

# A speculation on the nature of black hole horizon

Jianwei Mei (梅健伟)

Based on J. Mei, JHEP 1310 (2013) 195  
arXiv:1305.4461 [gr-qc]

IHEP - CAS, 06 March 2014

## Black hole horizon

$$\text{Schwarzschild: } ds^2 = -\Delta dt^2 + \frac{dr^2}{\Delta} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2.$$

Removing divergence in the metric:

$$\begin{aligned} -\Delta dt^2 + \frac{dr^2}{\Delta} &= -\Delta \left( dt^2 - \frac{dr^2}{\Delta^2} \right) = -\Delta du_+ du_-, \\ du_{\pm} &= dt \pm \frac{dr}{\Delta}. \end{aligned}$$

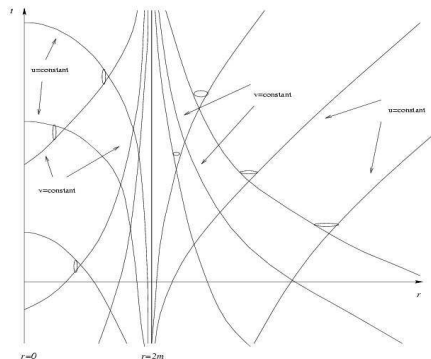


Figure: Light-cones of near the horizon of a Schwarzschild black hole. Figure: C.N.POPE

# Equivalence principle vs Schwinger effect

Equivalence principle (foundation of GR):

- ▶ local gravitational field  $\iff$  free fall;
- ▶ non-singular curved spacetime  $\iff$  locally flat.

Schwinger effect:

- ▶ Strong electric field  $\implies$  unstable electromagnetic vacuum,

$$eE_c \times \frac{1}{m_e} = m_e, \implies E_c = \frac{m_e^2}{e}.$$

The key quantity is acceleration.

- ▶ Applying to gravity:

$$|a| = \frac{M}{r^2} / \sqrt{1 - \frac{r_s}{r}}.$$

The acceleration diverges:

$\implies$  Breaking down of the gravitational vacuum (Hawking radiation).

Reason for concern:

The gravitational kick diverges

$\implies$  Breaking down of classical gravity (and also the equivalence principle)?

# The information paradox

Pure state  $\longrightarrow$  Black hole  $\longrightarrow$  Mixed state: Loss of information  
 $\implies$  Loss of unitarity in quantum mechanics + gravity

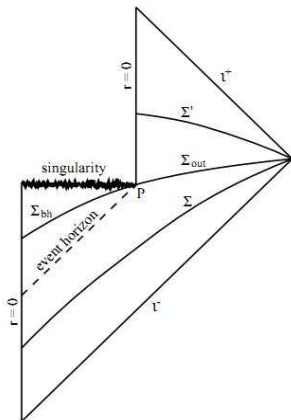
Possible solutions (Preskill, '92):

- ▶ information return through Hawking radiation;
- ▶ information saved in stable or long lived remnants;
- ▶ information return “at the end”;
- ▶ information in quantum hair;
- ▶ information in baby universe.

Black hole in a box: *AdS/CFT* correspondence

$\implies$  **No loss of information! Information back in Hawking radiation.**

## Black hole complementary



**Figure:** The existence of a state  $\Psi(\Sigma_P)$  that describes both the outside and inside of a black hole is suspicious. (Susskind, Thorlacius and Uglum '93)

# The firewall

Assuming (1) Purity, (2) EFT and (3) No drama, it is possible for Alice to see both  $S_{AB} = 0$  and  $S_{BC} = 0$ . (Almheiri, Marolf, Polchinski and Sully, 2012)

Strong subadditivity:

$$S_{ABC} + S_B \leq S_{AB} + S_{BC} .$$

One intriguing objection: Alice will not have enough time to verify  $S_{BC} = 0$ .

