4.7028
$\delta_S$ [GeV $^2$ ]	0.1534
$c_1$ [GeV]	1.12
$g_1$	-5.8943
$g_2$	-2.9960
$g_F$	4.9429

- with this set  $f_0^L = 1303$  GeV
- by setting  $\lambda_1 \rightarrow f_0^L$  mass can be lowered

## Polyakov loop

# Polyakov loops in Polyakov gauge

Polyakov loop variables:  $\Phi(\vec{x}) = \frac{\text{Tr}_c L(\vec{x})}{N_c}$  and  $\bar{\Phi}(\vec{x}) = \frac{\text{Tr}_c \bar{L}(\vec{x})}{N_c}$  with  
$$L(x) = \mathcal{P} \exp \left[ i \int_0^\beta d\tau A_4(\vec{x}, \tau) \right]$$

→ signals center symmetry ( $\mathbb{Z}_3$ ) breaking at the deconfinement

low  $T$ : confined phase,  $\langle \Phi(\vec{x}) \rangle, \langle \bar{\Phi}(\vec{x}) \rangle = 0$

high  $T$ : deconfined phase,  $\langle \Phi(\vec{x}) \rangle, \langle \bar{\Phi}(\vec{x}) \rangle \neq 0$

Polyakov gauge: the temporal component of the gauge field is time independent and can be gauge rotated to a diagonal form in the color space

## Effects of the gauge fields:

- In this gauge the effect of the gauge field on the quarks acts like an imaginary chemical potential  
→ modified quark distribution function.
- Polyakov potential:  $\mathcal{U}(\Phi, \bar{\Phi})$  models the free energy of a pure gauge theory, parameters are fitted to the pure gauge lattice data

Extremum equations for  $\phi_{N/S}$  and  $\Phi, \bar{\Phi}$ 

# $T/\mu_B$ dependence of the Polyakov-loops

$$\frac{\partial \Omega}{\partial \Phi} = \frac{\partial \Omega}{\partial \bar{\Phi}} \Big|_{\varphi_N = \phi_N, \varphi_S = \phi_S} = 0, \quad \Omega : \text{grand canonical potential}$$

$$-\frac{d}{d\Phi} \left( \frac{U(\Phi, \bar{\Phi})}{T^4} \right) + \frac{2N_c}{T^3} \sum_{q=u,d,s} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \left( \frac{e^{-\beta E_q^-(p)}}{g_q^-(p)} + \frac{e^{-2\beta E_q^+(p)}}{g_q^+(p)} \right) = 0$$

$$-\frac{d}{d\bar{\Phi}} \left( \frac{U(\Phi, \bar{\Phi})}{T^4} \right) + \frac{2N_c}{T^3} \sum_{q=u,d,s} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \left( \frac{e^{-\beta E_q^+(p)}}{g_q^+(p)} + \frac{e^{-2\beta E_q^-(p)}}{g_q^-(p)} \right) = 0$$

$$g_q^+(p) = 1 + 3 \left( \bar{\Phi} + \Phi e^{-\beta E_q^+(p)} \right) e^{-\beta E_q^+(p)} + e^{-3\beta E_q^+(p)}$$

$$g_q^-(p) = 1 + 3 \left( \Phi + \bar{\Phi} e^{-\beta E_q^-(p)} \right) e^{-\beta E_q^-(p)} + e^{-3\beta E_q^-(p)}$$

$$E_q^\pm(p) = E_q(p) \mp \mu_B/3, \quad E_{u/d}(p) = \sqrt{p^2 + m_{u/d}^2}, \quad E_s(p) = \sqrt{p^2 + m_s^2}$$

Extremum equations for  $\phi_{N/S}$  and  $\Phi, \bar{\Phi}$ 

# $T/\mu_B$ dependence of the condensates ( $\phi_{N/S}$ )

$$\frac{\partial \Omega}{\partial \phi_N} = \left. \frac{\partial \Omega}{\partial \phi_S} \right|_{\Phi, \bar{\Phi}} = 0, \quad (\text{after the SSB})$$

