Baryons in Sakai-Sugimoto model in D0-D4 background

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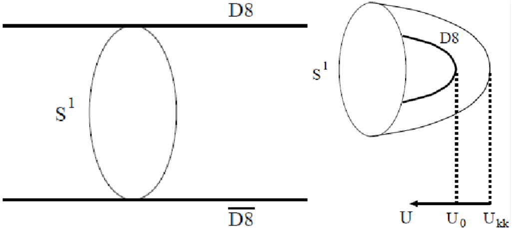
Intro. to S-S model in D0-D4

- The brane configuration:

	0	1	2	3	$4(\tau)$	$5(U)$	6	7	8	9
D4	-	-	-	-	-					
D8	-	-	-	-		-	-	-	-	-
D0	=	=	=	=	-					



- Massless Field content:
  $A_\mu^{(D4)}$: U(N) adj; Lorentz vector; $U_L(N_f) \times U_R(N_f)$, (**1**,**1**).
  q_L^f: U(N) fund.; Lorentz spinor 2_+; $U_L(N_f) \times U_R(N_f)$, (fund.,**1**).
  q_R^f: U(N) fund.; Lorentz spinor 2_-; $U_L(N_f) \times U_R(N_f)$, (**1**,fund.).

Intro. to S-S model in D0-D4

- The string frame metric in the limit $\alpha' = \ell_s^2 \to 0$, U/α', U_{KK}/α' finite.

$$
\begin{aligned}
ds^2 &= \left(\frac{U}{R}\right)^{3/2} \left(H_0^{1/2}(U)\eta_{\mu\nu}dx^\mu dx^\nu + H_0^{-1/2}(U)f(U)d\tau^2\right) \\
&\quad + H_0^{1/2}\left(\frac{R}{U}\right)^{3/2}\left(\frac{1}{f(U)}dU^2 + U^2 d\Omega_4^2\right), \\
e^\Phi &= g_s\left(\frac{U}{R}\right)^{3/4} H_0^{3/4}, \quad H_0 = 1 + \frac{U_{Q0}^3}{U^3}, \quad f(U) = 1 - \frac{U_{KK}^3}{U^3}, \\
U_{Q0}^3 &= \frac{1}{2}\left(-U_{KK}^3 + \sqrt{U_{KK}^6 + \left((2\pi)^5 \ell_s^7 g_s \tilde{\kappa} N_c\right)^2}\right), \quad \tilde{\kappa} = N_0/(N_c V_4).
\end{aligned}
$$

Bubble geometry ends at U_{KK}.

- Period of τ:

$$
\beta = \frac{4\pi}{3} U_{KK}^{-1/2} R^{3/2} b^{1/2}, \quad b \equiv H_0(U_{KK}).
$$

Intro. to S-S model in D0-D4

The relation to the gauge theory

- The four-dimensional gauge coupling: compactified from the five dim gauge theory $g_{YM}^2 = \frac{g_5^2}{\beta} = \frac{4\pi^2 g_s \ell_s}{\beta}$.
- The Chern-Simons term for D4

$$S_{CS} \sim \int d\tau C_\tau \wedge tr(F \wedge F) \Rightarrow \langle tr(F_{\mu\nu}\tilde{F}^{\mu\nu})\rangle = 8\pi^2 N_c \tilde{\kappa} \qquad (1)$$

  Poicaré invariance is preserved: $\langle F_{\mu\nu}\rangle = 0$.

- Expressed using 't Hooft coupling $\lambda = g_{YM}^2 N_c$:

$$\begin{aligned} R^3 &= \frac{\beta\lambda\ell_s^2}{4\pi}, \quad \beta = \frac{4\pi\lambda\ell_s^2}{9U_{KK}} b, \quad M_{KK} \equiv \frac{2\pi}{\beta} = \frac{9}{2}\frac{U_{KK}}{\lambda\ell_s^2 b}. \\ b &= \frac{1}{1 - \frac{(2\pi\ell_s^2)^8}{81U_{KK}^8}\lambda^4\tilde{\kappa}^2}. \end{aligned} \qquad (2)$$

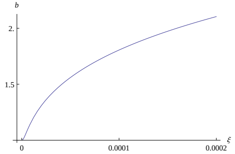
- A constraint on $\tilde{\kappa}$

$$|\tilde{\kappa}| \leq \frac{9U_{KK}^4}{(2\pi\ell_s^2)^4\lambda^2} = \frac{\lambda^2 M_{KK}^4 b^4}{9^3\pi^4}. \qquad (3)$$

- For fixed $\tilde{\kappa}$, the larger λ is , the smaller b is.