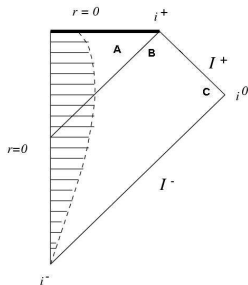


Figure: Collapsing to a black hole.

## A second (very different) motivation

Understanding the spectrum of elementary particles:

- ▶ Yukawa coupling;
- ▶ particles as excitations of strings (String theory);
- ▶ particles as excitations of other extended objects (Dirac '62).

$\iff$  Dynamics of extended objects (String theory and beyond)

An example: Dirac Brane (Trzetrzelewski, 2012),

$$\begin{aligned} S_p &= -\lambda \int \sqrt{|G|} d^{p+1}\sigma, \quad G = \det G_{\alpha\beta}, \quad G_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X^\nu g_{\mu\nu}, \\ \mathcal{P}_\mu &:= \frac{\delta S_p}{\delta \dot{X}^\mu} = -\lambda \sqrt{|G|} G^{\alpha\tau} g_{\mu\nu} \partial_\alpha X^\nu, \quad g^{\mu\nu} \mathcal{P}_\mu \mathcal{P}_\nu = \lambda^2 |G| G^{\tau\tau}, \\ &\implies \gamma^\mu \mathcal{P}_\mu + \lambda \sqrt{|G| G^{\tau\tau}} = 0, \quad \mathcal{P}_\mu = -i \frac{\delta}{\delta X^\mu}, \\ &\implies \left( -i \gamma^\mu \frac{\delta}{\delta X^\mu} + \lambda \sqrt{|G| G^{\tau\tau}} \right) \Psi = 0. \end{aligned}$$

# Horizon as a boundary

## Assumption:

- ▶ Horizon as a *physical* boundary to the spacetime;
- ▶ Dynamics determined like a brane (more detail later).

## Hints:

- ▶ Horizon is a causal boundary;
- ▶ Membrane paradigm;
- ▶ Conformal symmetry on the horizon (phase transition?);
- ▶ Firewall: making it physical .



## Boundary action

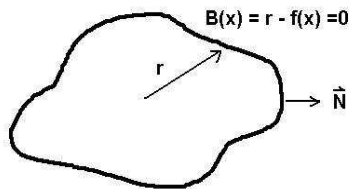


Figure: The profile of a boundary.

### Assumption:

The dynamics of a boundary brane is governed by  
the bulk action + boundary term.

The total action,

$$S_{tot} = \frac{1}{16\pi} \int d^n x \sqrt{-g} (R - 2\Lambda) + S_B,$$
$$S_B = \frac{1}{16\pi \ell_p^2} \int_B (d^{n-1}x)_\mu N^\mu \sqrt{-g} K,$$

- ▶  $K = g^{\mu\nu} K_{\mu\nu}$ ,  $K_{\mu\nu} = \nabla_\mu N_\nu + \nabla_\nu N_\mu$ ;
- ▶  $N_\mu = \partial_\mu B / |\partial B|$ ,  $|\partial B| = \sqrt{g^{\rho\sigma} \partial_\rho B \partial_\sigma B}$ ;
- ▶  $B(x) = 0$  defines the boundary.

## Boundary contributions

Varying the fields ( $\mathcal{P}_{\alpha\beta} = \mathbf{g}_{\alpha\beta} - N_\alpha N_\beta$ ,  $\tilde{\nabla}_\alpha = \mathcal{P}_\alpha^\beta \nabla_\beta$ ),

$$\delta S_{\text{tot}} = \frac{1}{16\pi} \int d^n x \sqrt{-g} \delta g^{\mu\nu} \left( R_{\mu\nu} - \frac{R - 2\Lambda}{2} g_{\mu\nu} \right) + \delta S'_B,$$

$$\delta S'_B = \frac{1}{16\pi} \int_B d^{n-1} x \sqrt{-h} \frac{\delta g^{\alpha\beta}}{2} \mathcal{T}_{\alpha\beta},$$

$$\mathcal{T}_{\alpha\beta} = \tilde{\nabla}_\alpha N_\beta + \tilde{\nabla}_\beta N_\alpha - K \mathcal{P}_{\alpha\beta}.$$

