6D $(1,0)$ SCFT and 5D SYM

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Outline

Introduction

- According to AdS/CFT, 6D (1,0) and (2,0) Superconformal field theories may be the dual gauge theories of multiple M5-branes, related to near-horizon AdS_7 geometries. Maldacena 9711200; Gubser-Klebanov-Polyakov 9802109; Witten 9802150
- The gauge description of M2 branes, 3D $\mathcal{N} = 6$, $U(N) \times U(N)$ theory of ABJM, has been constructed successfully; And the dual string/M- theories have been studied extensively. Aharony et al 0806.1218
- The success of the ABJM theory encourages us to construct **nonabelion** 6D (1,0) and (2,0) theories. Lambert-Papageorgakis, 1007.2982; Henning-Sezgin-Wimmer 1108.4060 and 1212.5199; FMC 1312.4330; and many others...

- However, it is difficult to construct an action for the (1,0) or (2,0) theory due to the self-duality of the three-form field strength $H_{\mu\nu\rho} = 3\partial_{[\mu}B_{\nu\rho]}$.
- But it is possible to write down a set of equations and the law of supersymmetry tansformations.
- Today we'll be talking about the construction of a 6D (1,0) theory, and its relation with the 5D SYM theory. FMC,1312.4330
- People are trying to argue that the 6D (2,0) theory is equivalent to the 5D maximally SYM theory quantum mechanically.

  Douglas,1012.2880; Hull-Lambert, 1403.4532; and others.

Free (r,0) (r=1,2)

- Our (1,0) theory will be the theory of *mini (1,0) tensor multiplet* **coupling** *hypermultiplet*.
- Also, the 6D free (2,0) theory = the mini (1,0) tensor multiplet + hypermultiplet. Here

$$\begin{aligned} \mathrm{mini}(1,0) &= (\phi, B_{\mu\nu}, \psi_+), && (2.1) \\ \mathrm{hyper} &= (X^i, \psi_-), \quad (i = 6, \ldots, 9) && (2.2) \end{aligned}$$

where $\psi_\pm$ are 32-component Majorana spinors, satisfying

$$\begin{aligned} \Gamma_{012345}\psi_\pm &= -\psi_\pm \quad (\mathrm{anti-chirality}) && (2.3) \\ \Gamma_{6789}\psi_\pm &= \pm\psi_\pm && (2.4) \end{aligned}$$

- Can see

$$\begin{aligned} & \mathrm{mini}(1,0) + \mathrm{hyper} \\ = \quad & (X^I, B_{\mu\nu}, \psi) \\ = \quad & (2,0) \end{aligned} \tag{2.5}$$

where $\psi = \psi_+ + \psi_-$, and $X^I = (X^i, \phi)$ $(I = 6, \ldots, 10)$. The R-symmetry is $SO(5)$.

- By linearizing the world-volume theory of an M5-brane, one obtains the free (2,0) theory

$$\begin{aligned} \delta X^I &= i\bar{\epsilon}\Gamma^I\psi, \\ \delta\psi &= \Gamma^\mu\Gamma^I\epsilon\partial_\mu X^I + \frac{1}{3!}\frac{1}{2!}\Gamma_{\mu\nu\lambda}\epsilon H^{\mu\nu\lambda} \\ \delta H_{\mu\nu\rho} &= 3i\bar{\epsilon}\Gamma_{[\mu\nu}\partial_{\rho]}\psi, \end{aligned} \tag{2.6}$$

  where $\Gamma_{012345}\epsilon = +\epsilon$ (chirality condition $\Rightarrow$ 16 real components $\Rightarrow$ (2,0) theory), and

- $$\Gamma^\mu\partial_\mu\psi = \partial^\mu\partial_\mu X^I = 0. \tag{2.7}$$

  The SUSY algebra is closed on-shell.

- Our free (1, 0) theory:

$$\begin{aligned} & \mathrm{mini}(1,0) + \mathrm{hyper} \\ = \quad & (X^i, \phi, B_{\mu\nu}; \psi_+, \psi_-) \\ = \quad & \mathrm{Our}(1,0) \end{aligned} \tag{2.8}$$

The free (2,0) is a special case of our (1,0).

