Finite-volume Hamiltonian for coupled channel interactions in lattice QCD

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Outline

- Introduction
- Hamiltonian for $\pi\pi$ scattering
- Finite-box Hamiltonian method
- Applications to Lattice QCD
- Lattice spectra from the experiment data
- Summary and Outlook

Introduction

Resonance Region

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QCD → Experiment Data (cross section)

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QCD ——?——> Experiment Data (cross section)

Nonperturbative

Introduction

Resonance Region

QCD → ? → Experiment Data (cross section)

Nonperturbative

↓ One way

Lattice QCD → Experiment Data (cross section)

Introduction

Resonance Region

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Lattice QCD → ? → Experiment Data (cross section)

Finite-volume &
Euclidean time

Introduction

Resonance Region

QCD → ? → Experiment Data (cross section)

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Lattice QCD → ? → Experiment Data (cross section)

Experiment Data (cross section)

↓ Partial Wave Analysis

Partial Wave S matrix (phase shift and inelasticity)

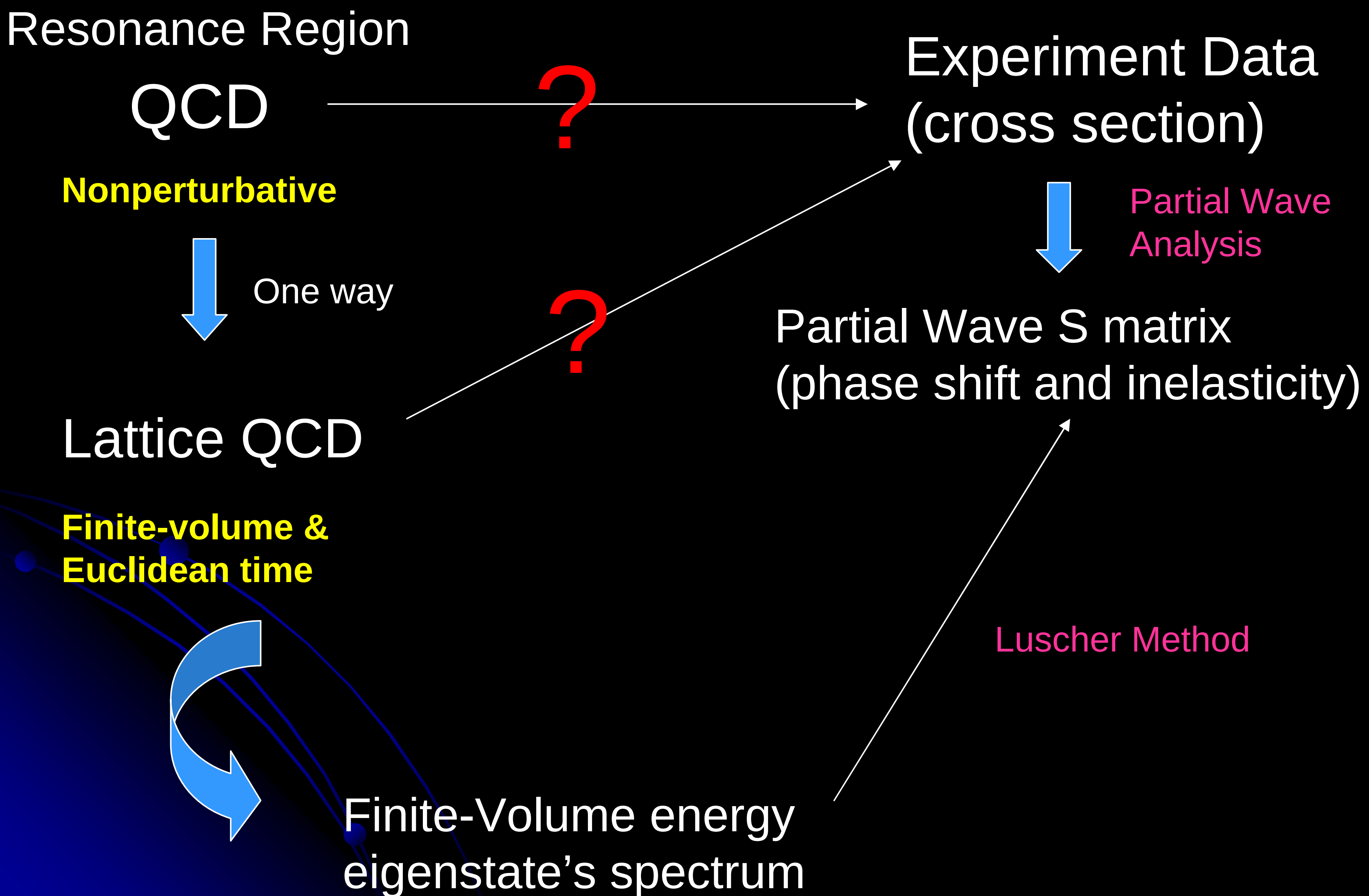
Lattice QCD

Finite-volume & Euclidean time

↓

Finite-Volume energy eigenstate's spectrum

Introduction



Introduction

Resonance Region

QCD

?

Experiment Data
(cross section)

Nonperturbative

One way

Partial Wave
Analysis

?

Partial Wave S matrix
(phase shift and inelasticity)

Lattice QCD

Finite-volume &
Euclidean time

Luscher Method

?

Finite-Volume energy
eigenstate’s spectrum

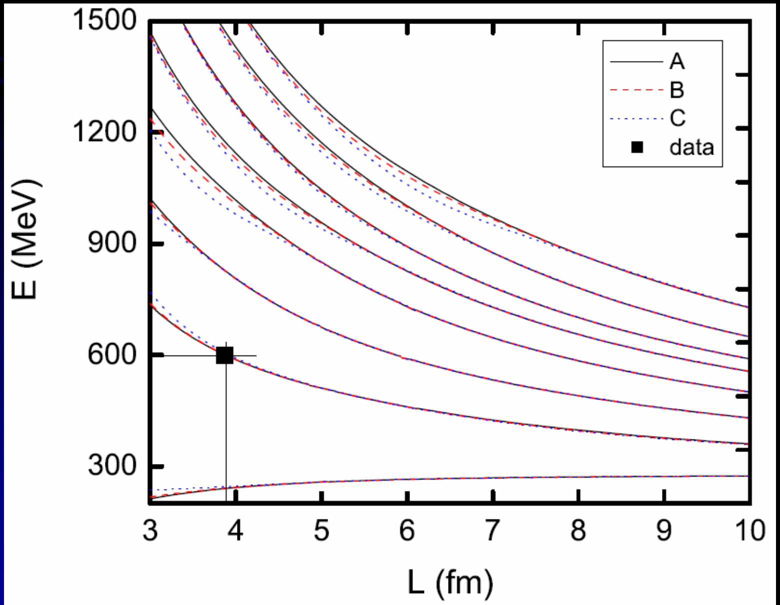
Introduction

- The limitation of Luscher method.
- One channel case:

one (E~L) ↔ one (E~δ)

- Two channel case:

Three (E ~ L_1, L_2, L_3) ↔ Three (E~δ_1, δ_2, η)



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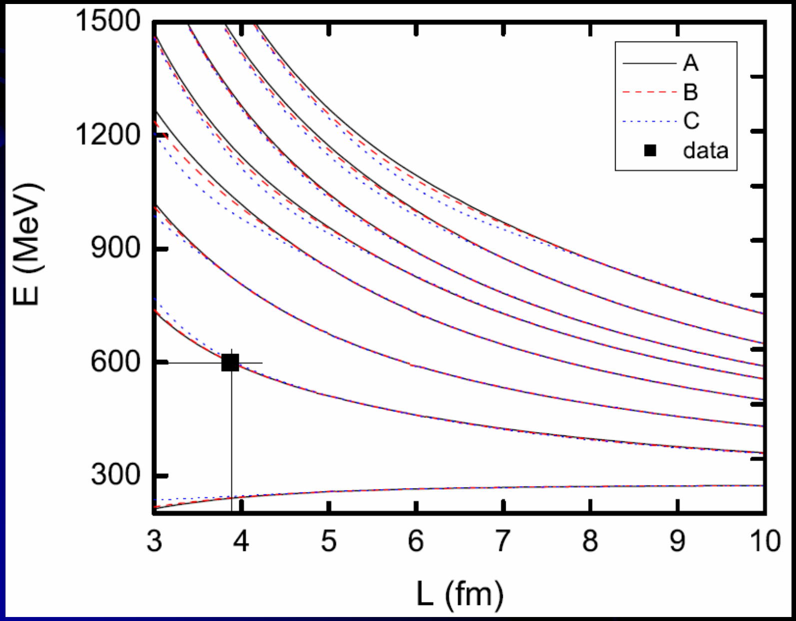

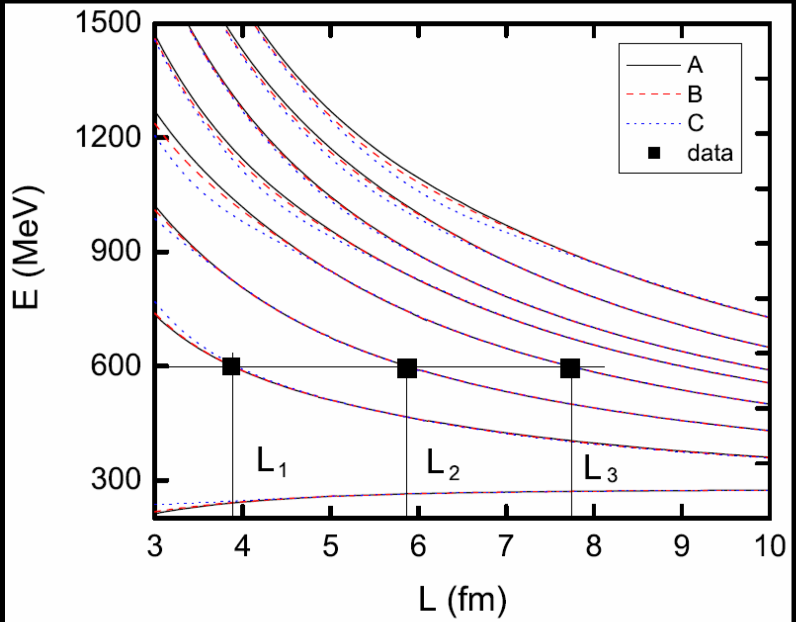

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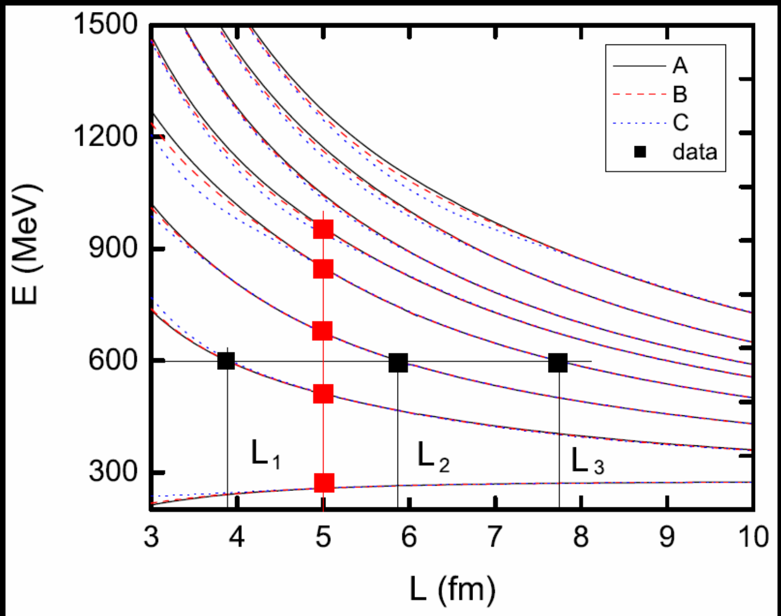
Difficult !