Hybrid approach: fermions at one-loop, mesons at tree-level (their effects are much smaller)

$$\begin{aligned} m_0^2 \phi_N &+ \left( \lambda_1 + \frac{1}{2} \lambda_2 \right) \phi_N^3 + \lambda_1 \phi_N \phi_S^2 - h_N + \frac{g_F}{2} N_c (\langle u\bar{u} \rangle_T + \langle d\bar{d} \rangle_T) = 0 \\ m_0^2 \phi_S &+ (\lambda_1 + \lambda_2) \phi_S^3 + \lambda_1 \phi_N^2 \phi_S - h_S + \frac{g_F}{\sqrt{2}} N_c \langle s\bar{s} \rangle_T = 0 \end{aligned}$$

$$\langle q\bar{q} \rangle_T = -4m_q \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \frac{1}{2E_q(p)} (1 - f_\Phi^-(E_q(p)) - f_\Phi^+(E_q(p)))$$

Extremum equations for  $\phi_{N/S}$  and  $\Phi, \bar{\Phi}$ 

# Masses

$$M_{i,ab}^2 = \left. \frac{\partial^2 \Omega(T, \mu_f)}{\partial \varphi_{i,a} \partial \varphi_{i,b}} \right|_{\min} = m_{i,ab}^2 + \Delta_0 m_{i,ab}^2 + \Delta_T m_{i,ab}^2,$$

$m_{i,ab}^2 \longrightarrow$  tree-level mass matrix,

$\Delta_0/T m_{i,ab}^2 \longrightarrow$  fermion **vacuum/thermal** fluctuation,

$$\Delta_0 m_{i,ab}^2 = \left. \frac{\partial^2 \Omega_{q\bar{q}}^{\text{vac}}}{\partial \varphi_{i,a} \partial \varphi_{i,b}} \right|_{\min} = -\frac{3}{8\pi^2} \sum_{f=u,d,s} \left[ \left( \frac{3}{2} + \log \frac{m_f^2}{M^2} \right) m_{f,a}^{2(i)} m_{f,b}^{2(i)} + m_f^2 \left( \frac{1}{2} + \log \frac{m_f^2}{M^2} \right) m_{f,ab}^{2(i)} \right],$$

$$\begin{aligned} \Delta_T m_{i,ab}^2 &= \left. \frac{\partial^2 \Omega_{q\bar{q}}^{\text{th}}}{\partial \varphi_{i,a} \partial \varphi_{i,b}} \right|_{\min} = 6 \sum_{f=u,d,s} \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2E_f(p)} \left[ (f_f^+(p) + f_f^-(p)) \left( m_{f,ab}^{2(i)} - \frac{m_{f,a}^{2(i)} m_{f,b}^{2(i)}}{2E_f^2(p)} \right) \right. \\ &\quad \left. + (B_f^+(p) + B_f^-(p)) \frac{m_{f,a}^{2(i)} m_{f,b}^{2(i)}}{2TE_f(p)} \right], \end{aligned}$$

where  $m_{f,a}^{2(i)} \equiv \partial m_f^2 / \partial \varphi_{i,a}$ ,  $m_{f,ab}^{2(i)} \equiv \partial^2 m_f^2 / \partial \varphi_{i,a} \partial \varphi_{i,b}$

