Axial charge generation at strong coupling

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Outline

- Axial charge in electroweak theory and QCD
- Review of axial charge generation at weak coupling and strong coupling
- A close look at anomaly equation
- Holographic realization of axial anomaly
- Correlators of axial charge and topological charge density at strong coupling
- Dynamics of axial charge fluctuation
- Conclusion

Axial anomalies in nature

Axial anomaly in Electroweak theory leads to baryogenesis

$$\partial_\mu j_B^\mu = \frac{N_f}{32\pi^2}[-2g^2 tr(F_{\mu\nu}\tilde{F}^{\mu\nu}) + g'^2 F'_{\mu\nu}\tilde{F}'^{\mu\nu}]$$

Axial anomaly in QCD leads to axial charge generation

$$\partial_\mu j_5^\mu = -\frac{g_s^2 N_f}{16\pi^2} tr(G_{\mu\nu}\tilde{G}^{\mu\nu})$$

For QCD, axial charge is generated through $\Delta Q_5 = \int dt d^3x j_5^0(x) = -\frac{g_s^2 N_f}{16\pi^2}\int d^4x tr(G\tilde{G})$

It averages to zero, but has nonvanishing fluctuations. $\left\langle trG\tilde{G}\right\rangle = 0, \left\langle \left(trG\tilde{G}\right)^2\right\rangle = 0$

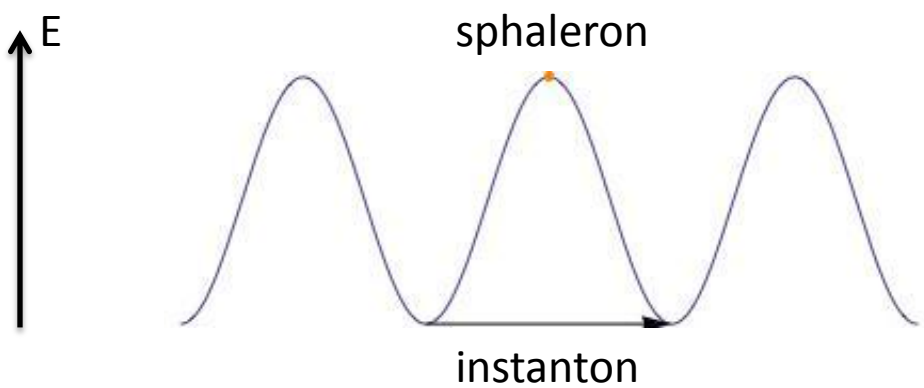
ΔQ_5: important input to anomaly induced effects in HIC, such as chiral magnetic effect and chiral vortical effect.

Sources of axial charge generation at weak coupling

Consider weakly coupled SU(N) gauge theory plasma, coupled to N_f flavors of quenched massless quarks

$$\partial_\mu j_5^\mu = -\frac{g^2 N_f}{16\pi^2} tr(G_{\mu\nu}\tilde{G}^{\mu\nu})$$

Nonperturbative contributions exponentially suppressed $e^{-\frac{\#}{g^2}}$



However, at finite temperature, there are fluctuations of arbitrary size, turning the exponential suppression into a power law suppression.

Arnold, McLerran PRD 1987

Perturbative generation of axial charge?

$$\partial_\mu j_5^\mu = -\frac{g^2 N_f}{16\pi^2} tr(G_{\mu\nu}\tilde{G}^{\mu\nu})$$

(e.g. parallel chromo-E and chromo-B field)?

Only topologically nontrivial field configurations contribute.
Perturbative contribution should be excluded.