Intro. to S-S model in D0-D4

- b monotonically depends on $\tilde{\kappa}$, $\xi \equiv \frac{|\tilde{\kappa}|}{\lambda^2 M_{KK}^4}$. The dependence on $\tilde{\kappa}$ is qualitatively the same as the dependence on b



Estimation:If $|\tilde{\kappa}| < M_{KK}^4$ and $\lambda \sim 100$(or 10^3), $0 < \xi < 10^{-4}$ (10^{-6}) and $1 < b < 1.81(1.005)$.
If $|\tilde{\kappa}| < (2M_{KK})^4$ and $\lambda = 100$, $1 < b < 3.41$.

- For the supergravity solution can be used, curvature should be small compared to the string scale: $1 \ll \left| \frac{1}{\mathbf{R}\ell_s^2} \right| \Rightarrow \lambda$ large.
- For the supergravity to be a low energy effective theory of string theory, to suppress the string loop effect : $e^{\Phi} \ll 1 \Rightarrow U \ll g_s^{-4/3} R \equiv U_{\text{crit}}$, and we require $U_{\text{crit}} \gg U_{KK}$, then

$$g_{YM}^4 \ll \frac{1}{g_{YM}^2 N_c} \ll 1 \tag{4}$$

Strong λ region.

Intro. to S-S model in D0-D4

Independent parameters:

- Gravity side: R^3, $U_{Q_0}^3$, U_{KK}, g_s.
- Gauge theory side: N_c, M_{KK}, λ,$\tilde{\kappa}$.
- Relations:

$$R^3 = \frac{\lambda \ell_s^2}{2M_{KK}} \quad g_s = \frac{\lambda}{2\pi M_{KK} N_c \ell_s}\,, \quad U_{KK} = \frac{2}{9} M_{KK} \lambda \ell_s^2 H_0(U_{KK})\,.$$

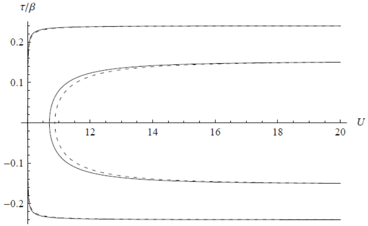
$\tilde{\kappa} \sim b$.

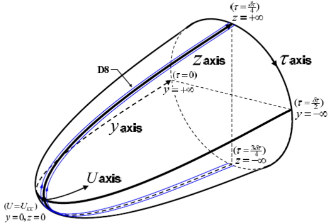
Intro. to S-S model in D0-D4

Put the D8 into the D4 geometry. The embeded metric on D8:

$$ds^2 = \left(\frac{U}{R}\right)^{3/2} H_0(U)^{-1/2} \left(f(U) + \left(\frac{R}{U}\right)^3 \frac{H_0(U)}{f(U)} U'^2\right) d\tau^2 + \left(\frac{U}{R}\right)^{3/2} H_0^{1/2}(U)\eta_{\mu\nu}dx^\mu dx^\nu + H_0^{1/2}(U)\left(\frac{R}{U}\right)^{3/2} U^2 d\Omega_4^2 \,. \quad (5)$$

Change of coordinate: $U^3 = U_{KK}^3 + U_{KK} r^2, y = r\cos\theta\,, \quad z = r\sin\theta.$





- Non-antipodal: Dashed line with non-zero $\tilde{\kappa}$.
- For simplicity: D8 at $y = 0$
- The configuration is stable w.r.t. y perturbations.
- Massless Goldstones still exist. Masses of mesons can be studied.

Baryons in S-S in D0-D4: Classical Soliton

Adding N_f D8 branes $\leftrightarrow$ adding N_f flavors

- Scalar mesons are the excitations of y coordinate, and the vector component A_z—Goldstone, $N_f \times N_f$ flavor matrix. We consider $N_f = 2$ here.
- Vector mesons are the excitations of A_μ, $\mu = 0, 1, 2, 3$, $N_f \times N_f$ flavor matrix.
- Baryons can be viewed as solitons on $D8$ like skymion. Chern-Simons term in

  $$S_{D8} \sim tr(\int A \wedge F \wedge F) \wedge F_4 \sim N_c \int dt\, \hat{A}_0 \wedge tr(F \wedge F)\,.$$

  F is the field strength of SU(2) gauge field A_μ. $\hat{A}$ is the U(1) field coupled to the $U(1)$ current, charge is the quark number.
- Soliton in x^1, x^2, x^3 and z, "Instanton" number $\int tr(F \wedge F) \sim N_q/N_c$=Baryon number.