Lattice spectrum

Several L → Several E

Several E $\not\rightarrow$ Several L



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Hamiltonian for ππ scattering

$$H = H_0 + H_I$$

$$H_0 = \sum_{i=1,n} |\sigma_i\rangle m_i \langle\sigma_i| + \sum_{\alpha} |\alpha(k_\alpha)\rangle \left[\sqrt{m_{\alpha 1}^2 + k_\alpha^2} + \sqrt{m_{\alpha 2}^2 + k_\alpha^2} \right] \langle\alpha(k_\alpha)|$$

$|\sigma_i\rangle$ bare state with mass m_i

$|\alpha(k_\alpha)\rangle$ the channels such as ππ, $\bar{K}$K, …

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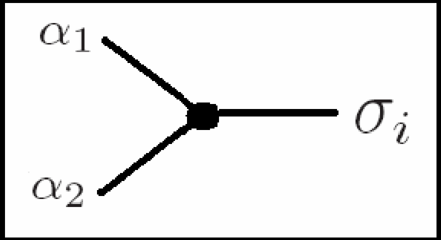
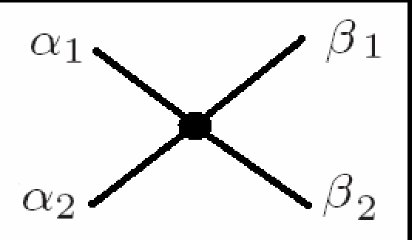
$$H_I = \hat{g} + \hat{v}$$



$$\hat{g} = \sum_{\alpha} \sum_{i=1,n} \left[|\alpha(k_\alpha)\rangle g_{i,\alpha}^{+} \langle\sigma_i| + |\sigma_i\rangle g_{i,\alpha} \langle\alpha(k_\alpha)| \right]$$

$$\hat{v} = \sum_{\alpha,\beta} |\alpha(k_\alpha)\rangle v_{\alpha,\beta} \langle\beta(k_\beta)|$$



Hamiltonian for ππ scattering

Scattering Equation: (Partial Wave)

$$t_{\alpha,\beta}(k_\alpha, k_\beta, E) = V_{\alpha,\beta}(k_\alpha, k_\beta) + \sum_\gamma \int k_\gamma{}^2 dk_\gamma \frac{V_{\alpha,\gamma}(k_\alpha, k_\gamma) t_{\gamma,\beta}(k_\gamma, k_\beta, E)}{E - \sqrt{m_{\gamma 1}^2 + k_\gamma^2} - \sqrt{m_{\gamma 1}^2 + k_\gamma^2} + i\varepsilon}$$

Hamiltonian for ππ scattering

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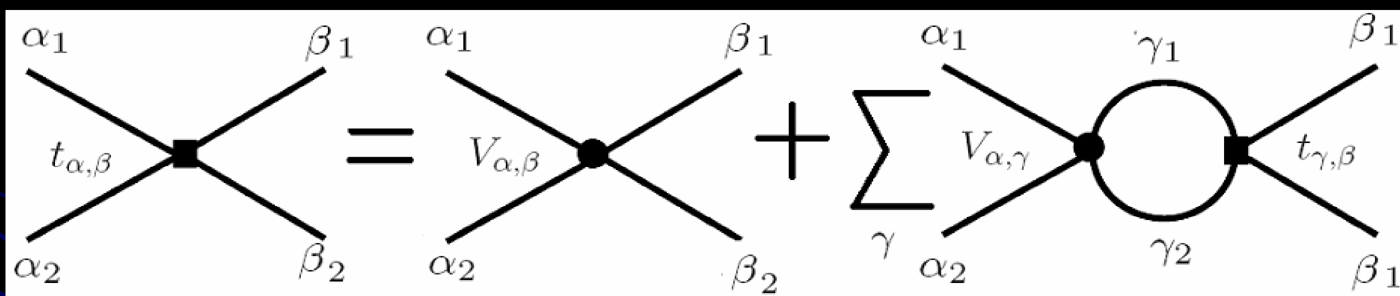

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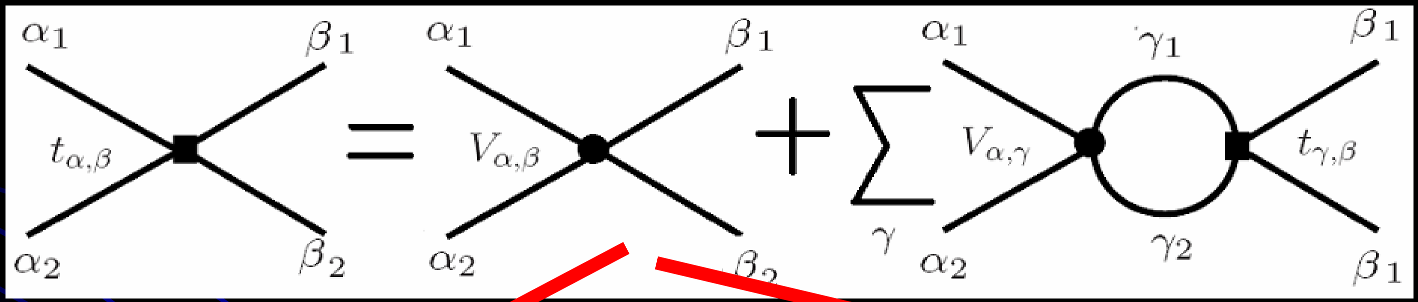

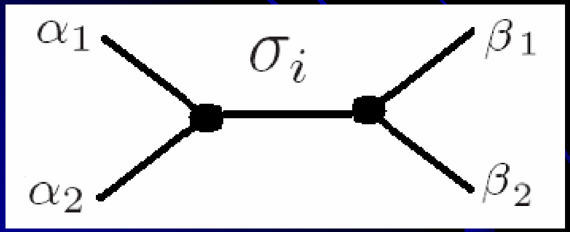

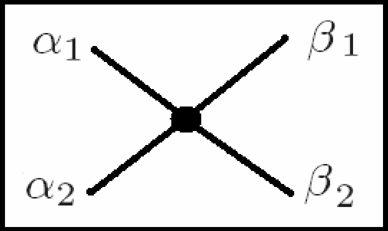

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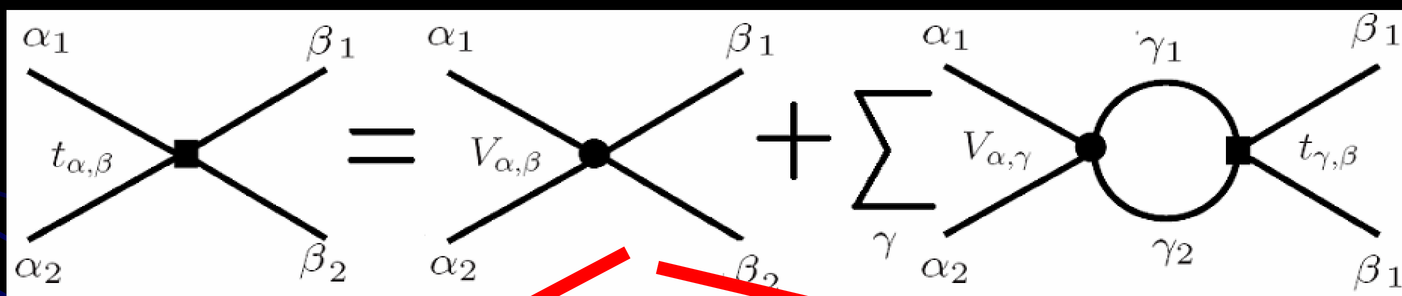
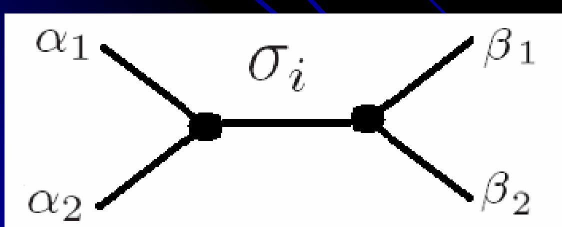
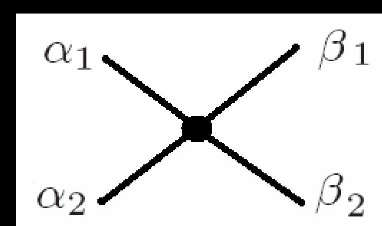
$$t_{\alpha,\beta}(k_\alpha,k_\beta,E)=V_{\alpha,\beta}(k_\alpha,k_\beta)+\sum_\gamma\int k_\gamma{}^2 dk_\gamma \frac{V_{\alpha,\gamma}(k_\alpha,k_\gamma)t_{\gamma,\beta}(k_\gamma,k_\beta,E)}{E-\sqrt{m_{\gamma1}^2+k_\gamma^2}-\sqrt{m_{\gamma1}^2+k_\gamma^2}+i\varepsilon}$$





$$g_{i,\alpha}^{*}\frac{1}{E-m_i}g_{i,\beta}$$



$$V_{\alpha,\beta}$$

Hamiltonian for ππ scattering

Observations & t martix

$$S_{\alpha,\beta} = 1 - i2\sqrt{\rho_{\alpha}}\,t_{\alpha,\beta}(k_{0\alpha}, k_{0\beta}, E)\sqrt{\rho_{\beta}}$$

$$\rho_{\alpha} = \frac{\pi k_{0\alpha}\sqrt{m_{\alpha 1}^{2} + k_{0\alpha}^{2}}\sqrt{m_{\alpha 1}^{2} + k_{0\alpha}^{2}}}{E}$$

$$\eta e^{2i\delta_{\alpha}} = S_{\alpha,\alpha}$$

Hamiltonian for ππ scattering

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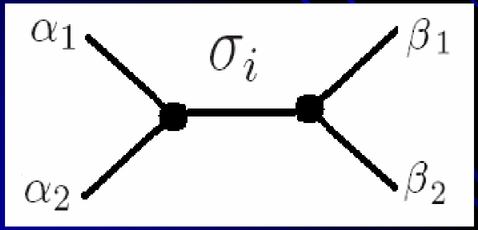
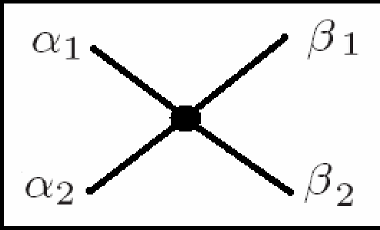
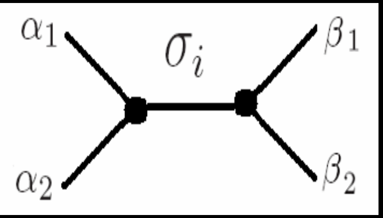


$$g_{i,\alpha}^{*}\frac{1}{E - m_i}g_{i,\beta} \qquad V_{\alpha,\beta}$$



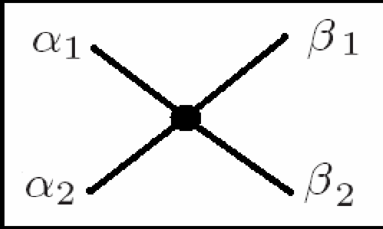
Hamiltonian for ππ scattering

$$g^*_{i,\alpha} \frac{1}{E - m_i} g_{i,\beta}$$



$$g_{i,\alpha}(k_\alpha) = \frac{\tilde{g}_{i,\alpha}}{\sqrt{\pi}} \frac{1}{(1+(c_\alpha k_\alpha)^2)}$$

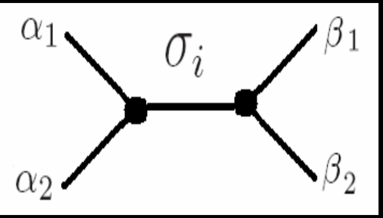
$$v_{\alpha,\beta}$$



$$v_{\alpha,\beta}(k_\alpha,k_\beta)=\frac{G_{\alpha,\beta}}{m_\pi^2}\frac{1}{(1+(d_\alpha k_\alpha)^2)^2}\frac{1}{(1+(d_\beta k_\beta)^2)^2}$$

Hamiltonian for ππ scattering

$$g^*_{i,\alpha}\frac{1}{E-m_i}g_{i,\beta}$$



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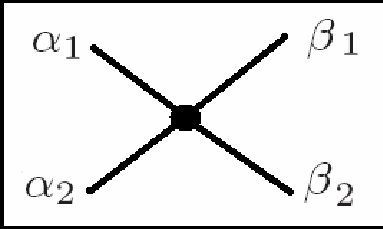

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	1b-1c	1b-2c
m_σ(MeV)	700.	700.00
$g_{\sigma\pi\pi}$	1.6380	2.0000
$c_{\sigma\pi\pi}$(fm)	1.0200	0.6722
$G_{\pi\pi,\,\pi\pi}$	0.5560	2.4998
$d_{\pi\pi}$(fm)	0.5140	0.2440
$g_{\sigma K\bar{K}}$	-	0.6451
$c_{\sigma K\bar{K}}$(fm)	-	1.0398
$G_{K\bar{K},\,K\bar{K}}$	-	0.0200
$d_{K\bar{K}}$(fm)	-	0.1000
$G_{\pi\pi,\,K\bar{K}}$	-	0.3500

Hamiltonian for ππ scattering

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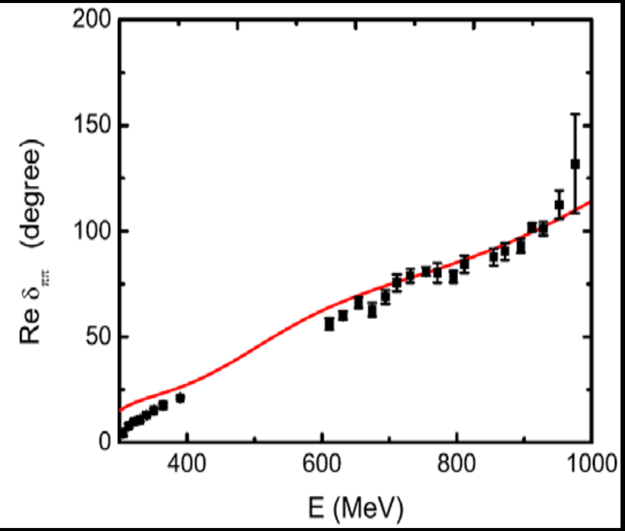
Hamiltonian for ππ scattering