Nonabelian Mini (1,0)

- To describe interacting multi-M5-branes, **we must construct nonabelian (1,0) theories**. Let's first try to construct a nonabelian Mini (1,0) theory.
- The nonabelian mini $(1,0)$ tensor multiplets in 6D are given by

$$(\phi_m, H_{\mu\nu\rho m}, \psi_{m+}), \tag{3.1}$$

where m is an adjoint index of the Lie algebra of gauge symmetry.

- We *postulate* the law of supersymmetry transformations as follows,

$$\begin{aligned}
\delta\phi_m &= -i\bar{\epsilon}_+\psi_{m+}, \\
\delta\psi_{m+} &= \Gamma^\mu\epsilon_+ D_\mu\phi_m + \frac{1}{3!}\frac{1}{2!}\Gamma_{\mu\nu\lambda}\epsilon_+ H_m^{\mu\nu\lambda}, \\
\delta A_\mu^m &= i\bar{\epsilon}_+\Gamma_{\mu\nu}\psi_+^m C^\nu + ic_1\bar{\epsilon}_+\psi_+^m C_\mu, \\
\delta C^\nu &= 0, \\
\delta H_{\mu\nu\rho m} &= 3i\bar{\epsilon}_+\Gamma_{[\mu\nu}D_{\rho]}\psi_{m+} + id_1\bar{\epsilon}_+\Gamma_{\mu\nu\rho\sigma}C^\sigma\psi_{n+}\phi_p f^{np}{}_m,
\end{aligned} \tag{3.2}$$

- C^μ is an abelian auxiliary field with scaling dimension -1.
- c_1 and d_1 are constants, to be determined.
-

$$\Gamma_{012345}\epsilon_+ = +\epsilon_+ \quad \text{(chirality)} \tag{3.3}$$

$$\Gamma_{6789}\epsilon_+ = +\epsilon_+ \tag{3.4}$$

ϵ_+ **has only 8 real independent components.** So we have only 8 SUSY generators (6D (1,0) theory).

Checking Closure For instance, let's look at the fermionic fields,

$$
\begin{aligned}
[\delta_1, \delta_2]\psi_{m+} &= v^\nu D_\nu \psi_{m+} + [\Lambda, \psi_+]_m \\
&+ \frac{3}{8}(1 - c_1 + d_1) v^{\mu\nu\rho ij} \Gamma_{\mu\nu\rho} \Gamma^{ij}_{-} C_\rho \psi_{n+} \phi_p f^{np}{}_m \\
&- \frac{1}{4} v^\nu D_\nu \psi_{m+} + \frac{1}{8}(7c_1 - 5d_1 - 3) v^\nu C_\nu \psi_{n+} \phi_p f^{np}{}_m \\
&+ \frac{1}{4} v^\nu \Gamma_{\mu\nu} D^\mu \psi_{m+} - \frac{1}{8}(c_1 + d_1 + 3) v^\nu \Gamma_{\mu\nu} C^\mu \psi_{n+} \phi_p f^{np}{}_m,
\end{aligned}
\tag{3.5}
$$

where

$$
\begin{aligned}
v^\mu &\equiv -2i\bar{\epsilon}_{2+}\Gamma^\mu \epsilon_{1+}, \\
\Lambda^m &\equiv (c_1 - 1) v^\nu C_\nu \phi^m, \\
v^{\mu\nu\rho ij} &\equiv -\frac{i}{12}(\bar{\epsilon}_{2+}\Gamma^{\mu\nu\rho}\Gamma^{ij}_{-}\epsilon_{1+}).
\end{aligned}
\tag{3.6}
$$

- The last **THREE** lines of (3.5) must vanish; This can be achieved by setting

$$
\begin{aligned}
0 &= 1 - c_1 + d_1, && (3.7)\\
0 &= \Gamma^\mu D_\mu \psi_{m+} - (d_1 + 2)\Gamma^\mu C_\mu \psi_{n+} \phi_p f^{np}{}_m. && (3.8)
\end{aligned}
$$

  The 1st equation determines the relation between c_1 and d_1; The second equation will be the EOM of fermions once d_1 is fixed.

- At the end of the day, we have

$$
[\delta_1, \delta_2]\psi_{m+} = v^\nu D_\nu \psi_{m+} + [\Lambda, \psi_+]_m + \mathrm{EOM}, \qquad (3.9)
$$

  which is the desired result.