$T$  dependence of the order parameters

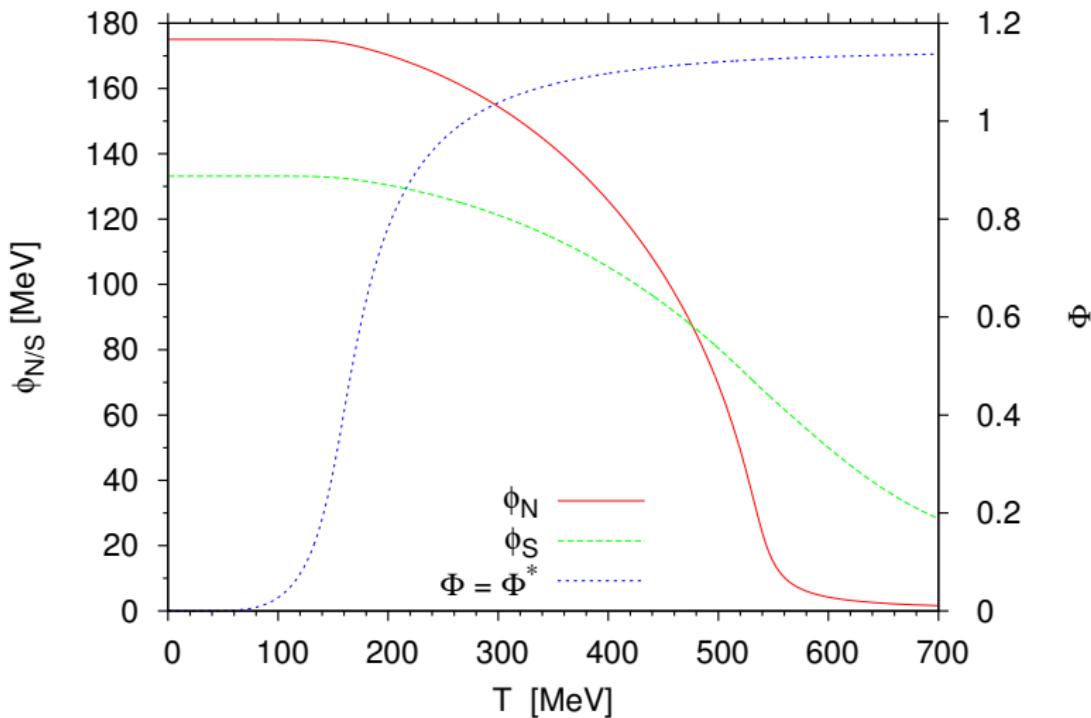
## Features of our approach

- D.O.F's: scalar, pseudoscalar, vector, axial vector nonets,
  - Polyakov loop variables,
  - constituent quarks
- Four order parameters  $(\phi_N, \phi_S, \Phi, \bar{\Phi}) \rightarrow$  four  $T/\mu$ -dependent equations
- Fermion **vacuum** fluctuations
- Fermion **thermal** fluctuations

## *T* dependence of the order parameters

$\phi_{N/S}(T), \Phi/\bar{\Phi}(T)$  **with** Polyakov loop  $m_{f_0^L} = 1326$  MeV

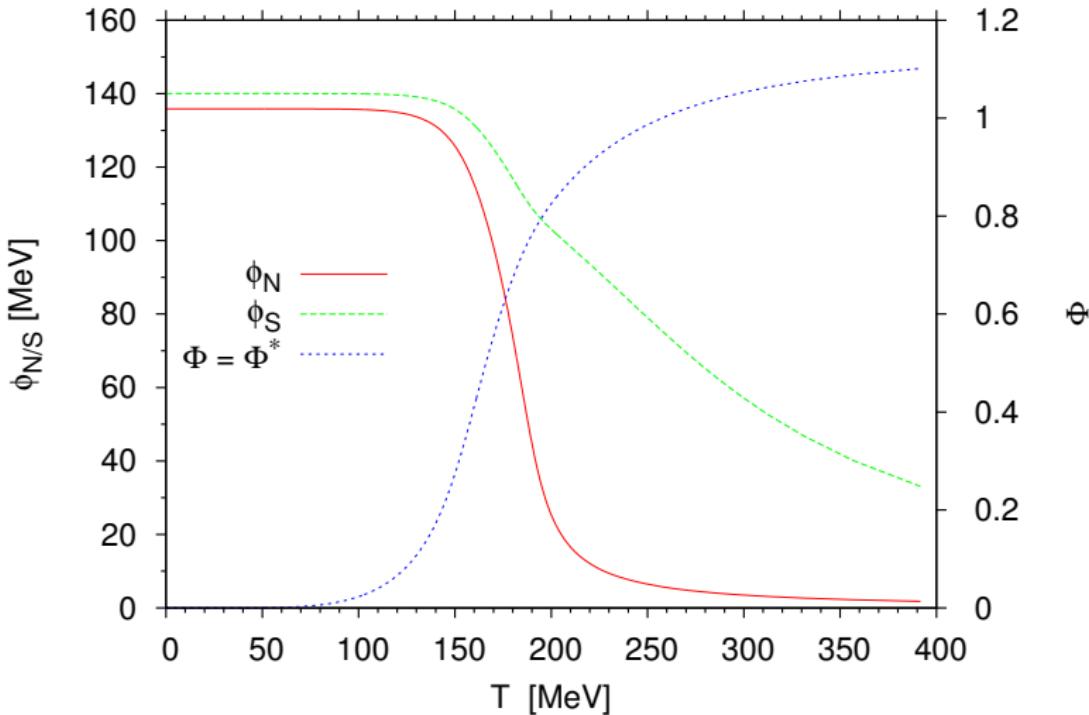
Condensates and Polyakov loop variables with vacuum fluctuations