N. Christ, PRD 1980

Bodeker's effective theory

Effective theory at scale g^2T and below

$$D_j F_{ji} + D_t E_i = -J_i \quad = -\sigma E_i + \zeta_i$$

σ: conductivity
ζ: noise due to interaction with field above scale g^2T

$$\int dt \zeta_i(\boldsymbol{x}, t) \zeta_j(y, 0) = 2\sigma T \delta_{ij} \delta^3(\boldsymbol{x} - \boldsymbol{y})$$

Different scales in weakly coupled SU(N) gauge theory
T: temperature scale
gT: electric screening scale, weakly coupled
g^2T: magnetic screening scale, strongly coupled

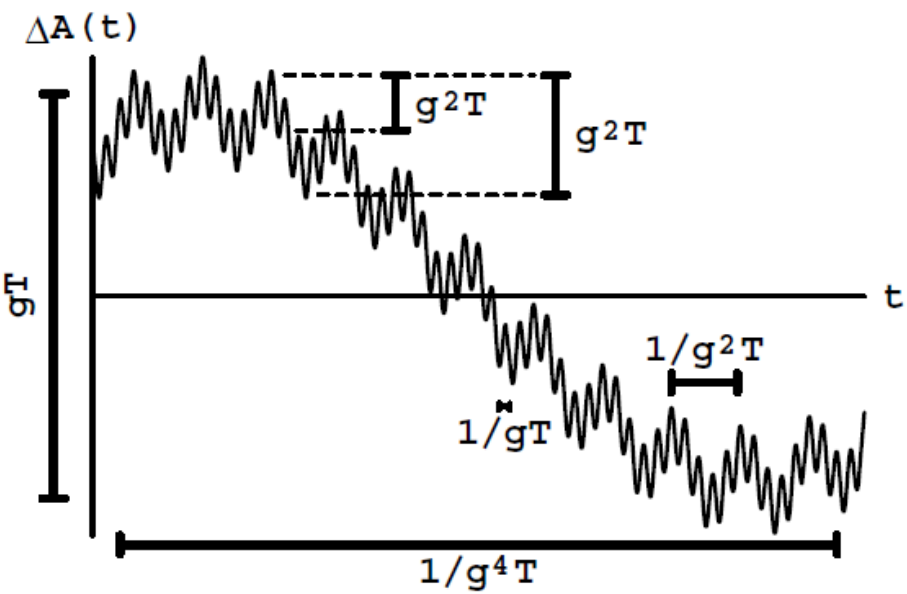
Bodeker, PLB 1998

Sphaleron rate from real time lattice simulation

$$\left\langle (Q_5(t) - Q_5(0))^2 \right\rangle = 4Vt\Gamma_{sph} + \ldots$$

$$\Gamma_{sph} = \int d^4x \left\langle \frac{g^2}{16\pi^2} trG\tilde{G}(x) \frac{g^2}{16\pi^2} trG\tilde{G}(0) \right\rangle \sim \frac{1}{\text{time} * \text{vol}} \sim \# g^{10}T^4$$

time ~ $1/(g^4T)$, volume ~ $1/(g^2T)$^3



Arnold, Son, Yaffe
Moore, Tassler, JHEP 2011

Sphaleron rate at strong coupling

large N_c $\mathcal{N}$=4 SYM theory

$$\Gamma_{sph} = \frac{g^4 N_c^2}{256\pi^3} T^4 = \frac{\lambda^2}{256\pi^3} T^4$$

Son, Starinets, JHEP 2002

time ~ $1/\sqrt{\lambda}T$, volume ~ $1/(\sqrt{\lambda}T)^3$??

Sphaleron rate from non-perturbative contribution in both weak coupling and strong coupling.

Is nontrivial topology necessary?

A close look at anomaly equation

$$\partial_\mu j_5^\mu = -\frac{e^2 N_f}{16\pi^2} tr(F_{\mu\nu}\tilde{F}^{\mu\nu})$$

Robustness

- Equation not modified at finite temperature, chemical potential etc
- Not renormalization by radiative correction

Adler, Bardeen, PR 1969
Adler, 2004 review

Subtleties

- Proof of the non-renormalization is done for matrix element at special kinematics
- Current operators implicitly time ordered in the equation

Anomaly for fully retarded correlator

$$Q_5 = \int d^3x j_5^0(t,x) = \int \frac{d\omega}{2\pi} e^{-i\omega t} j_5^0(\omega, q=0)$$
$$= \int \frac{d\omega}{2\pi} e^{-i\omega t} \frac{1}{i\omega} \partial_\mu j_5^\mu(\omega, q=0).$$