- We can do the same thing on the other fields.
- Final, we find that if

$$c_1 = 0 \quad \text{and} \quad d_1 = -1, \tag{3.10}$$

the Poincare superalgebra (3.2) will be closed on-shell; the EOMs are

$$\begin{aligned}
0 &= D^2\phi_p - \frac{i}{2}(\bar{\psi}_{m+}\Gamma_\nu\psi_{n+})C^\nu f^{mn}{}_p, \\
0 &= F^m_{\mu\nu} - H^m_{\mu\nu\rho}C^\rho \\
0 &= \Gamma^\mu D_\mu\psi_{m+} - \Gamma^\mu C_\mu\psi_{n+}\phi_p f^{np}{}_m \\
0 &= D_{[\mu}H_{\nu\rho\sigma]p} + \frac{i}{8}\varepsilon_{\mu\nu\rho\lambda\sigma\tau}(\bar{\psi}_{m+}\Gamma^\tau\psi_{n+})C^\lambda f^{mn}{}_p \\
&\quad + \frac{1}{4}\varepsilon_{\mu\nu\rho\lambda\sigma\tau}\phi_m D^\tau\phi_n C^\lambda f^{mn}{}_p. \\
0 &= C^\sigma D_\sigma\phi_m = C^\sigma D_\sigma\psi_{m+} = C^\sigma D_\sigma H_{\mu\nu\rho m} = \partial_\mu C^\nu. \qquad (3.11)
\end{aligned}$$

Coupling to Hypermultiplets

- Now we need to couple the nonabelian mini (1,0) tensor multiplets

$$(\phi_m, H_{\mu\nu\rho m}, \psi_{m+}), \tag{4.1}$$

to the (1,0) hypermultiplets:

$$(X^i_m, \psi_{m-}), \tag{4.2}$$

where m is an adjoint index of the Lie algebra of gauge symmetry.

We assume that the law of SUSY for the (1,0) theory are

$$
\begin{aligned}
\delta\phi_m &= -i\bar{\epsilon}_+\psi_{m+}, \\
\delta X^i_m &= i\bar{\epsilon}_+\Gamma^i\psi_{m-}, \\
\delta\psi_{m-} &= \Gamma^\mu\Gamma^i\epsilon_+ D_\mu X^i_m + a\Gamma_\lambda\Gamma^i\epsilon_+ C^\lambda X^i_n\phi_p f^{np}{}_m, \\
\delta\psi_{m+} &= \Gamma^\mu\epsilon_+ D_\mu\phi_m + \frac{1}{3!}\frac{1}{2!}\Gamma_{\mu\nu\lambda}\epsilon_+ H^{\mu\nu\lambda}_m + b\Gamma_\lambda\Gamma^{ij}_-\epsilon_+ C^\lambda X^i_n X^j_p f^{np}{}_m, \\
\delta A^m_\mu &= i\bar{\epsilon}_+\Gamma_{\mu\nu}\psi^m_+ C^\nu, \\
\delta C^\nu &= 0, \\
\delta H_{\mu\nu\rho m} &= 3i\bar{\epsilon}_+\Gamma_{[\mu\nu}D_{\rho]}\psi_{m+} - i\bar{\epsilon}_+\Gamma_{\mu\nu\rho\sigma}C^\sigma\psi_{n+}\phi_p f^{np}{}_m \\
&\quad + id\bar{\epsilon}_+\Gamma^i\Gamma_{\mu\nu\rho\sigma}\psi_{n-}C^\sigma X^i_p f^{pn}{}_m,
\end{aligned}
\tag{4.3}
$$

where a, b, and d are constants, to be determined later.