One channel case (**1b-1c**):

only ππ, fit up to 0.9 GeV

Include 5 parameters



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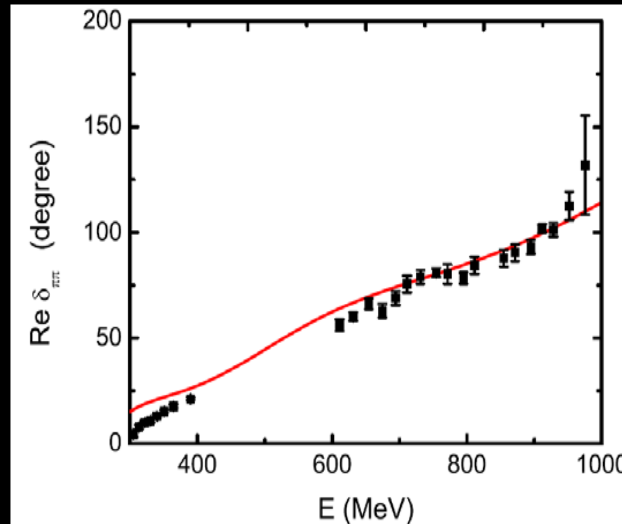
Hamiltonian for $\pi\pi$ scattering

One channel case (**1b-1c**):

only $\pi\pi$, fit up to 0.9 GeV

Include 5 parameters



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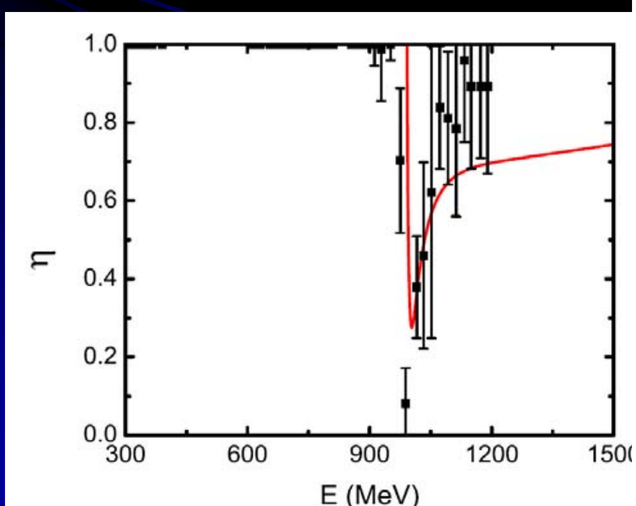
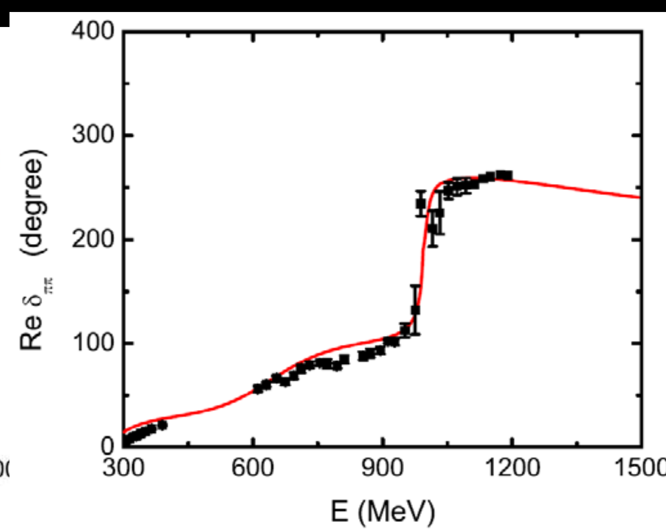
$d_{K\bar{K}}$(fm)	-	0.1000
$G_{\pi\pi,\,K\bar{K}}$	-	0.3500

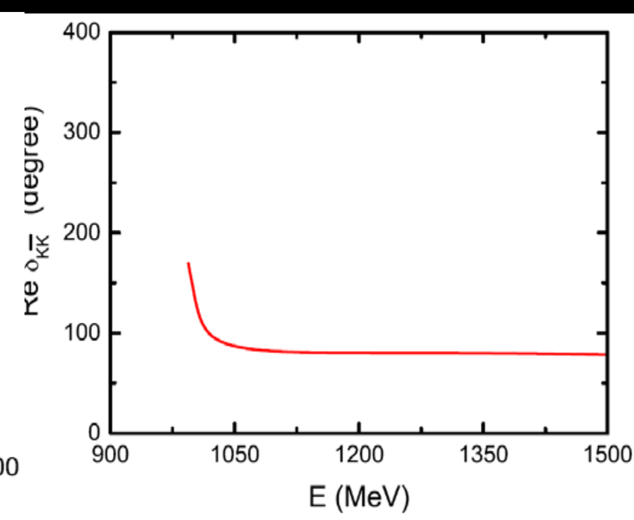
Two channel case (**1b-2c**):

$\pi\pi$ $\bar{K}K$, fit up to 1.2 GeV

Include 10 parameters







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Finite-box Hamiltonian method

$$H\left|\psi\right\rangle = E\left|\psi\right\rangle$$

$$\mathrm{Det}[H_0 + H_I - E\mathrm{I}]=0$$

$$\vec{k} = \vec{n}\frac{2\pi}{L} \qquad \vec{n} \in \mathrm{Z}^3$$

Finite-box Hamiltonian method

$$H|\psi\rangle = E|\psi\rangle$$

Eigenvalue
Energy

$$\mathrm{Det}[H_0 + H_I - E\,\mathrm{I}] = 0$$

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Lattice Size

Finite-box Hamiltonian method

$$H|\psi\rangle = E|\psi\rangle$$

Eigenvalue

Energy

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$$\vec{k} = \vec{n}\frac{2\pi}{L} \qquad \vec{n} \in Z^3$$

Lattice Size

One channel case (**1b-1c**):

$$H_0 = \begin{pmatrix} m_1 & 0 & 0 & . & . & . \\ 0 & 2\sqrt{k_0^2 + m_\pi^2} & 0 & . & . & . \\ 0 & 0 & 2\sqrt{k_1^2 + m_\pi^2} & . & . & . \\ . & . & . & . & & \\ . & . & . & & . & \\ . & . & . & & & . \end{pmatrix}$$

$$H_I = \begin{pmatrix} 0 & g_{\pi\pi}^{fin}(k_0) & g_{\pi\pi}^{fin}(k_1) & . & . & . \\ g_{\pi\pi}^{fin}(k_0) & v_{\pi\pi,\pi\pi}^{fin}(k_0, k_0) & v_{\pi\pi,\pi\pi}^{fin}(k_0, k_1) & . & . & . \\ g_{\pi\pi}^{fin}(k_1) & v_{\pi\pi,\pi\pi}^{fin}(k_1, k_0) & v_{\pi\pi,\pi\pi}^{fin}(k_1, k_1) & . & . & . \\ . & . & . & . & & \\ . & . & . & & . & \\ . & . & . & & & . \end{pmatrix}$$

$$g_{\pi\pi}^{fin}(k_n) = \sqrt{\frac{C_3(n)}{4\pi}}\left(\frac{2\pi}{L}\right)^{\frac{3}{2}} g_{\pi\pi}(k_n)$$

$$v_{\pi\pi,\pi\pi}^{fin}(k_n, k_m) = \sqrt{\frac{C_3(n)}{4\pi}}\sqrt{\frac{C_3(m)}{4\pi}}\left(\frac{2\pi}{L}\right)^{3} v_{\pi\pi,\pi\pi}(k_n, k_m)$$

Finite-box Hamiltonian method

$$H|\psi\rangle = E|\psi\rangle$$

Eigenvalue Energy

$$\mathrm{Det}[H_0 + H_I - E\mathrm{I}]=0$$

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Lattice Size

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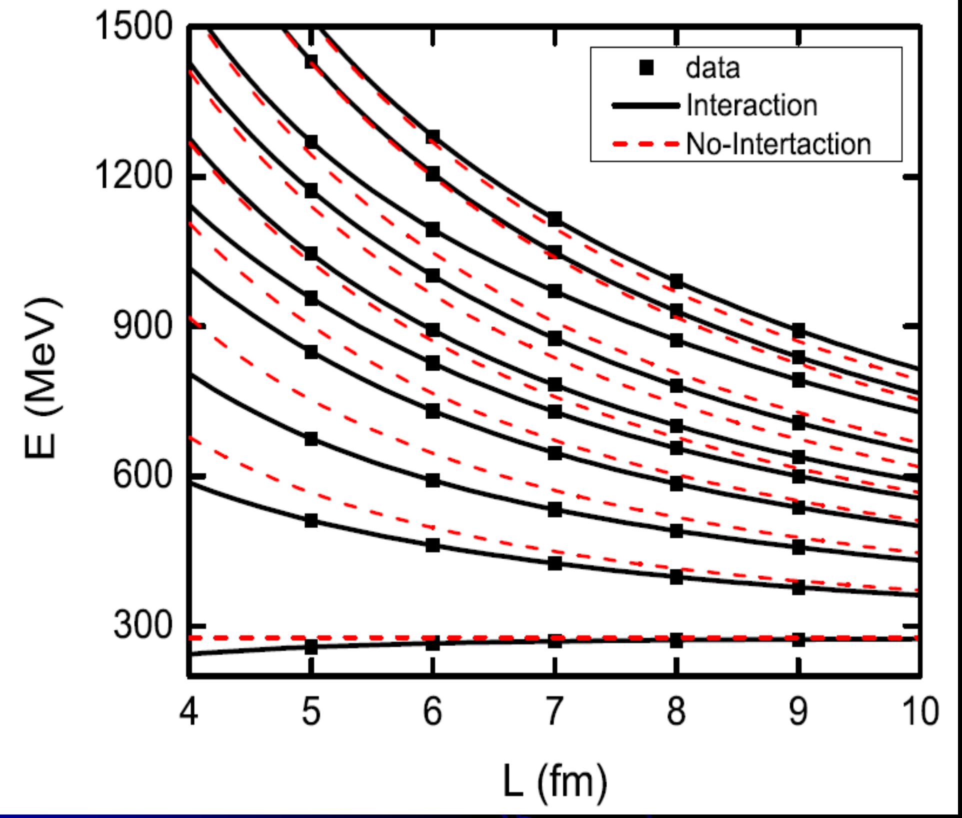
$C_3(n)$ The number of $\vec{n}$ when $|\vec{n}|^2 = n$