For simplicity, consider response of axial charge to external electromagnetic field

$$\partial_\mu j_5^\mu(\omega, q=0) = \frac{1}{2} \int \frac{d^4p d^4k}{(2\pi)^8} \delta(\omega - k^0 - p^0) \delta^{(3)}(\vec{k} + \vec{p}) \boxed{\langle \partial_\mu j_{5,r}^\mu(\omega) j_a^\nu(p) j_a^\lambda(k) \rangle} A_{\nu,r}(p) A_{\lambda,r}(k).$$

fully retarded correlator

In QED

$\langle \partial_\mu j_{5,r}^\mu(\omega) j_a^\nu(p) j_a^\lambda(k) \rangle$ vanishes identically when $\vec{k} + \vec{p} = 0.$

$\langle T \partial_\mu j_5^\mu(\omega) j^\nu(p) j^\lambda(k) \rangle = \frac{e^2}{4\pi^2} \epsilon^{\alpha\lambda\beta\nu} k_\alpha p_\beta.$ nonvanishing when $\vec{k} + \vec{p} = 0.$

Ordering of operators matters!

Implications

What can't generate axial charge

- turning on E and B fields simultaneously

What can generate axial charge

- turning on E field in a background B field (level crossing for fermions in LLL)
- exiciting nonperturbative field configuration, where operator ordering doesn't matter.

Nontrivial topology not a necessary condition for axial charge generation

More questions

1. The accumulation of net axial charge Q_5 creates a chiral imbalance μ_5, which disfavors further generation of Q_5. When does the saturation of Q_5 happen?

2. How does the accumulation of Q_5 effect the dynamics of gluons?

3. What is more complete time evolution of fluctuations beyond linear growth?

$$\left\langle (Q_5(t) - Q_5(0))^2 \right\rangle = 4Vt\Gamma_{sph} + \ldots$$

Try to seek answer in a holographic model

Direct access to correlators of axial charge and topological charge density

The Sakai-Sugimoto model (D4/D8)

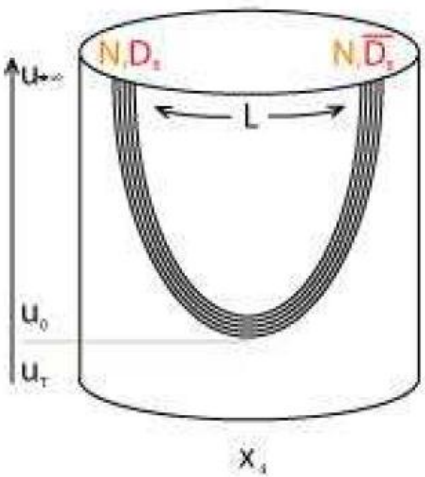
N_c D4 branes wrapped on S^1 circle + N_f D8/anti D8 branes being a point on S^1.

4+1D gluons

3+1D left/right handed quarks

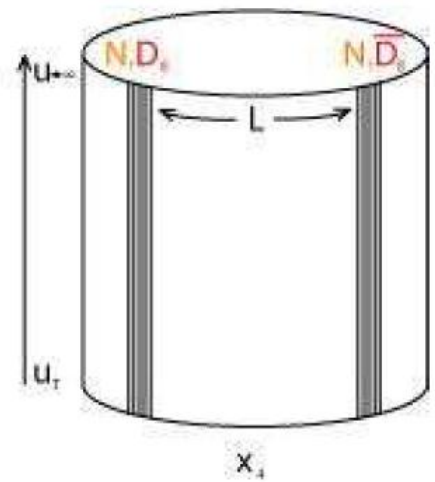
Intrinsic mass scale $M_{KK} = \frac{1}{R_4}$

Sakai, Sugimoto, Prog.Theor.Phys, 2005



$T < \frac{M_{KK}}{2\pi}$

Confined, chiral symmetry broken



$T > \frac{M_{KK}}{2\pi}$

Deconfined, chiral symmetry restored

Aharony, Sonnenschein, Yankielowicz, Annal. Phys. 2006

The deconfined background

S^1 circle

$$ds^2 = \left(\frac{U}{R}\right)^{3/2}\left(-f(U)dt^2 + d\vec{x}^2 + dx_4^2\right) + \left(\frac{R}{U}\right)^{3/2}\left(U^2 d\Omega_4^2 + \frac{dU^2}{f(U)}\right),$$

$$F_{(4)} = \frac{2\pi N_c \epsilon_4}{V_4}, \quad e^{\phi} = g_s\left(\frac{U}{R}\right)^{3/4}, \quad R^3 = \pi g_s N_c l_s^3, \quad f(u) = 1 - \left(\frac{U_T}{U}\right)^3$$