## $T$ dependence of the order parameters

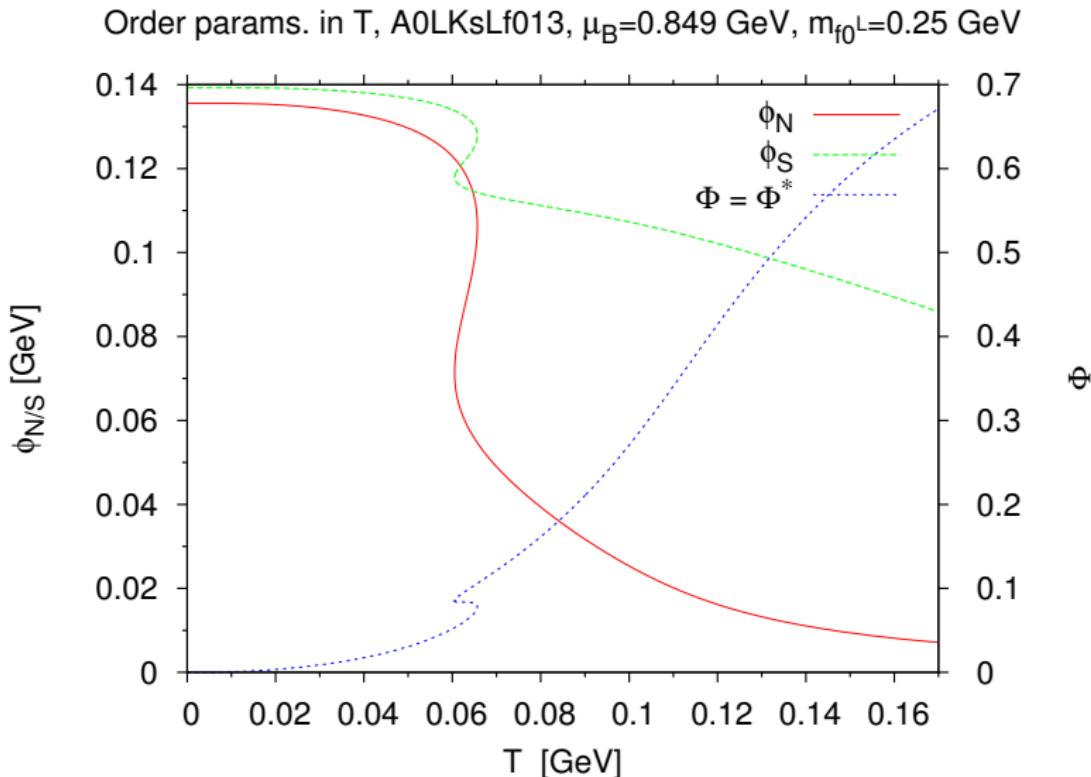
With low mass scalars,  $m_{f_0^L} = 402$  MeV