$$C_3(1)=6 \quad C_3(2)=12 \quad C_3(3)=8$$

Finite-box Hamiltonian method

1b-1c: $\mathrm{Det}[H_0 + H_I - E\mathrm{I}] = 0$

Spectrum from the Hamiltonian



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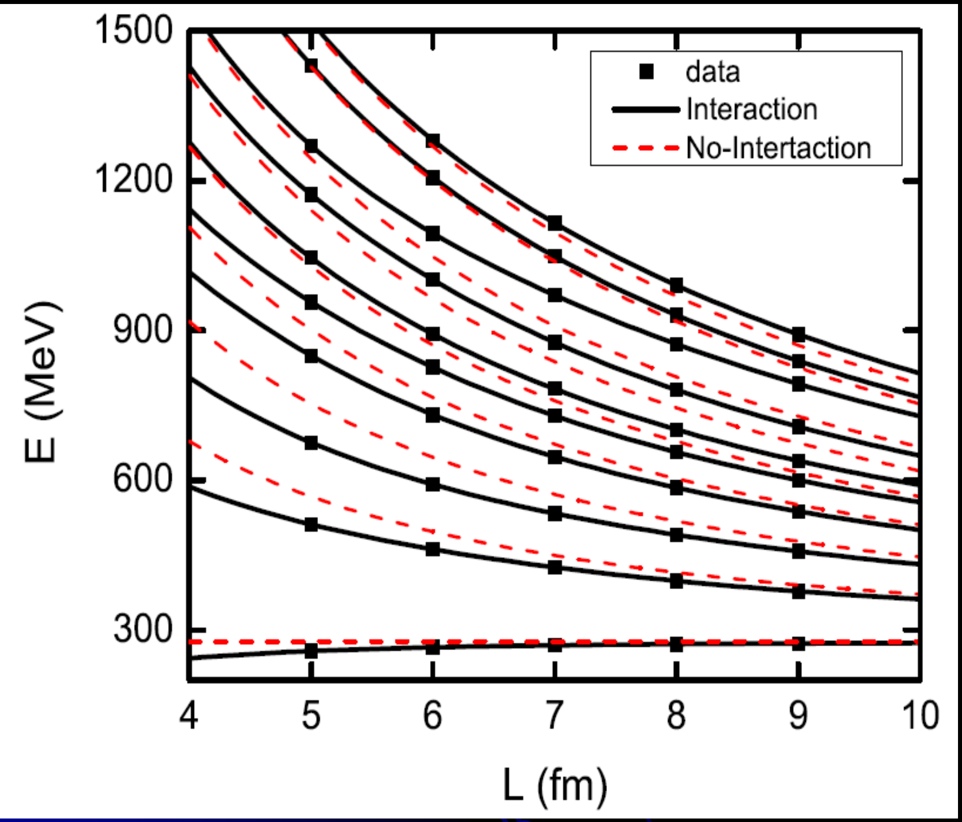
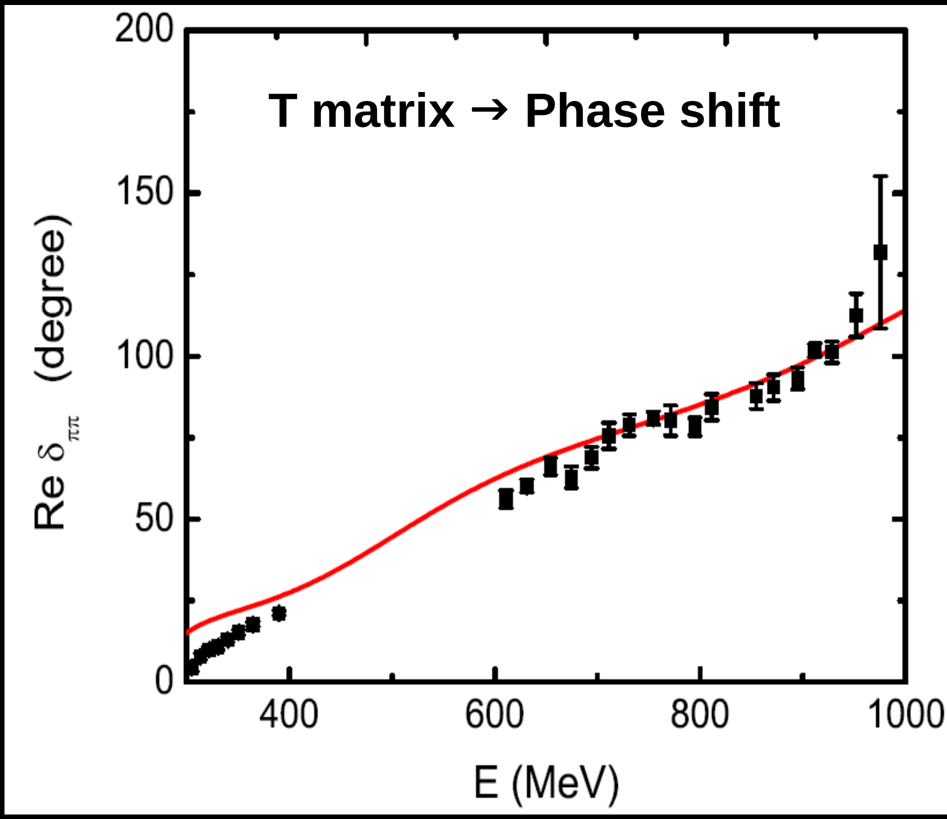


Phase shift from the Hamiltonian



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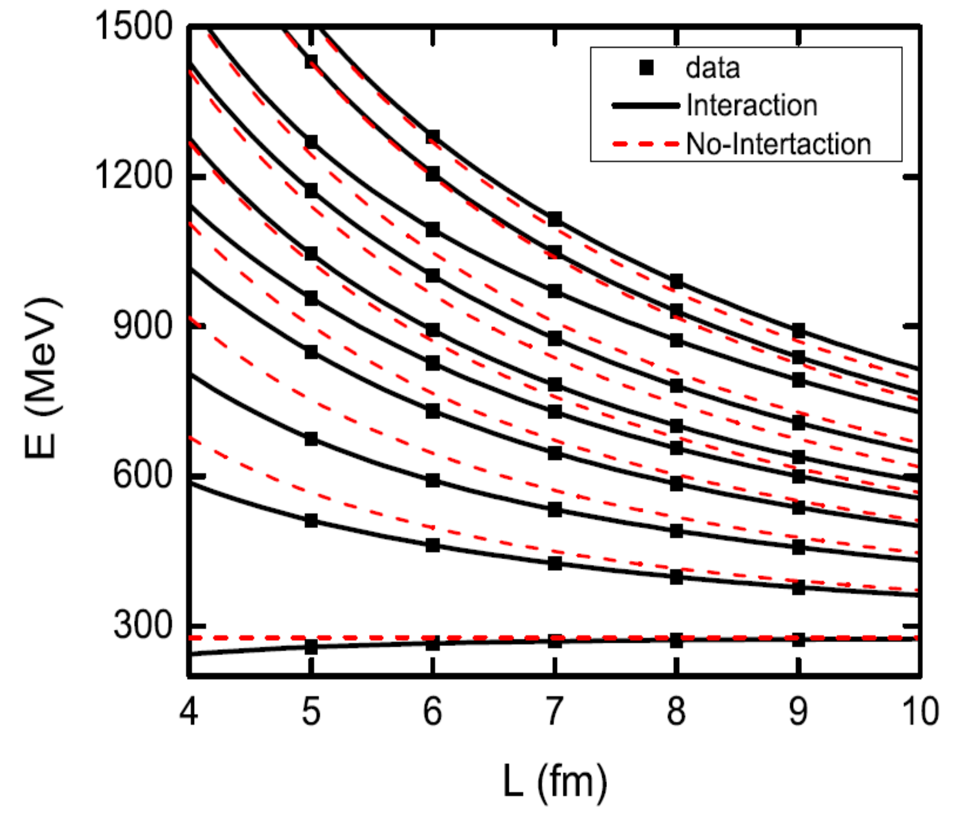
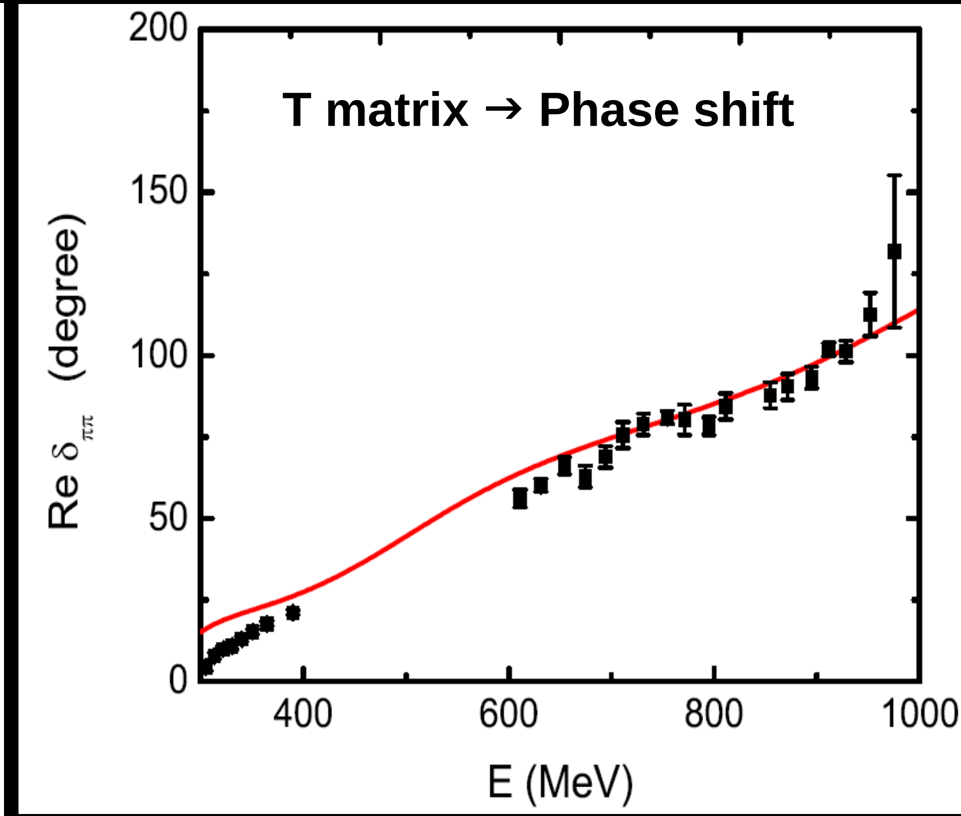
Spectrum from the Hamiltonian



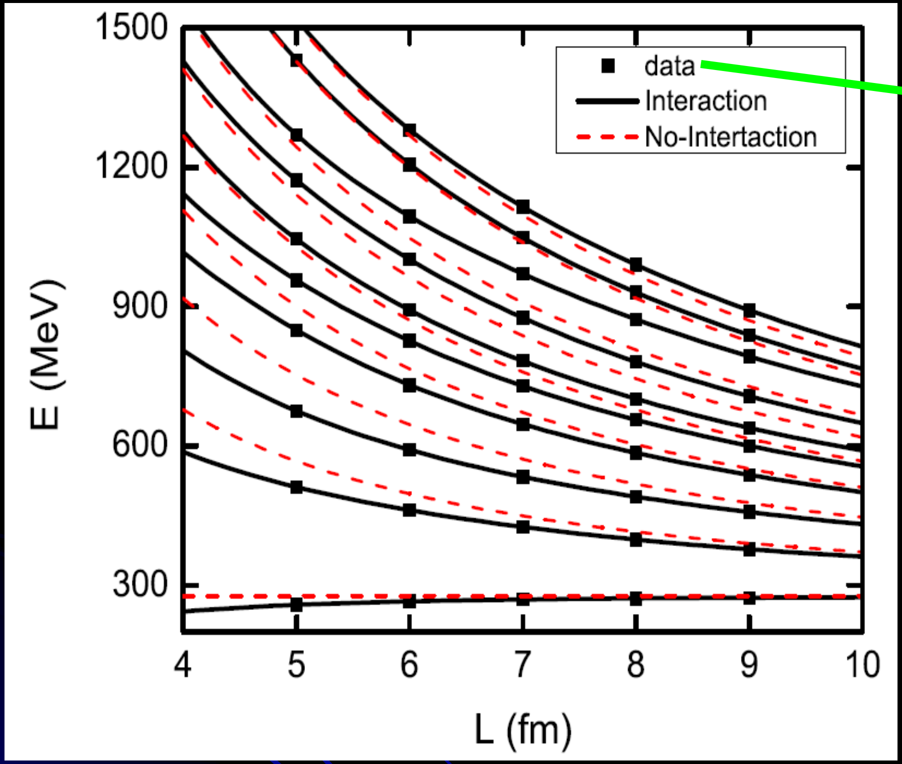
Phase shift from the Hamiltonian

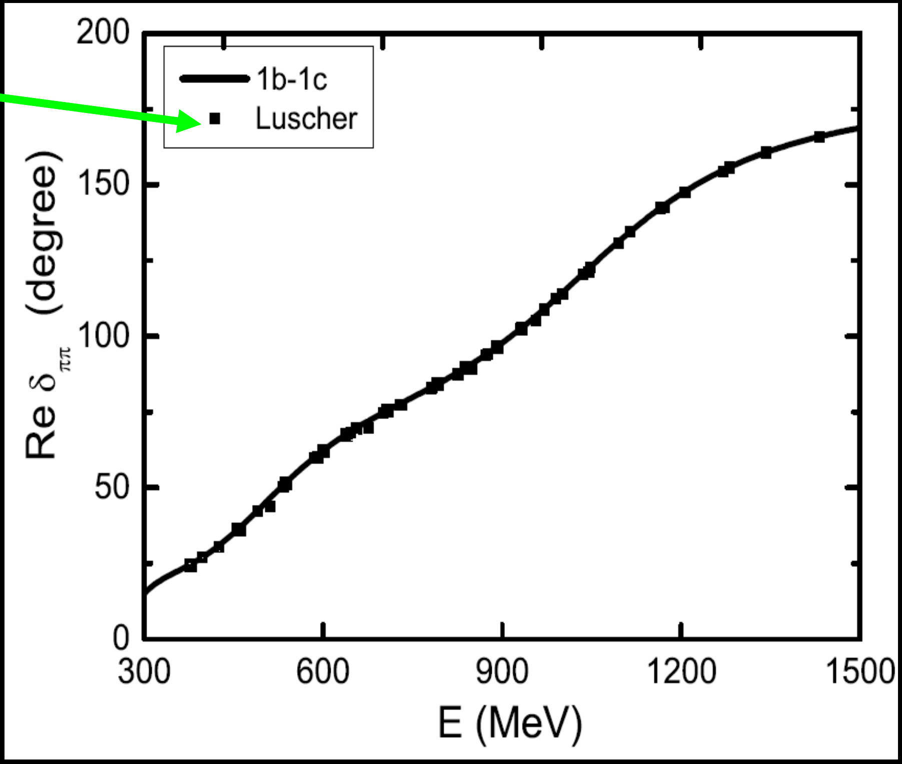


CONSISTENT OR NOT ??