Charges ( $T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta S'_B}{\delta g^{\mu\nu}}$ ),

$$H = -\frac{1}{16\pi} \int_B d^{n-1} x \sqrt{-h} \mathcal{T}_t^t, \quad J_\phi = -\frac{1}{16\pi} \int_B d^{n-1} x \sqrt{-h} \mathcal{T}_\phi^t.$$

# The Kerr background

The Kerr metric

$$\begin{aligned} ds^2 &= f \left( \frac{dr^2}{\Delta} - \frac{\Delta}{v^2} dt^2 \right) + \frac{fdx^2}{1-x^2} + \frac{v^2(1-x^2)}{f} (d\phi - wdt)^2, \\ \Delta &= (r-r_+)(r-r_-), \quad w = \frac{r}{v^2} \sqrt{r_+ r_-} (r_+ + r_-), \\ f &= r^2 + r_+ r_- x^2, \quad v^2 = (r^2 + r_+ r_-)^2 - \Delta r_+ r_- (1-x^2), \end{aligned}$$

where  $x = \cos \theta$ ,  $r_{\pm} = M \pm \sqrt{M^2 - J^2/M^2}$  and

$$\begin{aligned} T &= \frac{r_+ - r_-}{4\pi r_+ (r_+ + r_-)}, \quad \Omega = \frac{\sqrt{r_- r_+}}{r_+ (r_+ + r_-)}, \quad S = \pi r_+ (r_+ + r_-), \\ M &= \frac{r_+ + r_-}{2}, \quad J = M \sqrt{r_+ r_-}. \end{aligned}$$

## The configuration function

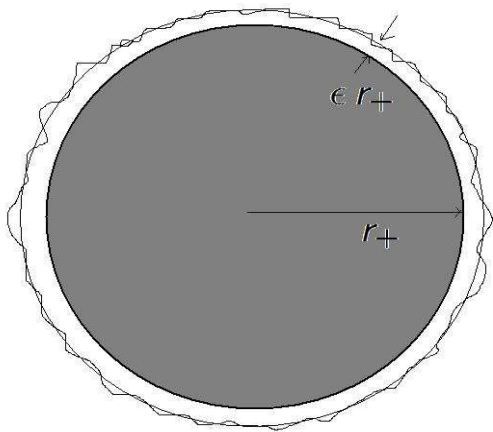


Figure:  $B = r - r_+ \{ 1 + \epsilon [ 1 + \Phi(x, \phi, t) ] \} = 0$ .

$$N_r = \frac{1}{|\partial B|}, \quad N_i = -\epsilon r_+ \frac{\partial_i \Phi}{|\partial B|}, \quad i = x, \phi, t,$$

$$|\partial B|^2 = \frac{\Delta}{f} + (\epsilon r_+)^2 \left\{ \frac{1-x^2}{f} (\partial_x \Phi)^2 + \frac{f(\partial_\phi \Phi)^2}{v^2(1-x^2)} - \frac{v^2}{f\Delta} [(\partial_t + w\partial_\phi)\Phi]^2 \right\}.$$

The induced metric on the boundary is

$$ds_H^2 = -\frac{f\Delta}{v^2} dt^2 + \frac{fdx^2}{1-x^2} + \frac{v^2(1-x^2)}{f} d\phi^2 + (\epsilon r_+)^2 \frac{f}{\Delta} \left( \partial_x \Phi dx + \partial_\phi \Phi d\phi + \partial_t \Phi dt \right)^2.$$