Describe plasma of strongly coupled 4+1D gluons, and 3+1D quarks as defect. Dynamics determined by:

$$S_{DBI} = -\mu_8 \int d^9x e^{-\phi}\sqrt{-det(g_{MN} + (2\pi\alpha')F_{MN})}$$

Set N_f=1

$$S_{WZ} = \int_{\Sigma 9} C_7 \wedge \frac{F}{2\pi}$$

$$S_{RR} = -\frac{1}{4\pi}(2\pi l_s)^6 \int_{10} dC_7 \wedge *dC_7.$$

Aharony, Sonnenschein, Yankielowicz, Annal. Phys. 2006

Holographic realization of axial anomaly

S_{DBI} and S_{RR} gauge invariant
S_{WZ} gauge invariant upto a boundary term → axial anomaly

$$S_{WZ}(F, C_7) = S_{WZ}(A, C_1)$$

$$\theta = \int dx_4 (C_1)_4$$ as in YM action $$S = -\int d^4x \frac{1}{4g^2} trGG + \int d^4x \frac{\theta}{16\pi^2} trG\tilde{G}$$

A_L/A_R source J_L/J_R, θ sources topological charge density:

$$J_L = \frac{\delta S}{\delta A_L};\ J_R = \frac{\delta S}{\delta A_R};\ \frac{1}{16\pi^2} trG\tilde{G} = \frac{\delta S}{\delta \theta}$$

Under flavor gauge transformation:
$\delta A_L = -d\lambda; \delta A_R = d\lambda; \delta\theta = -2\lambda$ →

$$\partial_\mu j_5^\mu = -\frac{g^2}{16\pi^2} tr(G\tilde{G})$$

No renormalization yet!

Naive expectation from anomaly equation

$$\partial_\mu j_5^\mu = q \longrightarrow$$

$$(-i\omega)\int dtd^3x e^{i\omega t}\theta(t)\langle[j_5^0(t,x), q(0,0)] = \int dtd^3x e^{i\omega t}\theta(t)\langle[q(t,x), q(0,0)],$$

$$(-i\omega)\int dtd^3x e^{i\omega t}\theta(t)\langle[j_5^0(t,x), j_5^0(0,0)] = \int dtd^3x e^{i\omega t}\theta(t)\langle[j_5^0(t,x), q(0,0)],$$

can be verified by LO calculation

Holographic renormalization

$$S(\text{on-shell}) = (\cdots)U^{5/2} + (\cdots)U^2 + (\cdots)U + (\cdots)U^{1/2}lnU + (\cdots)U^{1/2} + (\cdots)lnU + (\cdots)$$

U: holographic coordinate;
U = ∞ corresponds to UV
The divergence in on-shell should be removed by counter terms, which are polynomial in $\omega^2 - k^2$

source vev

$$\Psi(\omega) = a_2(\omega)U^2 + a_1(\omega)U + a_0(\omega)\ln U + b_1(\omega) + \cdots$$

$$-i\omega\theta - 2\mu_5 = \frac{4\pi R_4}{d_1 R^3} a_2.$$

$$\int dt d^3x e^{i\omega t}\theta(t)\langle[q(t,x), q(0,0)]\rangle = \frac{b_1}{a_2}\left(-\frac{4\pi^2 R_4}{3d_1 R^3}\right) 2\left(\frac{2\delta a_2}{\delta\theta}\right)^2,$$

$$\int dt d^3x e^{i\omega t}\theta(t)\langle[q(t,x), j_5^0(0,0)]\rangle = \frac{b_1}{a_2}\left(-\frac{4\pi^2 R_4}{3d_1 R^3}\right) 2\frac{2\delta a_2}{\delta\theta}\frac{\delta a_2}{\delta\mu_5},$$

$$\int dt d^3x e^{i\omega t}\theta(t)\langle[j_5^0(t,x), j_5^0(0,0)]\rangle = \frac{b_1}{a_2}\left(-\frac{4\pi^2 R_4}{3d_1 R^3}\right) 2\left(\frac{\delta a_2}{\delta\mu_5}\right)^2,$$