Finite-box Hamiltonian method

1b-1c:





Luscher Method

$$\delta(k) = -\phi(q) \bmod \pi$$

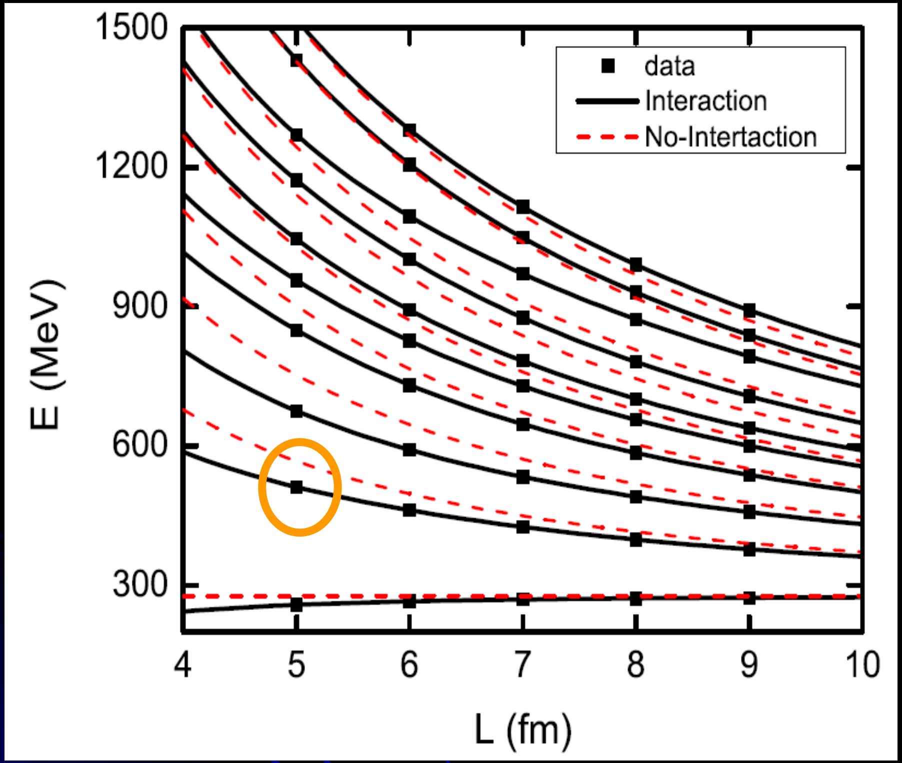
$$q = \frac{kL}{2\pi} = \frac{2\sqrt{E^2/4 - m_\pi^2}\,L}{2\pi}$$

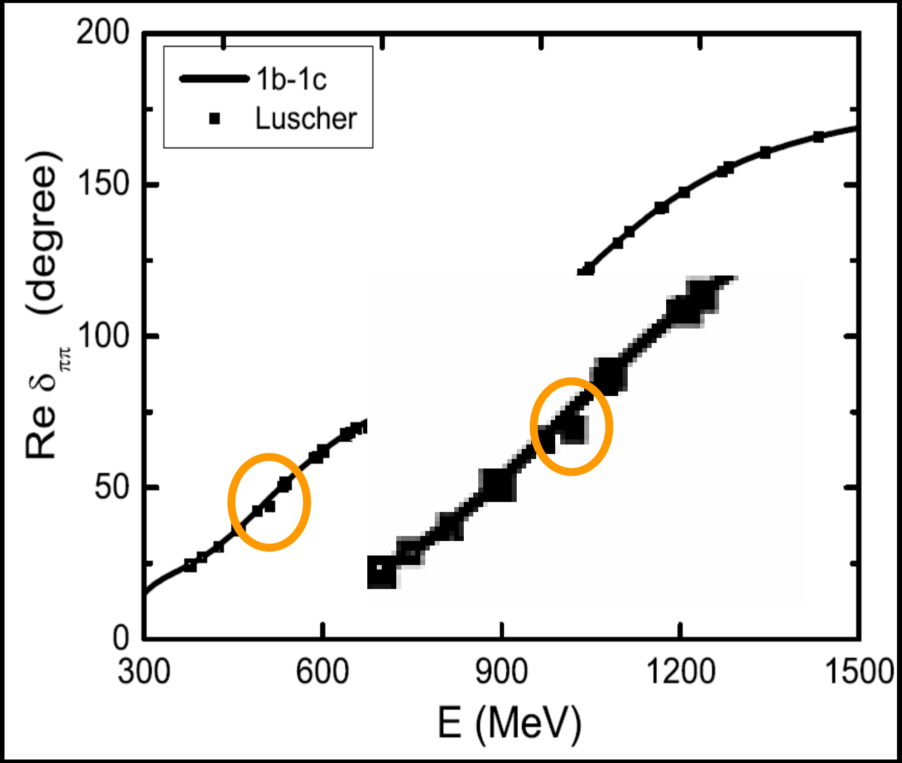
$$-\phi(q) = \tan^{-1}\left(\frac{q\pi^{3/2}}{Z_{00}(1;q^2)}\right)$$

$$Z_{00}(1;q^2) = \frac{1}{\sqrt{4\pi}} \sum_{\vec{n}\in Z^3} \frac{1}{\vec{n}^2 - q^2}$$

Finite-box Hamiltonian method

1b-1c:





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Finite-box Hamiltonian method

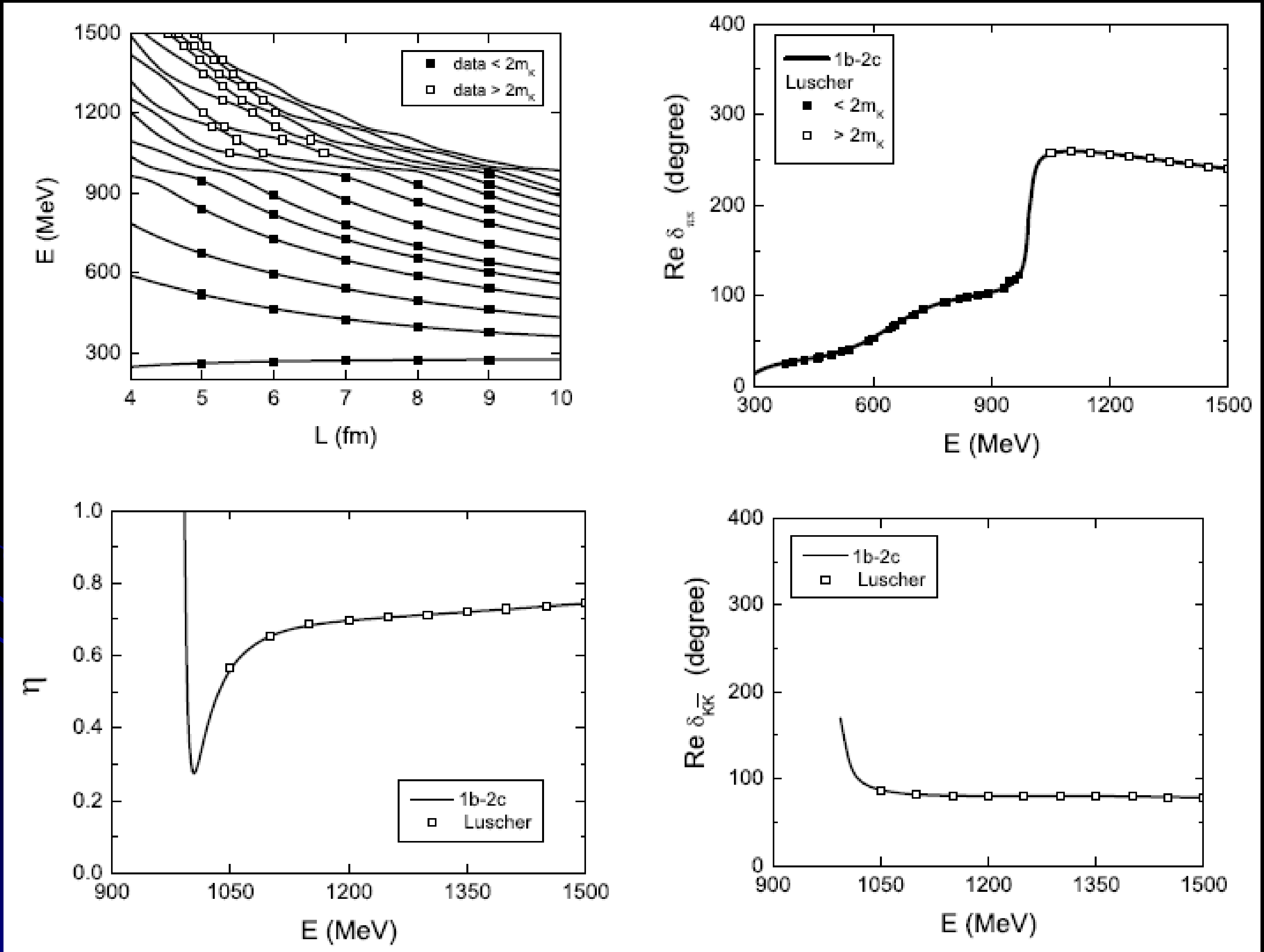
1b-2c: $\mathrm{Det}[H_0 + H_I - E\mathrm{I}] = 0$

$$H_0 = \begin{pmatrix} m_1 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 2\sqrt{k_0^2 + m_\pi^2} & 0 & 0 & 0 & \cdots \\ 0 & 0 & 2\sqrt{k_0^2 + m_K^2} & 0 & 0 & \cdots \\ 0 & 0 & 0 & 2\sqrt{k_1^2 + m_\pi^2} & 0 & \cdots \\ 0 & 0 & 0 & 0 & 2\sqrt{k_1^2 + m_K^2} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$H_I = \begin{pmatrix} 0 & g_{\pi\pi}^{fin}(k_0) & g_{KK}^{fin}(k_0) & g_{\pi\pi}^{fin}(k_1) & g_{KK}^{fin}(k_1) & \cdots \\ g_{\pi\pi}^{fin}(k_0) & v_{\pi\pi,\pi\pi}^{fin}(k_0,k_0) & v_{\pi\pi,KK}^{fin}(k_0,k_0) & v_{\pi\pi,\pi\pi}^{fin}(k_0,k_1) & v_{\pi\pi,KK}^{fin}(k_0,k_1) & \cdots \\ g_{KK}^{fin}(k_0) & v_{KK,\pi\pi}^{fin}(k_0,k_0) & v_{KK,KK}^{fin}(k_0,k_0) & v_{KK,\pi\pi}^{fin}(k_0,k_1) & v_{KK,KK}^{fin}(k_0,k_1) & \cdots \\ g_{\pi\pi}^{fin}(k_1) & v_{\pi\pi,\pi\pi}^{fin}(k_1,k_0) & v_{\pi\pi,KK}^{fin}(k_1,k_0) & v_{\pi\pi,\pi\pi}^{fin}(k_1,k_1) & v_{\pi\pi,KK}^{fin}(k_1,k_1) & \cdots \\ g_{KK}^{fin}(k_1) & v_{KK,\pi\pi}^{fin}(k_1,k_0) & v_{KK,KK}^{fin}(k_1,k_0) & v_{KK,\pi\pi}^{fin}(k_0,k_1) & v_{KK,KK}^{fin}(k_0,k_1) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Finite-box Hamiltonian method

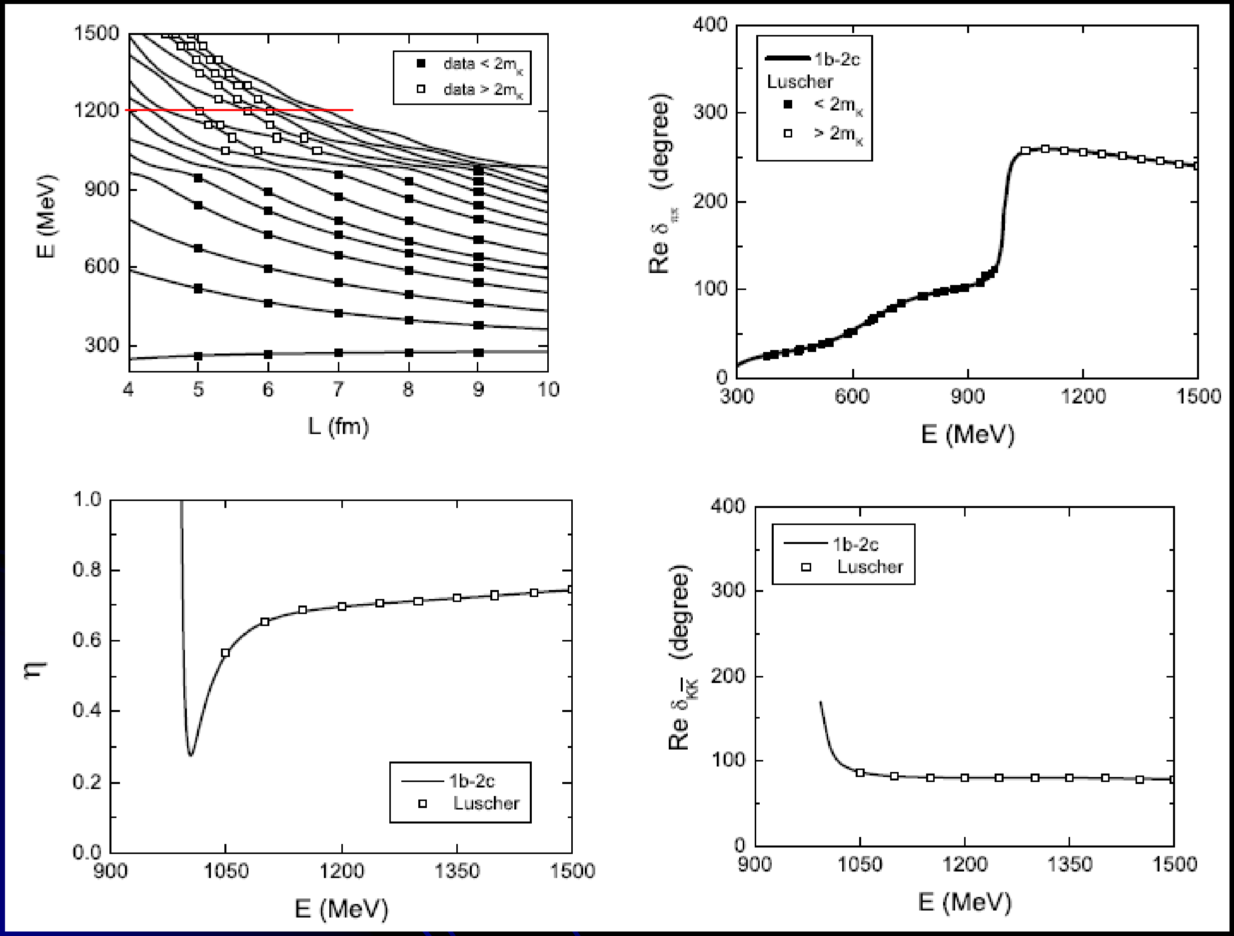
1b-2c:



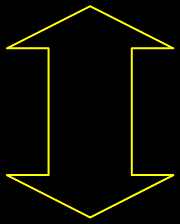
Finite-box Hamiltonian method

1b-2c:

$$\mathrm{Det}\ [H_0 + H_I - E\ \mathrm{I}] = 0$$



L_1, L_2, L_3 —— E



$$\delta_{\pi\pi}(E),\quad \delta_{K\bar{K}}(E),\quad \eta(E)$$

$$0 = \cos\left(\Delta_{\pi\pi}(L) + \Delta_{K\bar{K}}(L) - \delta_{\pi\pi}(E) - \delta_{K\bar{K}}(E)\right)$$
$$-\eta\cos\left(\Delta_{\pi\pi}(L) - \Delta_{K\bar{K}}(L) - \delta_{\pi\pi}(E) + \delta_{K\bar{K}}(E)\right)$$

$$\Delta_\alpha(L) = \tan^{-1}\left(\frac{q_\alpha \pi^{3/2}}{Z_{00}(1, q_\alpha^2)}\right)$$

Finite-box Hamiltonian method

1 channel and 2 channel

Finite-box Hamiltonian method →

Spectrum (L~E)

Hamiltonian → t matrix → observations

(δ_1, δ_2, η)

Finite-box Hamiltonian method

1 channel and 2 channel

Finite-box Hamiltonian method →

Spectrum (L~E)

Luscher Method

One channel

$$\delta(k) = -\phi(q) \bmod \pi$$

$$-\phi(q) = \tan^{-1}\left(\frac{q\pi^{3/2}}{Z_{00}(1;q^2)}\right)$$

Two channels

$$0 = \cos\left(\Delta_{\pi\pi}(L) + \Delta_{K\bar{K}}(L) - \delta_{\pi\pi}(E) - \delta_{K\bar{K}}(E)\right)$$

$$-\eta\cos\left(\Delta_{\pi\pi}(L) - \Delta_{K\bar{K}}(L) - \delta_{\pi\pi}(E) + \delta_{K\bar{K}}(E)\right)$$

Hamiltonian → t matrix → observations

(δ_1, δ_2, η)

Finite-box Hamiltonian method

1 channel and 2 channel

Finite-box Hamiltonian method →

Spectrum （L~E）

1. **Our approach is correct !**
2. **1 channel and 2 channel is almost the same**

Luscher Method

One channel

$$\delta(k) = -\phi(q) \bmod \pi$$

$$-\phi(q) = \tan^{-1}\left(\frac{q\pi^{3/2}}{Z_{00}(1;q^2)}\right)$$

Two channels

$$0 = \cos\left(\Delta_{\pi\pi}(L) + \Delta_{K\bar{K}}(L) - \delta_{\pi\pi}(E) - \delta_{K\bar{K}}(E)\right)$$
$$-\eta\cos\left(\Delta_{\pi\pi}(L) - \Delta_{K\bar{K}}(L) - \delta_{\pi\pi}(E) + \delta_{K\bar{K}}(E)\right)$$

Hamiltonian → t matrix → observations

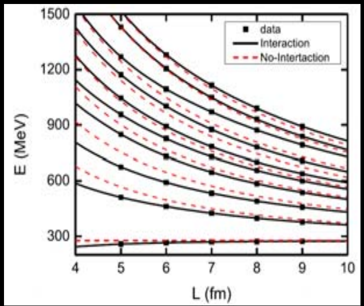
(δ_1, δ_2, η)

Outline

- Introduction
- Hamiltonian for $\pi\pi$ scattering
- Finite-box Hamiltonian method
- Applications to Lattice QCD
- Lattice spectra from the experiment data
- Summary and Outlook

Applications to Lattice QCD

If there are some Lattice data of spectrum, how can we change them to the observations?



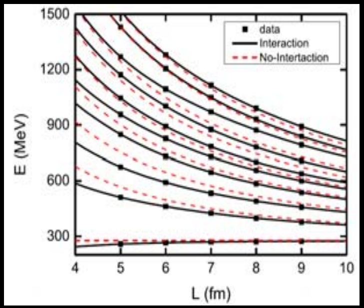
By our method: Fitting

By Luscher method: Solving Equation

Can (1 channel) or Can not (2 or multi-channel)

Applications to Lattice QCD

If there are some Lattice data of spectrum, how can we change them to the observations?



By our method: Fitting

By Luscher method: Solving Equation

Can (1 channel) or Can not (2 or multi-channel)

Fitting bring one problem: would the form of the “g” and “v” influence the last result or not ?

Check this problem: we will produce some Lattice spectrum data by the 1b-1c and 1b-2c models, then we will use different form of potential to fit these data, then using fitted parameters to compute the observations to check the dependence of the form of interaction.

Applications to Lattice QCD

A $$g_{i,\alpha}(k_\alpha) = \frac{\tilde{g}_{i,\alpha}}{\sqrt{\pi}} \frac{1}{(1+(c_\alpha k_\alpha)^2)}$$

$$v_{\alpha,\beta}(k_\alpha,k_\beta) = \frac{G_{\alpha,\beta}}{m_\pi^2} \frac{1}{(1+(d_\alpha k_\alpha)^2)^2} \frac{1}{(1+(d_\beta k_\beta)^2)^2}$$

B $$g_{i,\alpha}(k_\alpha) = \frac{\tilde{g}_{i,\alpha}}{\sqrt{\pi}} \frac{1}{(1+(c_\alpha k_\alpha)^2)^2}$$

$$v_{\alpha,\beta}(k_\alpha,k_\beta) = \frac{G_{\alpha,\beta}}{m_\pi^2} \frac{1}{(1+(d_\alpha k_\alpha)^2)^4} \frac{1}{(1+(d_\beta k_\beta)^2)^4}$$

C $$g_{i,\alpha}(k_\alpha) = \frac{\tilde{g}_{i,\alpha}}{\sqrt{\pi}} e^{-(c_\alpha k_\alpha)^2}$$

$$v_{\alpha,\beta}(k_\alpha,k_\beta) = \frac{G_{\alpha,\beta}}{m_\pi^2} e^{-(d_\alpha k_\alpha)^2} e^{-(d_\beta k_\beta)^2}$$

Applications to Lattice QCD

One channel case:

A $g_{i,\alpha}(k_\alpha) = \frac{\tilde{g}}{\sqrt{}}$

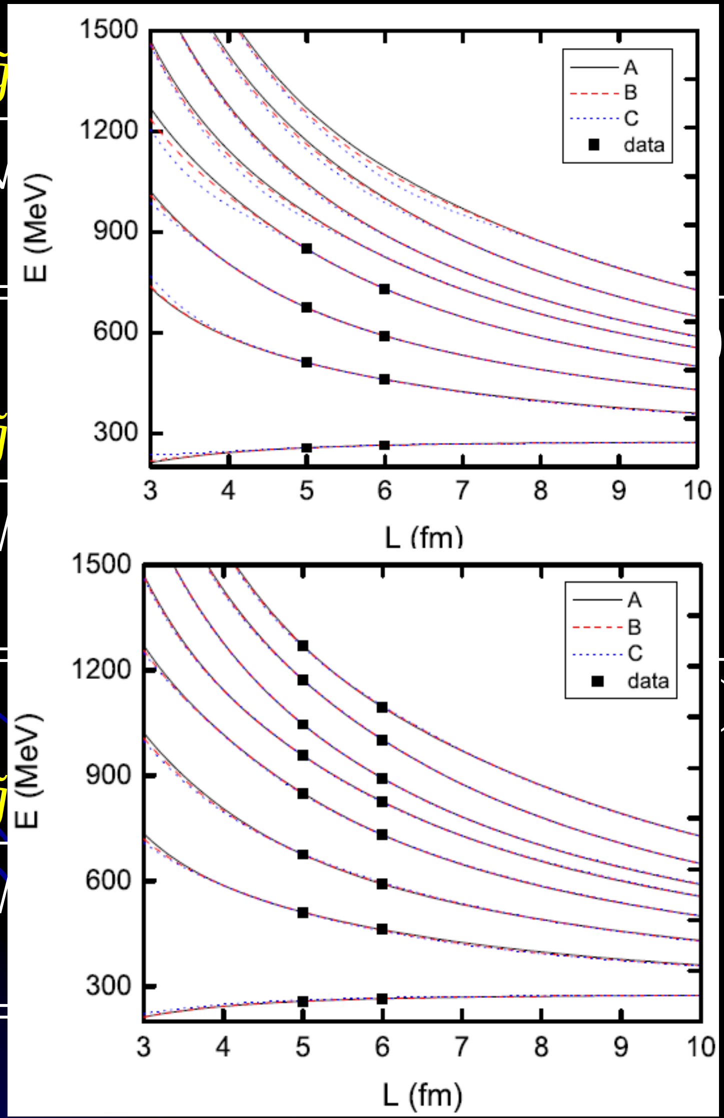
$v_{\alpha,\beta}(k_\alpha, k_\beta) =$ ... $)^2)^2$

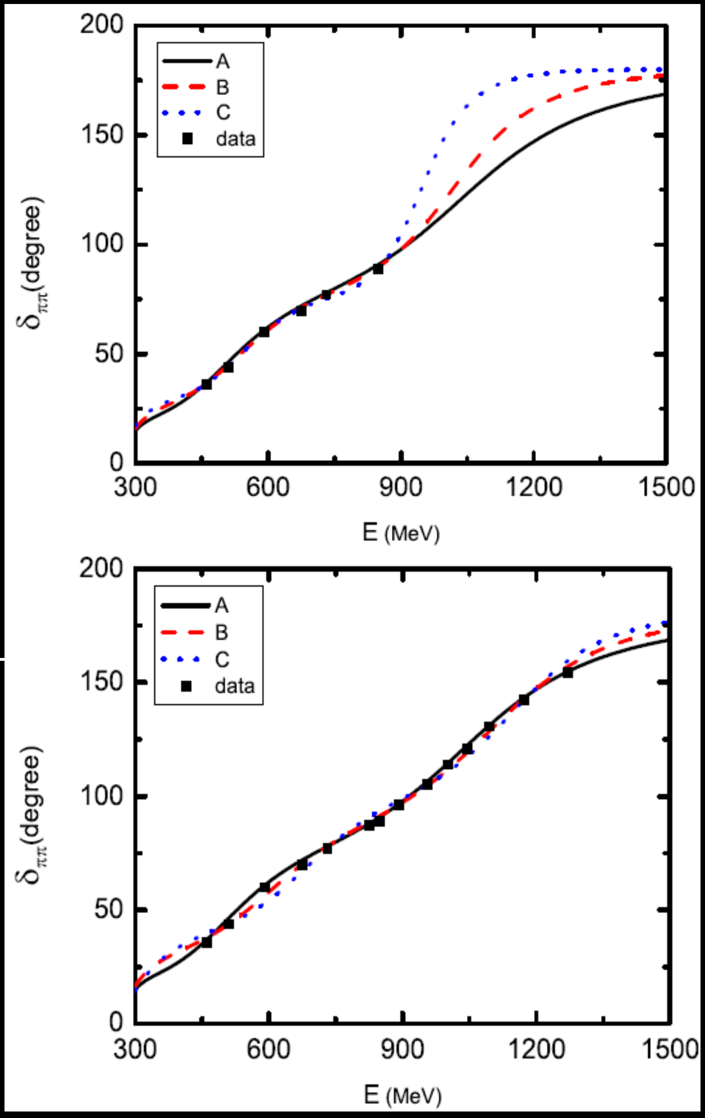
B $g_{i,\alpha}(k_\alpha) = \frac{\tilde{g}}{\sqrt{}}$