Using the same method, we find that if

$$a = 1 \quad \text{and} \quad d = 2b, \tag{4.4}$$

the Poincare superalgebra (4.3) will be closed on-shell; the EOMs are

$$\begin{aligned}
0 &= D^2 X^i_p + i(\bar{\psi}_{m+}\Gamma_\nu\Gamma^i\psi_{n-})C^\nu f^{mn}{}_p - C^2(\phi_m X^i_n \phi_q + 2b X^j_m X^i_n X^j_q) f^{mn}{}_o f^{oq}{}_p, \\
0 &= D^2\phi_p - \frac{i}{2}[(\bar{\psi}_{m+}\Gamma_\nu\psi_{n+}) \\
&\quad -2b(\bar{\psi}_{m-}\Gamma_\nu\psi_{n-})]C^\nu f^{mn}{}_p - 2bC^2 X^i_m \phi_n X^i_q f^{mn}{}_o f^{oq}{}_p, \\
0 &= F^m_{\mu\nu} - H^m_{\mu\nu\rho}C^\rho, \\
0 &= \Gamma^\mu D_\mu\psi_{m-} + \Gamma^\mu C_\mu\psi_{n-}\phi_p f^{np}{}_m + \Gamma^\mu\Gamma^i C_\mu\psi_{n+}X^i_p f^{np}{}_m, \\
0 &= \Gamma^\mu D_\mu\psi_{m+} - \Gamma^\mu C_\mu\psi_{n+}\phi_p f^{np}{}_m + 2b\Gamma^\mu\Gamma^i C_\mu\psi_{n-}X^i_p f^{np}{}_m, \\
0 &= D_{[\mu}H_{\nu\rho\sigma]p} + \frac{i}{8}\varepsilon_{\mu\nu\rho\lambda\sigma\tau}[(\bar{\psi}_{m+}\Gamma^\tau\psi_{n+}) + 2b(\bar{\psi}_{m-}\Gamma^\tau\psi_{n-})]C^\lambda f^{mn}{}_p \\
&\quad + \frac{1}{4}\varepsilon_{\mu\nu\rho\lambda\sigma\tau}(\phi_m D^\tau\phi_n + 2bX^i_m D^\tau X^i_n)C^\lambda f^{mn}{}_p, \\
0 &= C^\sigma D_\sigma\phi_m = C^\sigma D_\sigma X^i_m = C^\sigma D_\sigma\psi_{m-} = C^\sigma D_\sigma\psi_{m+} \\
&= C^\sigma D_\sigma H_{\mu\nu\rho m} = \partial_\mu C^\nu.
\end{aligned} \tag{4.5}$$

b still survives.

- b is a free continuous parameter; it indicates that the theories of tensor multiplets and hypermultiplets are **separately supersymmetric.** (See also Henning-Sezgin-Wimmer 1212.5199.)
- However, if $b = 1/2$, the $\mathcal{N} = (1, 0)$ supersymmetry will be enhanced to $(2, 0)$ (1312.4330, FMC.), which is equivalent to the (2,0) LP theory, constructed in terms of 3-algebras (Lambert-Papageorgakis, 1007.2982).

- The gauge field is **non-dynamical**. (The gauge field of the (1,0) theory of Henning-Sezgin-Wimmer is dynamical.) Recall that the gauge field of ABJM is also non-dynamical!
- The theory is invariant under scale transformation. We expect that the **classical scale invariance (plus super-poincare invariance) implies superconformal invariance**. It would be interesting to examine the supereconformal invariance explicitly.

Relating to 5D SYM

- Adopting the method in Lambert-Papageorgakis, 1007.2982, we choose the space-like vector vev

$$\langle C^\mu \rangle = g(0, \ldots, 0, 1) = g\delta_5^\mu, \tag{5.1}$$

- The equations of motion of gauge fields (the 3rd equation of (4.5)) now become

$$F_{\alpha\beta m} = gH_{\alpha\beta 5m}, \tag{5.2}$$

where we have decomposed μ into $\mu = (\alpha, 5)$, with $\alpha = 0, 1, \ldots, 4$.

- On the other hand, since $F_{5\beta m} = gH_{5\beta 5m} = 0$, we have

$$F_{5\beta} = \partial_5 A_\beta - \partial_\beta A_5 + [A_5, A_\beta] = 0. \tag{5.3}$$