## Effective action

Action at  $r = r_+ \{1 + \epsilon[1 + \Phi(x, \phi, t)]\}$ ,

$$S_B = \frac{r_+ - r_-}{4} - \frac{\mu}{2} \int_B dx d\phi dt \left[ \mathcal{L}_0 + \frac{2r_+(r_+ + r_-)^2 \Phi (\partial_t + \Omega \partial_\phi)^2 \Phi}{r_+ - r_-} + \mathcal{O}(\Phi^3) \right] + \mathcal{O}(\sqrt{\epsilon}),$$

$$\mathcal{L}_0 = \frac{2r_+(r_+ + r_-)^2}{r_+ - r_-} [(\partial_t + \Omega \partial_\phi) \Phi]^2 - (1 - x^2) (\sqrt{\epsilon} \partial_x \Phi)^2 - \frac{(r_+ + r_- x^2)^2 (\sqrt{\epsilon} \partial_\phi \Phi)^2}{(r_+ + r_-)^2 (1 - x^2)},$$

where  $\mu = r_+ / (8\pi)$ .

At  $r = r_+(1 + \epsilon)$ ,

$$S_H = -\frac{r_+ - r_-}{4} + \frac{\mu}{2} \int_H dx d\phi dt \left[ \mathcal{L}_0 + \mathcal{O}(\Phi^3) \right] + \mathcal{O}(\sqrt{\epsilon}).$$

# Charges

Charges at  $r = r_+(1 + \epsilon)$ ,

$$H = -\Omega J_\phi + \frac{\mu}{2} \int_H dx d\phi dt \left\{ \frac{2r_+(r_+ + r_-)^2}{r_+ - r_-} [(\partial_t + \Omega \partial_\phi)\Phi]^2 - \mathcal{L}_0 \right\}, \quad (1)$$

$$\begin{aligned} J_\phi &= \frac{\mu}{2} \int_H dx d\phi dt \left\{ \frac{2r_+(r_+ + r_-)^2}{r_+ - r_-} (\partial_t + \Omega \partial_\phi)\Phi \partial_\phi \Phi \right. \\ &\quad \left. + \Omega \frac{(r_+ + r_-)^2 (1 - x^2)}{(r_+ + r_- x^2)^2} [3r_+^2 - r_-^2 x^2 + r_+ r_- (1 + x^2)] \right\} \\ &= J + \frac{\mu}{2} \int_H dx d\phi dt \left[ \frac{2r_+(r_+ + r_-)^2}{r_+ - r_-} (\partial_t + \Omega \partial_\phi)\Phi \partial_\phi \Phi \right]. \end{aligned}$$

The canonical momentum

$$\Pi_\Phi = \mu \frac{r_+(r_+ + r_-)^2}{r_+ - r_-} (\partial_t + \Omega \partial_\phi)\Phi = \frac{1}{2} \frac{\delta S_H}{\delta(\partial_t \Phi)}.$$

## Classical solutions

In the small rotation limit ( $\Omega \rightarrow 0 \Rightarrow \rho \equiv r_-/r_+ \rightarrow 0$ ),

$$\mathcal{L}_0 = 2r_+^2(1+3\rho)[(\partial_t + \Omega \partial_\phi)\Phi]^2 - (1-x^2)(\sqrt{\epsilon} \partial_x \Phi)^2 - \left(\frac{1}{1-x^2} - 2\rho\right)(\sqrt{\epsilon} \partial_\phi \Phi)^2,$$

$$H = -\Omega J_\phi + \frac{\mu}{2} \int_H dx d\phi \left\{ (1-x^2)(\sqrt{\epsilon} \partial_x \Phi)^2 + \left(\frac{1}{1-x^2} - 2\rho\right)(\sqrt{\epsilon} \partial_\phi \Phi)^2 \right\},$$

$$J_\phi = J + \int_H dx d\phi \mu r_+^2(1+3\rho)(\partial_t + \Omega \partial_\phi)\Phi \partial_\phi \Phi.$$