Condensates and Polyakov loop variables with vacuum fluctuations



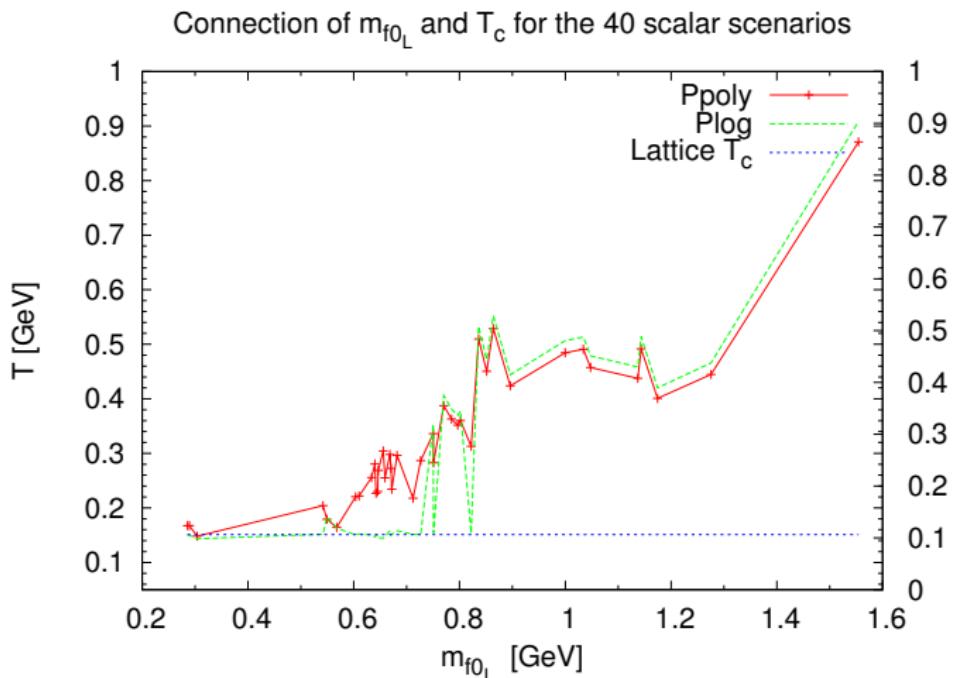
$T$  dependence of the order parameters

# With low mass scalars 1st order phase transition



$T$  dependence of the order parameters

# $T_c$ at $\mu_B = 0$ for various parameterizations



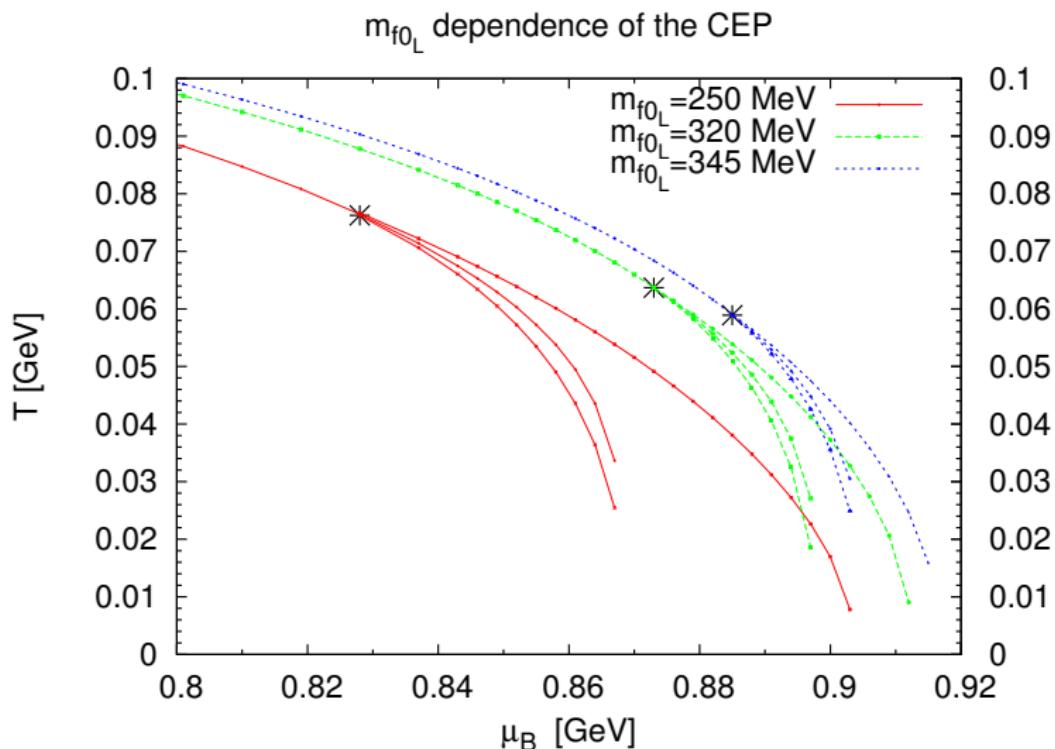
40 parameterizations → all the possible combinations for the masses of the scalar sector