Baryons in S-S in D0-D4: Classical Soliton

Solving the EOM at large λ approximation.
Action of excitations on D8: $S_{D8} = S_{YM} + S_{CS}$

$$S_{\rm YM} \sim -\int d^4x dz\, 2\, H_0^{1/2}(U){\rm tr}\left[\frac{1}{4}\frac{R^3}{U}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + \frac{9}{8}\frac{U^3}{U_{KK}}\mathcal{F}_{z\mu}\mathcal{F}^{z\mu}\right]$$

$$S_{CS} = \frac{N_c}{24\pi^2}\int_{M^4\times R}\omega_5(\mathcal{A}), \quad \omega_5(\mathcal{A}) = {\rm tr}\left(\mathcal{A}\wedge\mathcal{F}\wedge\mathcal{F} - \frac{1}{2}\mathcal{A}^3\wedge\mathcal{F} + \frac{1}{10}\mathcal{A}^5\right).$$

- Rescaling: $z \to zU_{KK}$, $\mathcal{A}_z \to \mathcal{A}_z/U_{KK}$, $x_\mu \to x_\mu/M_{KK}$, $\mathcal{A}_\mu \to \mathcal{A}_\mu M_{KK}$
  $\to$ dimensionless fields
- Extracting the λ orders: $M, N = 1, 2, 3, z$.

$$x^0 \to x^0 \quad , \quad x^M \to \lambda^{-1/2}x^M$$
$$\mathcal{A}_0(t,x) \to \mathcal{A}_0(t,x) \quad , \quad \mathcal{A}_M(t,x) \to \lambda^{1/2}\mathcal{A}_M(t,x)$$
$$\mathcal{F}_{0M}(t,x) \to \lambda^{1/2}\mathcal{F}_{0M}(t,x) \quad , \quad \mathcal{F}_{MN}(t,x) \to \lambda\mathcal{F}_{MN}(t,x),$$

- Expand the action to $O(\lambda^0)$

$$S_{\rm YM} = -aN_c b^{3/2}\int d^4x dz\left[\frac{\lambda}{4}(F^a_{MN})^2 - \frac{b\,z^2}{2}\left(\frac{5}{12} - \frac{1}{4b}\right)(F^a_{ij})^2 + \frac{b\,z^2}{4}\left(1 + \frac{1}{b}\right)(F^a_{iz})^2 - \frac{1}{2}(F^a_{0N})^2\right.$$
$$\left. + \frac{\lambda}{4}\hat{F}^2_{MN} - \frac{b\,z^2}{2}\left(\frac{5}{12} - \frac{1}{4b}\right)\hat{F}^2_{ij} + \frac{b\,z^2}{4}\left(1 + \frac{1}{b}\right)\hat{F}^2_{iz} - \frac{1}{2}\hat{F}^2_{0N} + O(\lambda^{-1})\right]$$
$$S_{CS} = \frac{N_c}{24\pi^2}\epsilon_{MNPQ}\int d^4x dz\left[\frac{3}{8}\hat{A}_0 tr(F_{MN}F_{PQ}) - \frac{3}{2}\hat{A}_M tr(\partial_0 A_N F_{PQ})\right.$$
$$\left. + \frac{3}{4}\hat{F}_{MN} tr(A_0 F_{PQ}) + \frac{1}{16}\hat{A}_0\hat{F}_{MN}\hat{F}_{PQ} - \frac{1}{4}\hat{A}_M\hat{F}_{0N}\hat{F}_{PQ} + (\ldots)\right] \qquad (6)$$

Baryons in S-S in D0-D4: Classical Soliton

EOM's: F_{MN} SU(2) part; $\hat{F}_{MN}$, U(1) part.

$$D_M F_{MN} = 0\,, \quad D_M F^a_{0M} + \frac{\epsilon_{MNPQ}}{64\pi^2 a b^{3/2}} \hat{F}_{MN} F^a_{PQ} = 0 \tag{7}$$

$$\partial_M \hat{F}_{MN} = 0\,, \quad \partial_M \hat{F}_{0M} + \frac{\epsilon_{MNPQ}}{128\pi^2 a b^{3/2}} F^a_{MN} F^a_{PQ} = 0 \tag{8}$$

Soliton solution without time dependence:

- BPST solution for F_{MN}:

$$A_M = -if(\xi) g^{-1}(x) \partial_M g(x),$$

$$f(\xi) = \frac{\xi^2}{\xi^2 + \rho^2}, \quad \xi^2 = (\vec{x} - \vec{X})^2 + (z - Z)^2, \quad g(x) = \frac{1}{\xi}((z - Z) - i(\vec{x} - \vec{X}) \cdot \vec{\sigma})\,, \tag{9}$$

- U(1) part $\hat{A}_M = 0$ up to a pure gauge.
- SU(2) A_0: $D^2_M A_0 = 0$, with vanishing boundary condition, $A_0 = 0$.
- U(1) $\hat{A}_0$:

$$-\partial^2_M \hat{A}_0 + \frac{3\rho^4}{\pi^2 a b^{3/2} (\xi^2 + \rho^2)^4} = 0$$

$$\hat{A}_0 = -\frac{1}{8\pi^2 a b^{3/2}} \frac{\xi^2 + 2\rho^2}{(\rho^2 + \xi^2)^2}\,.$$

Baryons in S-S in D0-D4: Classical Soliton

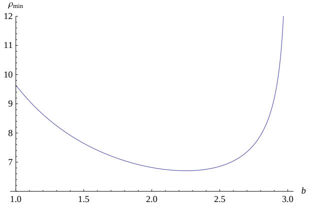
Substitute the solution into the action: the soliton mass from the on-shell action $S = -\int dt M$:

$$M = 8\pi^2 a b^{3/2} N_c \left(\lambda + \underbrace{\frac{1}{12}(3-b)(2Z^2+\rho^2)}_{\text{Attractive potential}} + \underbrace{\frac{1}{320\pi^4 \rho^2 a^2 b^3}}_{\text{Repulsive potential}} + O\left(\frac{1}{\lambda}\right) \right)$$

- For $b < 3$, at the minimum, ρ_{min} characterizes the size of the baryon

- b small: the attractive potential dominant: ρ decreases
- b large: the repulsive potential dominant: ρ increases

$$\rho_{min}^2 = \frac{1}{4\pi^2}\sqrt{\frac{3}{5}}\frac{1}{a b^{3/2}\sqrt{3-b}} .$$

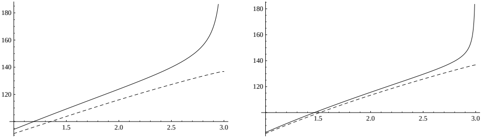


- Minimum mass:

$$M_{min} = 8\pi^2 a b^{3/2} N_c \left(\lambda + 9\pi\sqrt{\frac{3}{5}}\frac{\sqrt{3-b}}{b^{3/2}} + \frac{1}{6}(3-b)Z^2 + O\left(\frac{1}{\lambda}\right) \right) .$$

Baryons in S-S in D0-D4: Classical Soliton

- The size decreases first.
- At $b = 3$, the size blows up : the instability of the baryon
- At $b = 3$, the second and the third term in M_{min} vanish.
- In fact at $b = 3$, the $1/\lambda$ expansion may not be valid.
- Estimate how much we could trust the $1/\lambda$ expansion:
  (1)Expansion of S_{YM}: Compare S_{YM} with expansion and without expansion $\Rightarrow$ the larger λ is, the larger the region of b is. $\lambda \sim 100$ (or 250), $1 < b < 1.5$ (1.8).
  (2) From previous discussion, for $\tilde{\kappa} < M_{KK}^4$ and $\lambda \sim 100$ (or 250), $1 < b < 1.81(1.25)$. The larger λ is, the smaller the region of b is.
  (3) If we allow the massive mode to come in, abandoning $\tilde{\kappa} < M_{KK}^4$, the constraint of (2) no longer exists. As long as we choose large enough λ, b can approach 3 without invalidating the expansion. The size of the baryon really blows up and the baryon is unstable.



Left: $\lambda = 100$; Right:$\lambda = 250$

Baryons in S-S in D0-D4: collective coordinate quantization

- The moduli for the one instanton BPST solution, $\mathcal{M} = \mathbb{R}^4 \times \mathbb{R}^4/\mathbb{Z}_2$: $\vec{X}$, Z, size ρ, $SU(2)$ orientation a_I, $I = 1,2,3,4$, $\sum_I a_I^2 = 1$. Collectively: $X^\alpha = (\vec{X}(t), Z(t), y_I(t))$
- Make them time dependent: $A_M^{cl}(x, X^\alpha(t))$ the BPST solution

$$\Phi = -iV^{-1}\dot{V}, \quad A_M(t,x) = VA_M^{cl}(x, X^\alpha(t))V^{-1} - iV\partial_M V^{-1}.$$

$$F_{MN} = VF_{MN}^{cl}V^{-1}, \quad F_{0M} = V(\dot{X}^\alpha \partial_\alpha A_M^{cl} - D_M^{cl}\Phi)V^{-1},$$