The solution is

$$\begin{aligned} \Phi &= P_\ell^m(x) \exp\{i[m(\phi - \Omega t) - \mathcal{E}_{\ell,m} t]\}, \\ 2r_+^2(1+3\rho)\mathcal{E}_{\ell,m}^2 f_\ell^m + \epsilon \partial_x \left[ (1-x^2) \partial_x f_\ell^m \right] - \left( \frac{1}{1-x^2} - 2\rho \right) \epsilon m^2 f_\ell^m &= 0, \\ \mathcal{E}_{\ell,m} &= \sqrt{\epsilon \frac{\ell(\ell+1) - 2m^2\rho}{2r_+^2(1+3\rho)}} \approx \frac{\ell}{r_+} \sqrt{\frac{\epsilon}{2}} \left[ 1 - \rho \left( \frac{3}{2} + \frac{m^2}{\ell^2} \right) \right], \\ \ell &= 0, 1, \dots, \infty, \quad m = -\ell, \dots, \ell. \end{aligned}$$



# Quantization

General solution,  $\Phi = \sum_{\ell,m} \Phi_{\ell}^m$ ,

$$\Phi_{\ell}^m = N_{\ell}^m P_{\ell}^m(x) \left\{ a_{\ell,m} e^{i[m(\phi - \Omega t) - \mathcal{E}_{\ell,m} t]} + a_{\ell,m}^{\dagger} e^{-i[m(\phi - \Omega t) - \mathcal{E}_{\ell,m} t]} \right\},$$

$$N_{\ell}^m = \sqrt{\frac{1}{2\mu r_+^2 (1 + 3\rho) \mathcal{E}_{\ell,m}} \frac{2\ell + 1}{4\pi} \frac{(\ell - m)!}{(\ell + m)!}},$$

Commutators,

$$\begin{aligned} [\hat{\Phi}(x, \phi, t), \hat{\Pi}_{\Phi}(x', \phi', t)] &= i\delta(x - x')\delta(\phi - \phi'), \\ [\hat{\Pi}_{\Phi}(x, \phi, t), \hat{\Pi}_{\Phi}(x', \phi', t)] &= [\hat{\Phi}(x, \phi, t), \hat{\Phi}(x', \phi', t)] = 0, \\ [\hat{a}_{\ell,m}, \hat{a}_{p,q}^{\dagger}] &= \delta_{\ell,p} \delta_{m,q}, \quad [\hat{a}_{\ell,m}, \hat{a}_{p,q}] = [\hat{a}_{\ell,m}^{\dagger}, \hat{a}_{p,q}^{\dagger}] = 0. \end{aligned}$$

## Fock space and charges

- ▶  $\hat{a}_{\ell,m} |0\rangle = 0$ ;
- ▶ multiparticle states,  $|N_{\ell,m}\rangle \sim (\hat{a}_{\ell,m}^\dagger)^{N_{\ell,m}} |0\rangle$ ,

$$\hat{N}_{\ell,m} |N_{\ell',m'}\rangle = \delta_{\ell\ell'} \delta_{mm'} N_{\ell,m} |N_{\ell,m}\rangle, \quad \langle N_{\ell,m} | N_{\ell',m'} \rangle = \delta_{\ell\ell'} \delta_{mm'},$$

where  $\hat{N}_{\ell,m} = \hat{a}_{\ell,m}^\dagger \hat{a}_{\ell,m}$ ;

The charges become

$$\begin{aligned} \hat{H} &= -\Omega \hat{J}_\phi + \sum_{\ell,m} \mathcal{E}_{\ell,m} \left( \hat{N}_{\ell,m} + \frac{1}{2} \hat{N}_0 \right) \\ &= \left( -\Omega J + \sum_{\ell,m} \frac{\mathcal{E}_{\ell,m}}{2} \right) \hat{N}_0 + \sum_{\ell,m} \mathcal{E}'_{\ell,m} \hat{N}_{\ell,m}, \\ \hat{J}_\phi &= J \hat{N}_0 - \sum_{\ell,m} m \hat{N}_{\ell,m}, \end{aligned}$$