Understanding anomaly equation

$$(-i\omega)\int dtd^3xe^{i\omega t}\theta(t)\langle[j_5^0(t,x),q(0,0)]=\int dtd^3xe^{i\omega t}\theta(t)\langle[q(t,x),q(0,0)],$$

$$(-i\omega)\int dtd^3xe^{i\omega t}\theta(t)\langle[j_5^0(t,x),j_5^0(0,0)]=\int dtd^3xe^{i\omega t}\theta(t)\langle[j_5^0(t,x),q(0,0)],$$

verified, disappointing result, maybe not surprising.

At strong coupling, all gluonic fluctuations are non-perturbative, operator ordering doesn't matter.

Renormalization doesn't change anomaly equation

Fluctuation $\langle \Delta Q_5(t)^2 \rangle$ in terms of spectral density $\mathrm{Im}G_R$

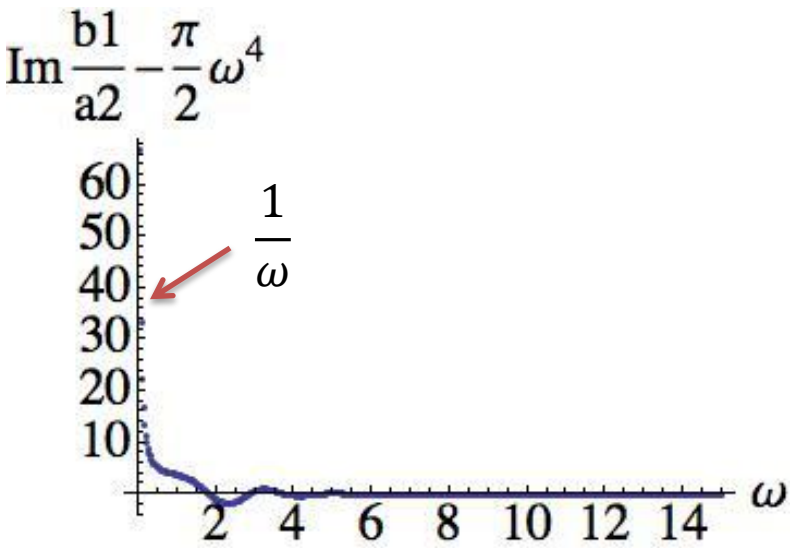
$$\langle \Delta Q_5^2(t) \rangle = \langle (Q_5(t) - Q_5(0))^2 \rangle$$
$$= V \int \frac{d\omega}{2\pi} \left(1 - e^{-i\omega t}\right) \coth \frac{\beta\omega}{2} 2 Im G_R(\omega)$$

Nontrivial time dependence

Spectral density

$$G_R(\omega) = \int dt d^3x e^{i\omega t} \theta(t) \langle \theta[j_5^0(t,x), j_5^0(0,0)] \rangle.$$