$T$  dependence of the order parameters

## Important remarks

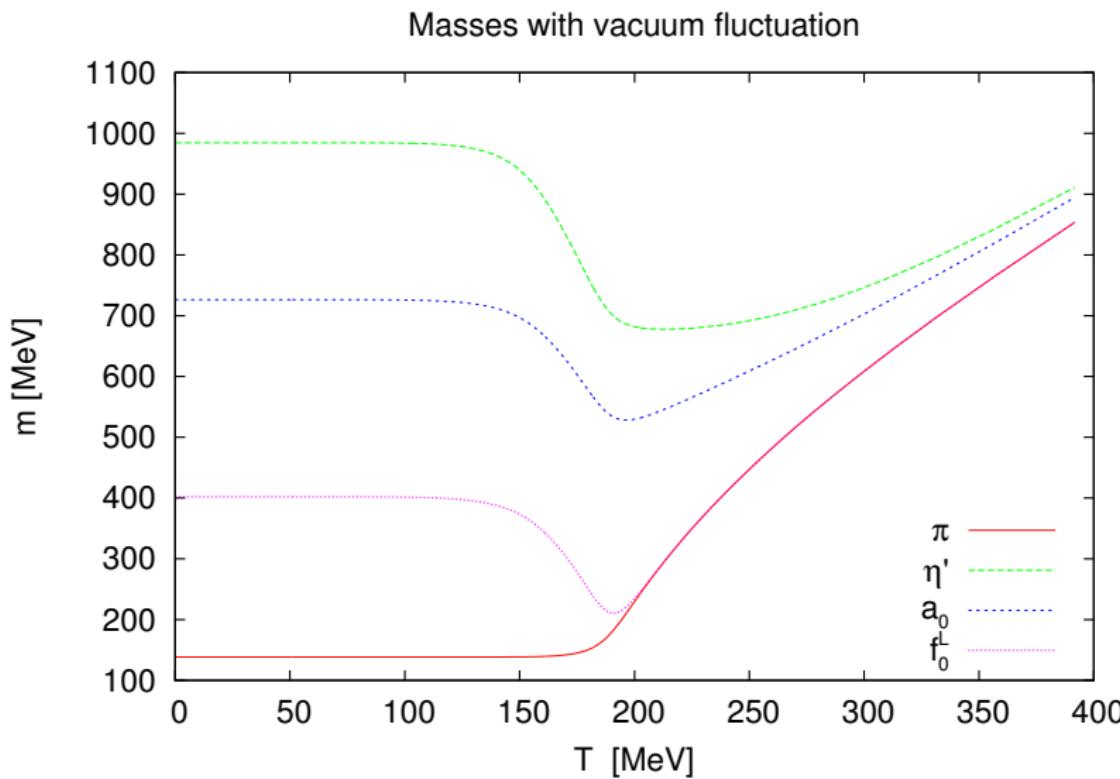
- In all 40 cases the best  $\chi^2$  solution was chosen
- Only parameterizations, which produced  $m_{f_0^L} \lesssim 800$  MeV can have  $T_c \approx 151$  MeV (lattice data)
- Only parameterizations, which produced  $m_{f_0^L} \lesssim 400$  MeV can have 1<sup>st</sup> order transition in  $\mu_B \implies$  there is CEP
- If  $T_c \approx 150$  MeV and the CEP exists  $\implies m_{a_0}$  and  $m_{K_S}$  is also below 1 GeV