So locally, we may set the flat connection

$$A_5 = 0. \tag{5.4}$$

The rest equations in (4.5) can be reduced to those of 5D SYM theory with 8 supersymmetries:

$$
\begin{aligned}
0 &= D^\alpha D_\alpha X^i_p + ig(\bar{\psi}_{m+}\Gamma_5\Gamma^i\psi_{n-})f^{mn}{}_p \\
&\quad -g^2(\phi_m X^i_n \phi_q + 2bX^j_m X^i_n X^j_q)f^{mn}{}_o f^{oq}{}_p, \\
0 &= D^\alpha D_\alpha \phi_p - \frac{i}{2}g[(\bar{\psi}_{m+}\Gamma_5\psi_{n+}) \\
&\quad -2b(\bar{\psi}_{m-}\Gamma_5\psi_{n-})]f^{mn}{}_p - 2bg^2 X^i_m \phi_n X^i_q f^{mn}{}_o f^{oq}{}_p, \\
0 &= \Gamma^\alpha D_\alpha \psi_{m-} + g\Gamma^5\psi_{n-}\phi_p f^{np}{}_m + g\Gamma^5\Gamma^i\psi_{n+}X^i_p f^{np}{}_m, \\
0 &= \Gamma^\alpha D_\alpha \psi_{m+} - g\Gamma^5\psi_{n+}\phi_p f^{np}{}_m + 2bg\Gamma^5\Gamma^i\psi_{n-}X^i_p f^{np}{}_m, \\
0 &= gD_{[\alpha}H_{\beta\gamma]5p} = D_{[\alpha}F_{\beta\gamma]p} \qquad (5.5) \\
0 &= D^\alpha F_{\alpha\beta p} - \frac{i}{2}g^2[(\bar{\psi}_{m+}\Gamma_\beta\psi_{n+}) + 2b(\bar{\psi}_{m-}\Gamma_\beta\psi_{n-})]f^{mn}{}_p \\
&\quad -g^2(\phi_m D_\beta \phi_n + 2bX^i_m D_\beta X^i_n)f^{mn}{}_p, \\
0 &= \partial_5\phi_m = \partial_5 X^i_m = \partial_5\psi_{m-} = \partial_5\psi_{m+} = \partial_5 H_{\mu\nu\rho m} = \partial_5 g.
\end{aligned}
$$

- The EOM of $H_{\mu\nu\rho m}$ are converted into:
- (1) the equations of motion of Yang-Mills fields in 5D;
- (2) the Bianchi identity for the field strength $F_{\beta\gamma}$ in 5D.

- The supersymmetry transformations (4.3) become

$$\begin{aligned}
\delta\phi_m &= -i\bar{\epsilon}_+\psi_{m+}, \\
\delta X^i_m &= i\bar{\epsilon}_+\Gamma^i\psi_{m-}, \\
\delta\psi_{m-} &= \Gamma^\alpha\Gamma^i\epsilon_+ D_\alpha X^i_m + g\Gamma_5\Gamma^i\epsilon_+ X^i_n\phi_p f^{np}{}_m, \\
\delta\psi_{m+} &= \Gamma^\alpha\epsilon_+ D_\alpha\phi_m + \frac{1}{2g}\Gamma_{\alpha\beta}\Gamma_5\epsilon_+ F^{\alpha\beta}_m + bg\Gamma_5\Gamma^{ij}_-\epsilon_+ X^i_n X^j_p f^{np}{}_m, \\
\delta A^m_\alpha &= ig\bar{\epsilon}_+\Gamma_\alpha\Gamma_5\psi^m_+.
\end{aligned} \tag{5.6}$$

- The coupling constant of the SYM theory is given by

$$g = g^2_{YM}. \tag{5.7}$$

- These are the law of SUSY of the 5D SYM theory of 8 supersymmetries.
- Eqns (5.5) and (5.6) are the main results of the 5D SYM theory of 8 supersymmetries: (1) They can be simplified by using new notations; (2) It is possible to construct a Lagrangian.

Discussions

- It would be interesting to construct the gravity duals of these 6D (1,0) SCFT and 5D SYM.
- Is it possible that the 6D (1,0) SCFT and the 5D SYM of 8 SUSYs are the same thing quantum mechanically?
- Wilson loops? Integrability? (For 5D SYM of 8 or 16 SUSYs.)
- 5D SYM of 8 SUSYs $\Rightarrow$ 4D $\mathcal{N} = 2$ SYM? or 6D (1,0) SCFT $\Rightarrow$ 4D $\mathcal{N} = 2$ SYM?
- Other possible 6D (1,0) or (2,0) theories? Can our theories be derived in F-theory? Heckman-Morrison-Vafa, 1312.5746

Thanks

Many Thanks !