$v_{\alpha,\beta}(k_\alpha, k_\beta) =$ ... $)^2)^4$

C $g_{i,\alpha}(k_\alpha) = \frac{\tilde{g}}{\sqrt{}}$

$v_{\alpha,\beta}(k_\alpha, k_\beta) =$

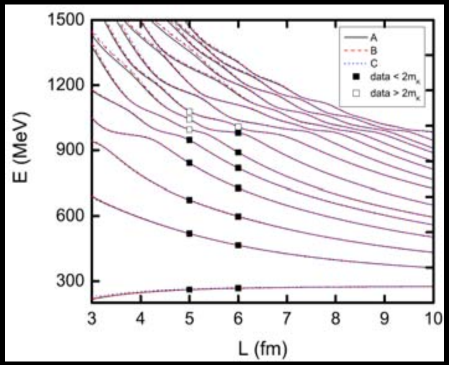


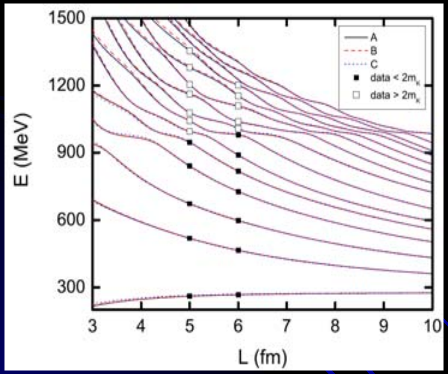


Applications to Lattice QCD

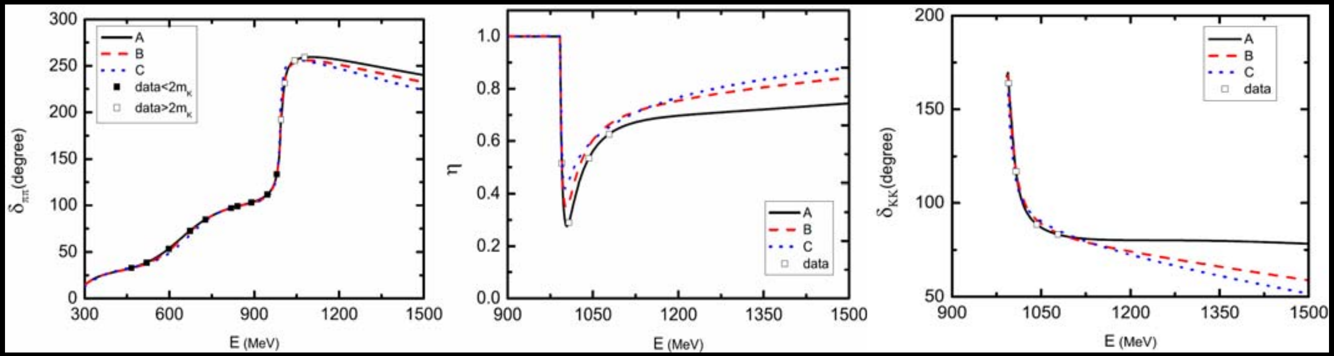
Two channels case:

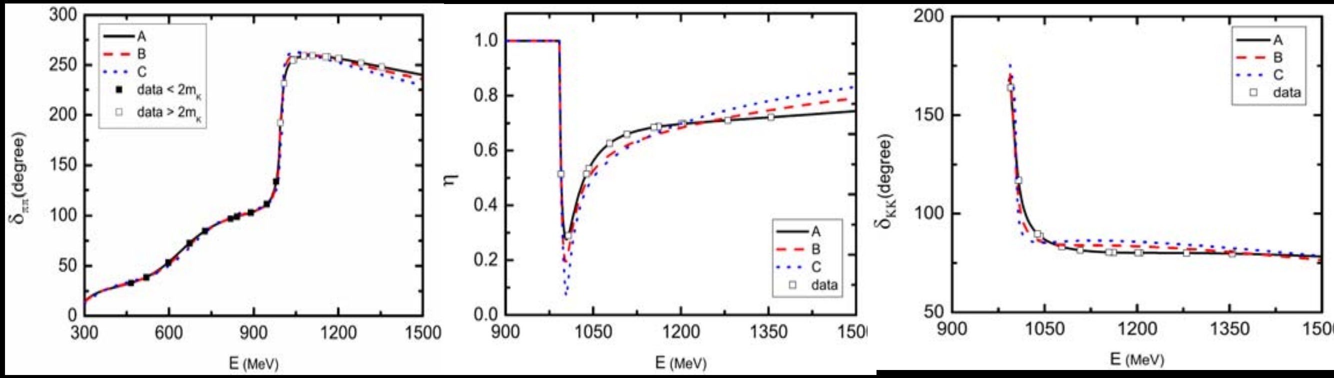
FITTING

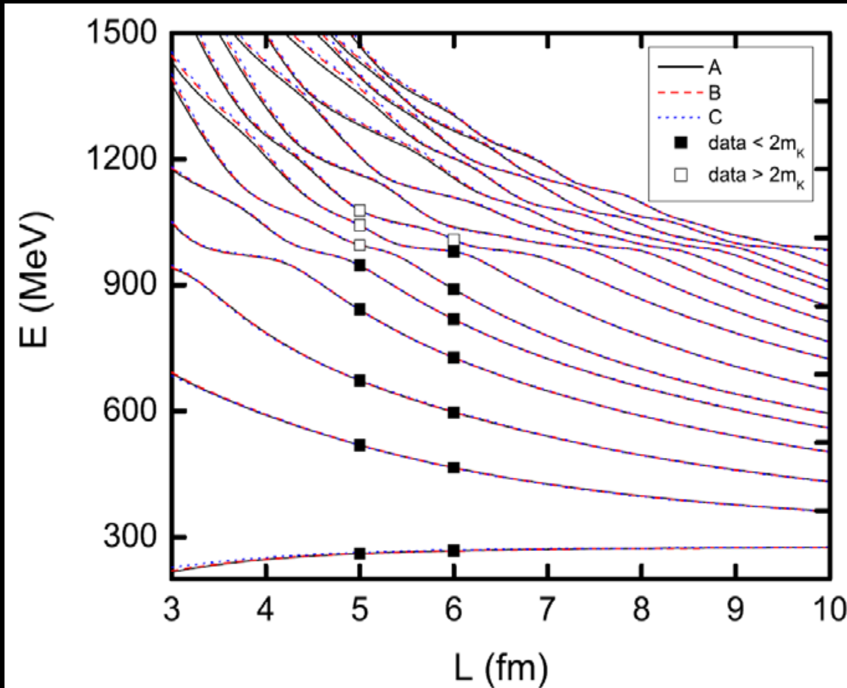


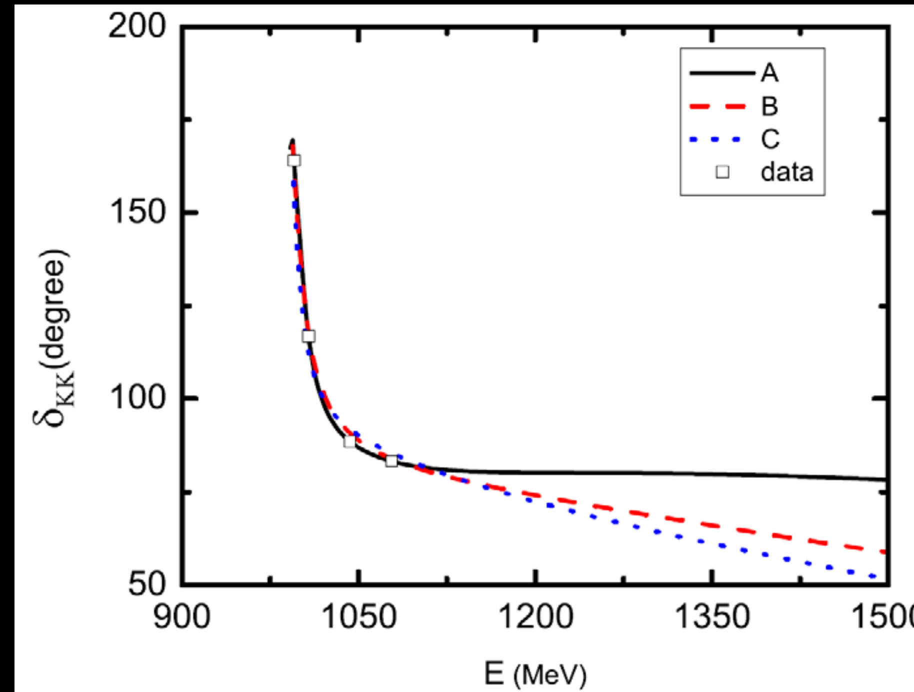


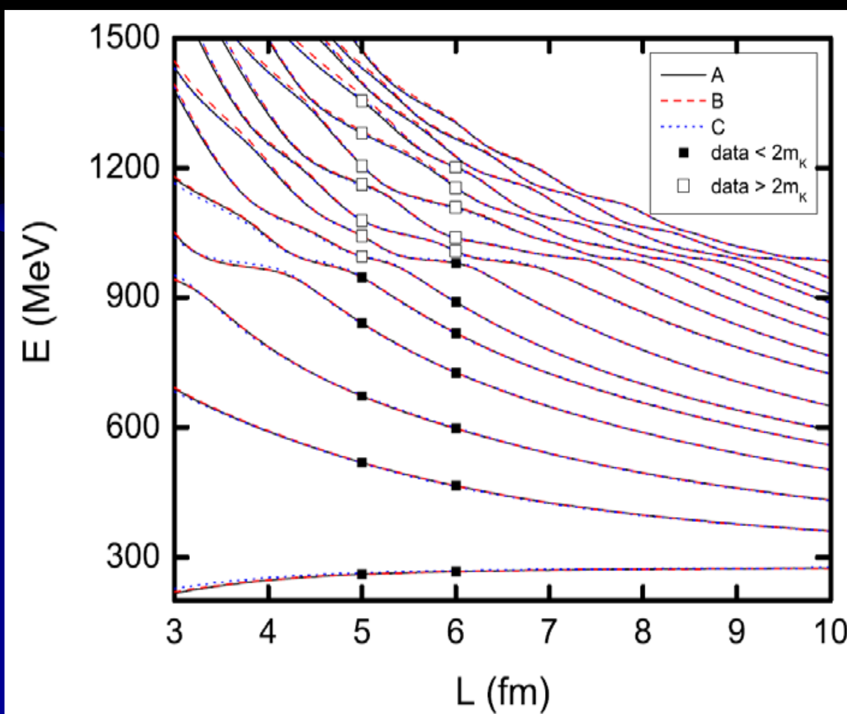
COMPUTING






E (MeV)
L (fm)
1500
1200
900
600
300
3
4
5
6
7
8
9
10
A
B
C
data < 2m$_K$
data > 2m$_K$