Critical endpoint

CEP for different  $f_0^L$  masses

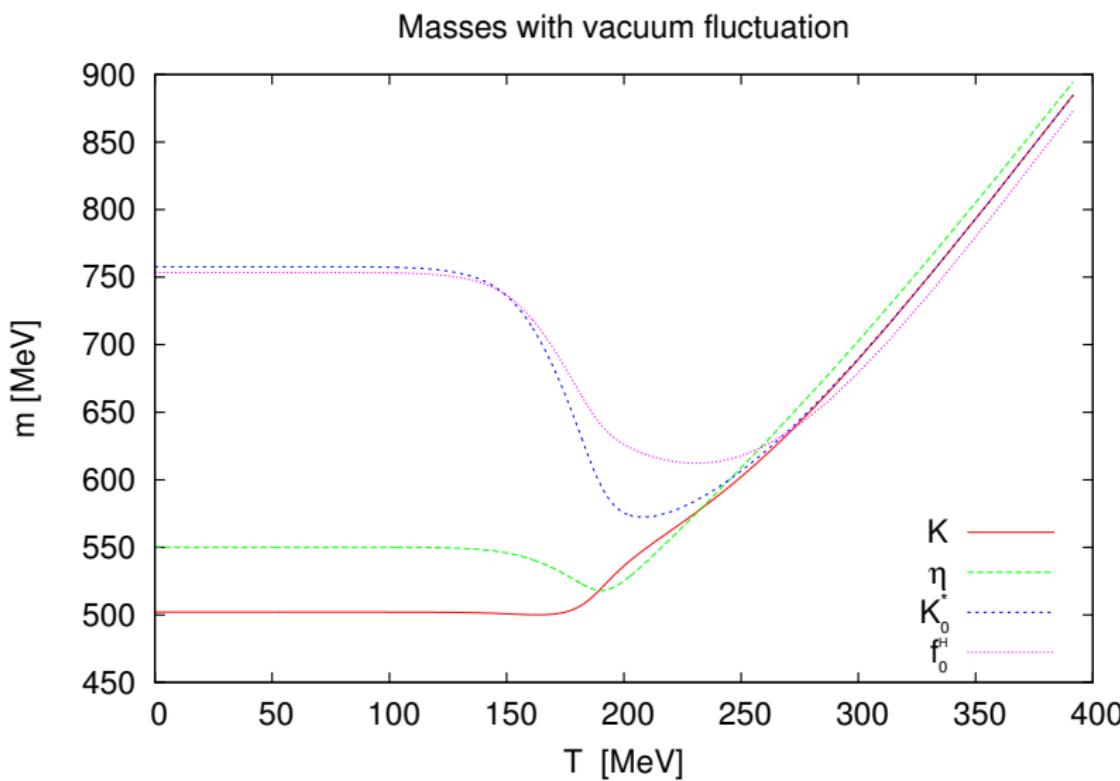
## $T$ dependence of the (pseudo)scalar masses

$\pi, \eta', a_0, f_0^L$  masses



$T$  dependence of the (pseudo)scalar masses

$K, \eta, K^*, f_0^H$  masses



# Summary

- An extended linear  $\sigma$ -model was shown with constituent quarks and Polyakov-loops
  - The meson phenomenology was very well described by scalars above 1 GeV
  - We used hybrid approach at  $T = 0$ : only fermion loops, since it has the largest contribution
  - At finite  $T/\mu_B$  there were 4 coupled equations for the 4 order parameters
  - The phase transition temperature requires low mass  $f_0$
- To do ...
- Improve the vacuum phenomenology by tetraquarks (and glueballs)

Thank you for your attention!