where  $\mathcal{E}'_{\ell,m} = \mathcal{E}_{\ell,m} + m\Omega$ , and  $\hat{N}_0$  is the number operator for the vacuum state,

$$\hat{N}_0 |0\rangle = |0\rangle, \quad \hat{N}_0 |N_{\ell,m}\rangle = 0.$$

## Zero point energy

- ▶ Zero point energy contributes to gravity;
- ▶ Need a cutoff to render the contribution finite;
- ▶ Existence of substructure  $\iff$  cutoff

$$N_c = \ell_{max} = m_{max} \approx \frac{2\pi r_+}{a},$$

where  $a$  is lattice spacing of substructure of spacetime.

With the cutoff, the contribution from the zero point energy is

$$M_0 = \sum_{\ell=0}^{N_c} \sum_{m=-\ell}^{\ell} \frac{\mathcal{E}_{\ell,m}}{2} \approx \frac{N_c^3}{3r_+} \sqrt{\frac{\epsilon}{2}} \left(1 - \frac{11\rho}{6}\right).$$

## The thermal state

System with temperature  $T$  and angular velocity  $\Omega$  is

$$\begin{aligned} |\Psi\rangle &= |0\rangle + \sum_{\ell,m} \sum_{N_{\ell,m}} \left( \frac{e^{-N_{\ell,m}(\beta\mathcal{E}'_{\ell,m} + \alpha m)}}{\Xi_{\ell,m}} \right)^{1/2} |N_{\ell,m}\rangle \\ &= |0\rangle + \sum_{\ell,m} \sum_{N_{\ell,m}} \left( \frac{e^{-N_{\ell,m}\beta\mathcal{E}_{\ell,m}}}{\Xi_{\ell,m}} \right)^{1/2} |N_{\ell,m}\rangle, \end{aligned}$$

where  $\Xi_{\ell,m} = \sum_{N_{\ell,m}} e^{-N_{\ell,m}\beta\mathcal{E}_{\ell,m}} = 1/(1 - e^{-\beta\mathcal{E}_{\ell,m}})$ ,  $\beta = 1/T$  and  $\alpha = -\beta\Omega$ . (The minus sign in  $\alpha$  is due to the fact that  $\beta\Omega$  is a chemical potential.) Note

$$\langle \Psi | \hat{N}_0 | \Psi \rangle = 1, \quad \langle \Psi | \hat{N}_{\ell,m} | \Psi \rangle = \sum_{N_{\ell,m}} \frac{e^{-N_{\ell,m}\beta\mathcal{E}_{\ell,m}}}{\Xi_{\ell,m}} N_{\ell,m}.$$

## Localization of charges

$\mathcal{E}_{\ell,m}$  is an even function in  $m$ . So “particles” with  $m > 0$  and  $m < 0$  are evenly excited. Hence

$$\begin{aligned} E &= \langle \Psi | \hat{H} | \Psi \rangle = -\Omega J + \sum_{\ell,m} \frac{\mathcal{E}_{\ell,m}}{2} + \sum_{\ell,m} \sum_{N_{\ell,m}} \mathcal{E}'_{\ell,m} N_{\ell,m} \frac{e^{-N_{\ell,m}\beta\mathcal{E}_{\ell,m}}}{\Xi_{\ell,m}} \\ &= -\Omega J + \sum_{\ell,m} \frac{\mathcal{E}_{\ell,m}}{2} - \partial_\beta \sum_{\ell,m} \ln \Xi_{\ell,m}, \\ J_\phi &= \langle \Psi | \hat{J}_\phi | \Psi \rangle = J. \end{aligned}$$

The result for  $J_\phi$  suggests that all the angular momentum of the black hole is carried by the boundary, i.e., the horizon.