δ_{KK}(degree)
E (MeV)
200
150
100
50
900
1050
1200
1350
1500
A
B
C
data


E (MeV)
L (fm)
1500
1200
900
600
300
3
4
5
6
7
8
9
10
A
B
C
data < 2m$_K$
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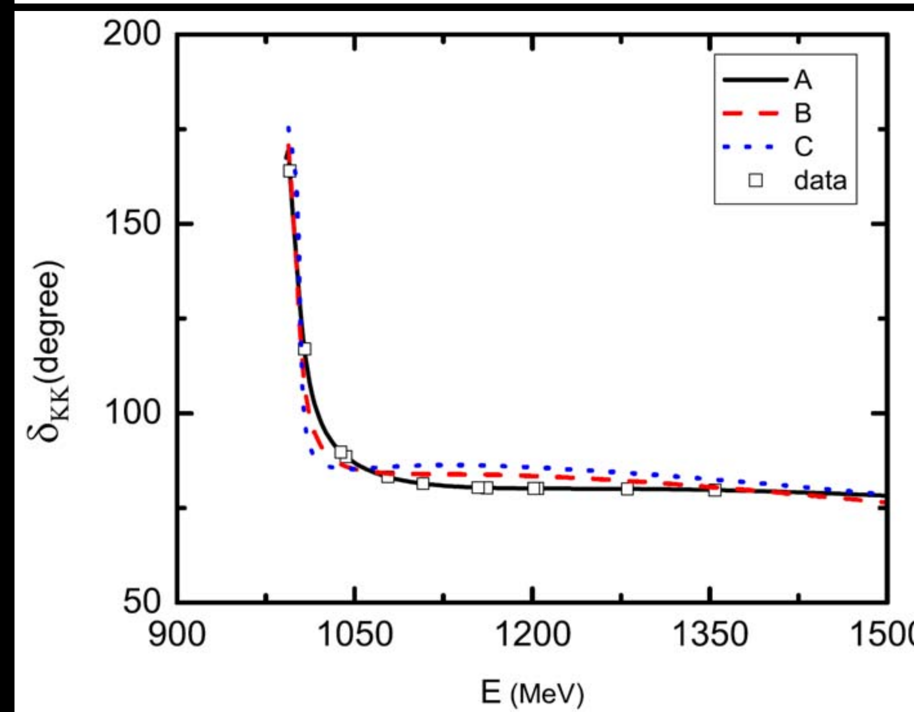
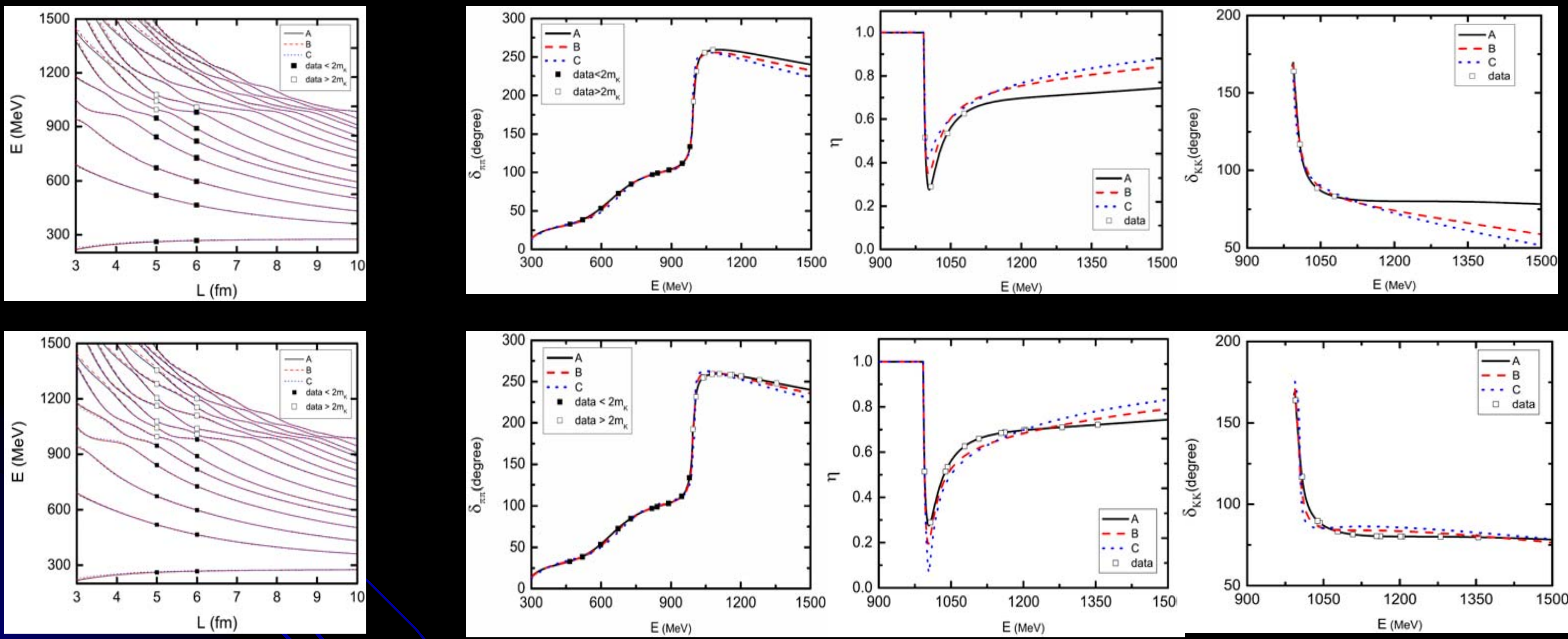

δ_{KK}(degree)
E (MeV)
200
150
100
50
900
1050
1200
1350
1500
A
B
C
data

Applications to Lattice QCD

Two channels case:



By 16 or 24 points on the two different L, Luscher method can tell us NOTHING, but our approach can give a good description of observations.

Applications to Lattice QCD

Summary

Fitting approach with our Hamiltonian method:

1. It is valid in the energy region where the spectrum data are fitted.
2. It is valid not only for one-channel case, but also for two-channels case, then we believe it would be also valid for multi-channel case.
3. It is independent of the form of the Hamiltonian.

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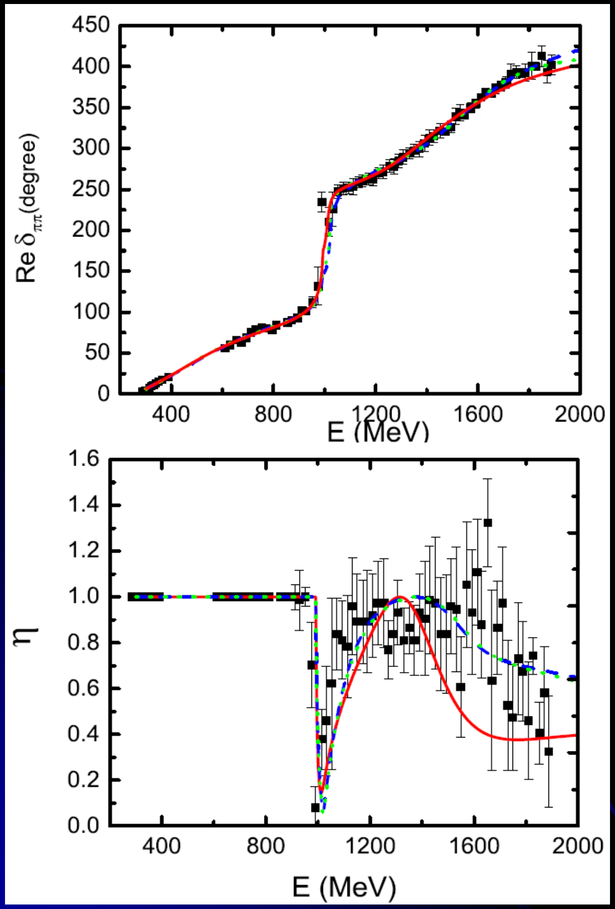
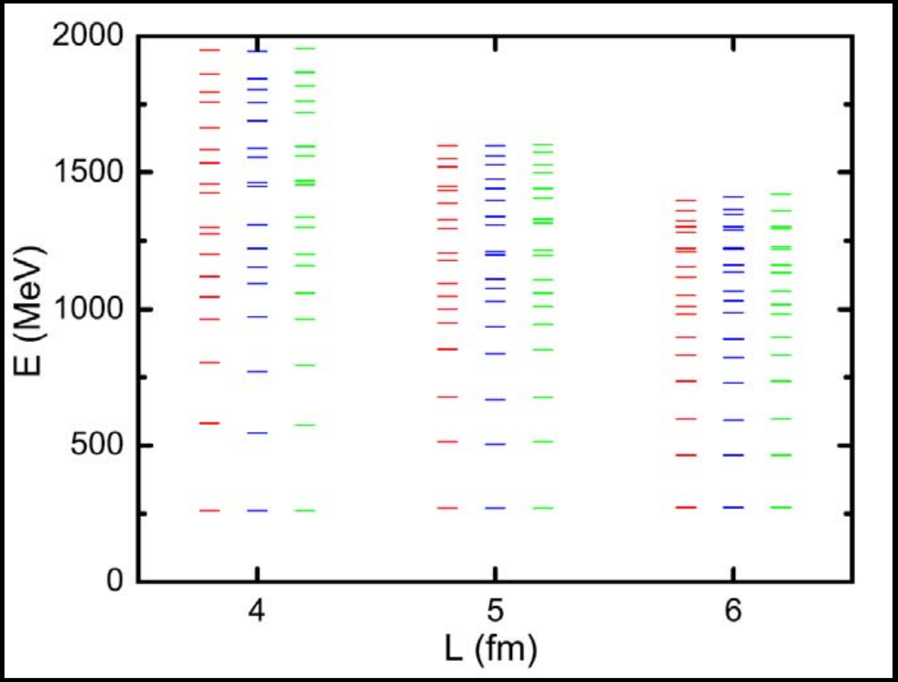
Lattice spectra from the experiment data

Spectra → Observations ➡ Observations → Spectra





Lattice spectra from the experiment data

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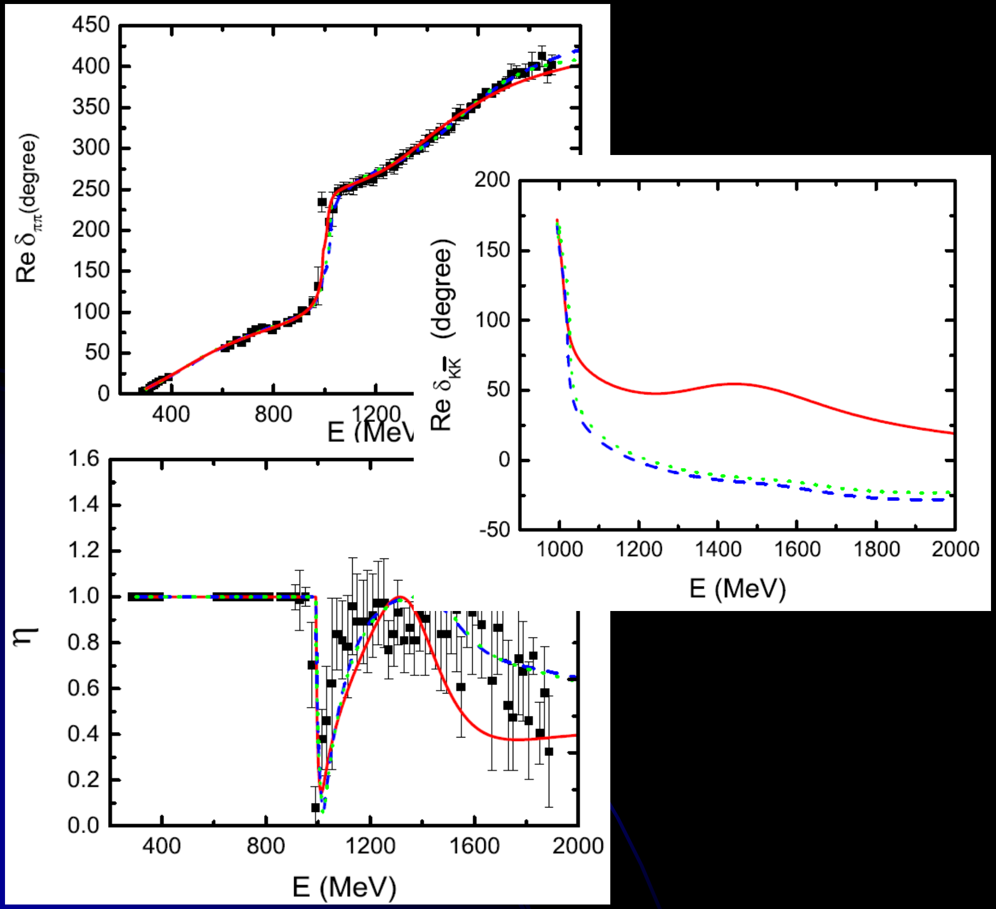

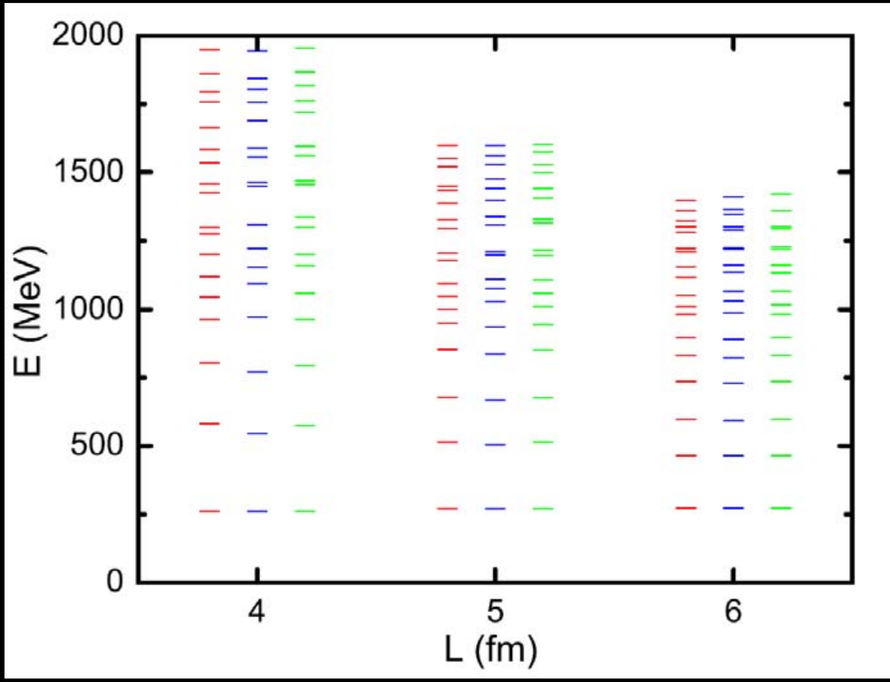

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Summary

- We apply the finite-volume Hamiltonian method to the $\pi\pi$ - KK scattering.
- The finite-volume Hamiltonian method is as accurate as the approach based on Luscher method in both the one-channel and two-channel cases.
- The finite-box Hamiltonian method can give correct prediction of scattering observables in the energy region where the spectrum data are fitted, independent of the form of the Hamiltonian.
- In the two channel cases, this Hamiltonian method need much less LQCD efforts than Luscher method.

Outlook

- Our approach is only in the S-wave and Center Mass system (C. M.). It is the simplest system.
- P-wave → Consider the Spin and angular momentum interaction
- C. M. → boost system

  → High eigenvalue of energy

  & Three body case

  & Electromagnetic form factor

Thank you very much