$V(x,t)$ is a $SU(2)$ 2×2 matrix which is asymptotic to $\mathbf{a} = a_4(t) + ia_a(t)\sigma^a$ at $z \to \infty$

- Insert into the Lagrangian: to $O(1/\lambda)$

$$L = \frac{1}{2} m_X g_{\alpha\beta} \dot{X}^\alpha \dot{X}^\beta - U(X^\alpha) \quad (10)$$

$$= \frac{1}{2} m_X \dot{\vec{X}}^2 + \frac{1}{2} m_Z \dot{Z}^2 + \frac{1}{2} m_y \dot{y}_I \dot{y}_I - U(X^\alpha) \quad (11)$$

$ds^2 = g_{\alpha\beta} dX^\alpha dX^\beta = d\vec{X}^2 + dZ^2 + 2dy_I^2$
$m_X = m_Z = \frac{1}{2} m_y = 8\pi^2 ab^{3/2} N_c$

Baryons in S-S in D0-D4: collective coordinate quantization

- Hamiltonian:

$$
\begin{aligned}
H &= H_X + H_Z + H_y\,, \\
H_X &= \frac{1}{2m_X}P_X^2 + M_0\,, \quad M_0 = 8\pi^2\lambda a b^{3/2} N_c \\
H_Z &= \frac{1}{2m_Z}P_Z^2 + \frac{1}{2}m_Z\omega_Z^2 Z^2\,, \quad \omega_Z = \frac{1}{3}(3-b), \\
H_y &= \frac{1}{2m_y}P_y^2 + \frac{1}{2}m_y\omega_y^2\rho^2 + \frac{Q}{\rho^2}\,, \quad Q = \frac{N_c}{40\pi^2 a b^{3/2}}\,, \quad \omega_y = \frac{1}{12}(3-b),.
\end{aligned}
$$

- Quantization: $P_X \to -i\partial_X, P_y \to -i\partial_y, P_Z \to -i\partial_Z$
- Fermion wave function: $\psi(X)$, anti-periodic boundary condition.

Baryons in S-S in D0-D4: collective coordinate quantization

- Solve the Shrödinger eq: energy eigenvalues:

$$
\begin{aligned}
M &= M_0 + E_y + E_Z\,, \quad M_0 = \frac{\lambda N_c b^{3/2}}{27\pi} && (12)\\
E_y &= \omega_y(\tilde{l} + 2n_\rho + 2) = \omega_y(\sqrt{(l+1)^2 + 2m_y Q} + 2n_\rho + 1)\\
&= \frac{1}{2\sqrt{3}}\sqrt{3-b}\left(\sqrt{(l+1)^2 + \frac{4}{5}N_c^2} + 2n_\rho + 1\right)\,, && (13)\\
E_Z &= \omega_Z(n_z + \frac{1}{2}) = \frac{1}{\sqrt{3}}\sqrt{3-b}(n_z + \frac{1}{2}) && (14)
\end{aligned}
$$

$I = J = l/2$.

- The zero point energy can not be determined to order N_c^0 order.[Prog.Theor.Phys.117:1157,2007]
- We look at the mass difference between baryons:

$$M = M_0 + \sqrt{\frac{3-b}{2}}\big(E_y(b=1) + E_Z(b=1)\big), \quad \Delta M = \sqrt{\frac{3-b}{2}}\Delta M(b=1) \quad (15)$$

- The mass difference decreases with $\tilde{\kappa}$: vanishes at $b = 3$ at λ^0.
- For $|\tilde{\kappa}| < M_{KK}^4$, only b near 1 can be trusted.
- For $|\tilde{\kappa}| > M_{KK}^4$, b can be in a larger region. For $b > 3$, ΔM become imaginary — an indication of the instability of baryons.
- The scale factor $\sqrt{3-b}$ is independent of different quantum numbers of the baryons.

Summary

- We studied the effect of the smeared D0 charges on the baryons in the S-S model.
- The classical analysis: The size of the baryon shrinks first and then increases to blow up at $b = 3$ in the leading λ expansion.
- The quantum analysis: The mass difference decreases.
- Only b near 1 can be trusted for massless gauge theory.
- If we also interested in the massive mode in the gauge theory, the baryon may be unstable for large b.
- Large N_c limit: for the $1/\lambda$ expansion to be valid, N_c should not be small comparable to 3.