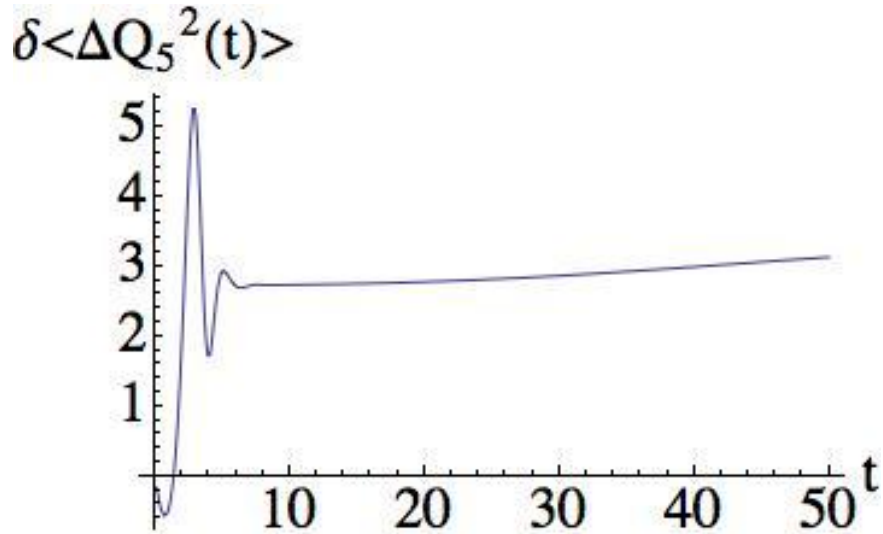
Time evolution of $\langle \Delta Q_5(t)^2 \rangle$



spectral density of j_5^0

$$Im\frac{b_1}{a_2} \to -\frac{2}{\omega R^{3/2}}, \quad \text{as } \omega \to 0$$

pole at the origin



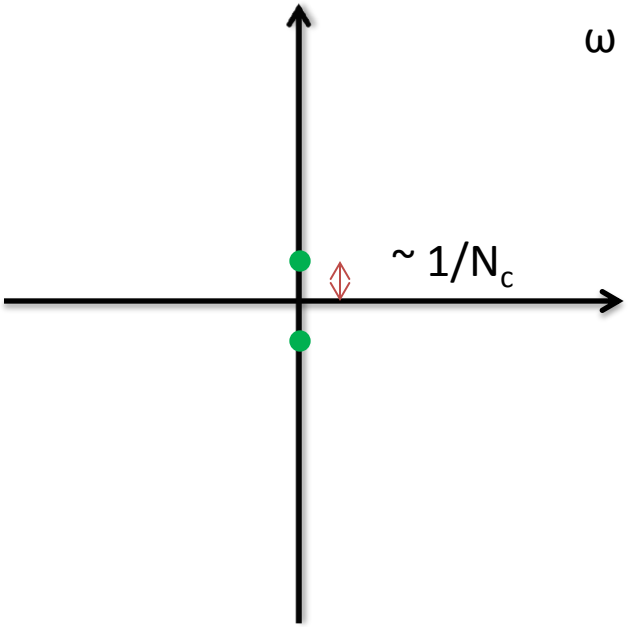
$$\langle \Delta Q_5(t)^2 \rangle = \delta\langle \Delta Q_5(t)^2 \rangle + \frac{4\lambda^3 T^6}{3^6 \pi {M_{KK}}^2} Vt$$

correction

long time limit
Sphaleron rate

Speculated finite N_c pole structure

$$2ImG^R(\omega) = G^R(\omega) - G^A(\omega)$$



G^R/G^A has poles in the lower/upper half plane of ω

It can be shown the pole contribution gives the Sphaleron rate.

Effects invisible at large N_c

1. Dissipation of axial charge:

$$\frac{dQ_5}{dt} = -Q_5 \frac{(2N_{\rm f})^2}{\chi_Q} \frac{\Gamma_{\rm sphal}}{2T}$$

$$\chi_Q = N_{\rm f}\, N_{\rm c} T^2/3$$

Dissipation time scale ~ $N_c/N_f \to \infty$

2. Backreaction of axial charge to dynamics of gluon plasma, suppressed by N_f/N_c , need to consider backreaction of D8/anti D8 branes

Conclusion

- We investigated the issue of operator ordering for anomaly equation in perturbative regime. Operator ordering does matters!
- We suggested nontrivial topology is not a necessary condition for axial charge generation.
- We verified the naive anomaly equation holds at leading order in N_c and strong coupling.
- We presented the full time evolution of axial charge fluctuation, and identified sphaleron rate only as long time behavior.

Thank you!

Sphaleron rate in Sakai-Sugimoto model

$$\Gamma_{sph} = \frac{2\lambda^3 T^6}{3^6 \pi {M_{KK}}^2}$$

Reminiscent of a 6D theory, from which SS model descents from

Craps, Hoyos, Surowka, Taels, JHEP 2012