## The partition function

$$\ln \Xi = \sum_{\ell, m} \ln \Xi_{\ell, m} = - \sum_{\ell=0}^{N_c} \sum_{m=-\ell}^{\ell} \ln \left( 1 - e^{-\beta \mathcal{E}_{\ell, m}} \right).$$

Calculate in the small rotation limit:

$$M \approx \frac{r_+}{2}(1 + \rho), \quad J \approx \frac{r_+^2}{2} \sqrt{\rho}(1 + \rho), \quad \Omega \approx \frac{\sqrt{\rho}}{r_+}(1 - \rho),$$
$$T \approx \frac{1 - 2\rho}{4\pi r_+}, \quad S \approx \pi r_+^2(1 + \rho), \quad \rho = \frac{r_-}{r_+} \rightarrow 0.$$

The result is (note  $y = kn$  and  $k = \frac{\beta}{r_+} \sqrt{\frac{\epsilon}{2}}$ )

$$\ln \Xi \approx -\frac{2}{k^2} \int_0^{kN_c} dy f_1(y) = -\frac{2}{k^2} \left[ f_2(kN_c) - f_2(0) \right],$$

$$f_1(y) = f_2'(y) = y \ln(1 - e^{-y}) - \frac{11\rho y^2}{6(e^y - 1)}, \quad f_1(0) = 0,$$

$$f_2(y) = y Li_2(e^{-y}) + Li_3(e^{-y}) + \frac{11\rho}{3} f_3(y), \quad f_2(0) = \zeta(3) \left( 1 + \frac{11\rho}{3} \right),$$

$$f_3(y) = y Li_2(e^{-y}) + Li_3(e^{-y}) - \frac{y^2}{2} \ln(1 - e^{-y}), \quad f_3(0) = \zeta(3),$$

where  $Li_s(z)$  is the polylogarithm.

# Energy and Entropy

The total energy and the entropy are

$$H = -\Omega J + M_0 - \partial_\beta \ln \Xi = \frac{\ell_p^2}{4\pi^3 r_+^2 \epsilon_0} \left[ \zeta(3) - f_3(c_0) + \frac{c_0^3}{12} + \mathcal{O}(\rho) \right] M,$$

$$S' = (1 - \beta \partial_\beta) \ln \Xi = \frac{3\ell_p^2}{4\pi^3 r_+^2 \epsilon_0} \left[ \zeta(3) - f_3(c_0) - \frac{c_0^2}{6} \ln(1 - e^{-c_0}) + \mathcal{O}(\rho) \right] S,$$

where we have let  $\epsilon = \epsilon_0 + \epsilon_1 \rho + \mathcal{O}(\rho^2)$ ,  $N_c = N_0 + N_1 \rho + \mathcal{O}(\rho^2)$ , and  $c_0 = 2\pi\sqrt{2\epsilon_0} N_0$ . Assuming  $H = M$  and  $S' = S$ ,

$$\sqrt{\epsilon_0} \approx \frac{\ell_p}{7.27 r_+}, \quad N_0 \approx \frac{2\pi r_+}{2.84 \ell_p}$$

- ▶ The putative lattice spacing is  $a \approx 2.84 \ell_p$ .
- ▶ Coordinate deviation:  $\delta \approx r_+ \epsilon_0 \sim \frac{\ell_p^2}{r_+}$ ;  
 $\implies$  Physical deviation:  $\delta' \approx r_+ \sqrt{\epsilon_0} \approx \frac{\ell_p}{7.27}$ .
- ▶ The cutoff energy:  $\mathcal{E}_{N_0, m} \approx \frac{N_0}{r_+} \sqrt{\frac{\epsilon_0}{2}} \approx \frac{0.22}{r_+}$ .

# Summary

- ▶ An effective action for the black hole horizon, treated as a physical boundary of the spacetime, is derived;
- ▶ The model is capable of explain the charges of the black hole in a natural way;
- ▶ Such a model presumably does not induce loss of information;
- ▶ The relation to a CFT description is not clear so far.