Cosmic desolation: A pedestrian view of the long term Universe

Bing-Lin Young
杨炳麟
Iowa State University

Institute of High Energy Physics November 14, 2013

Contents

I Introduction
II 5 minutes general relativity
III The FLRW metric
IV Cosmic compositions & the ΛCDM
V Distance & time measurements
VI The long term future of the universe
VII Summary and comments

I. Introduction

- The presently known cosmic parameters have established a comprehensive picture of the history of the universe and the future cosmos.
- Some important facts are:
  - There exists a microwave background radiation, largely homogeneous and isotropic, as the cosmic "initial" condition.
  - The universe described by ΛCMD is now dominated by the dark energy and dark matter and has a flat geometry.
- Some of the conspicuous conclusions (see below)
  - The universe causally connected to an observer is finite.
  - An observer has a light cone out of which all distant objects eventually exit, the earlier the sooner. The observer will lose contact with all entities which are not gravitationally bound to it.
  - Earth will continue to receive lights from a limited region of the cosmos, but will go darker as distant lights are reshifted away.

II. 5 minutes general relativity

- Given a 4-dimemsional space-time $x^\mu : x^0 \equiv t, (x^1, x^2, x^3) \equiv \vec{r}$ with the metric tensor $g_{\mu\nu} = g_{\nu\mu}, g^{\mu\lambda} g_{\lambda\nu} = \delta^\mu_\nu$. The metric tensor has vanishing covariant derivatives

$$g^{\mu\nu}_{;\lambda} = g_{\mu\nu;\lambda} = 0$$

- The line element is given by (in Weinberg's notation)

$$ds^2 = -g_{\mu\nu} dx^\mu dx^\nu$$

- The *affine connection* and *Ricci tensor* can be defined

$$\Gamma^\lambda_{\mu\nu} = g^{\lambda\sigma} \left(\frac{\partial g_{\sigma\nu}}{\partial x^\mu} + \frac{\partial g_{\sigma\mu}}{\partial x^\nu} - \frac{\partial g_{\mu\nu}}{\partial x^\sigma} \right) / 2$$

$$R_{\mu\nu} = \frac{\partial \Gamma^\lambda_{\lambda\mu}}{\partial x^\nu} - \frac{\partial \Gamma^\lambda_{\mu\nu}}{\partial x^\lambda} + \Gamma^\lambda_{\mu\sigma} \Gamma^\sigma_{\lambda\nu} - \Gamma^\lambda_{\mu\nu} \Gamma^\sigma_{\lambda\sigma}$$

- They are determined by the geometry of the given space-time. No dynamics are explicitly shown yet.
- The manifested dynamic input is the conserved *energy-momentum tensor* of the system $T^{\mu\nu}$:

$$T^{\mu\nu}_{;\lambda} = \frac{\partial}{\partial x^\nu} T^{\mu\nu} + \Gamma^\mu_{\nu\lambda} T^{\lambda\nu} + \Gamma^\nu_{\nu\lambda} T^{\mu\lambda} = 0$$

The Einstein equation

- The Einstein equation is given by, including a cosmological constant Λ (Einstein constant, vacuum energy) (G_N is the Newtonian gravitational constant),

$$R_{\mu\nu} = -8\pi \mathrm{G}_N (T_{\mu\nu} - \frac{1}{2} T^{\lambda}_{\lambda} g_{\mu\nu}) - \Lambda g_{\mu\nu}$$

- Or, equivalently redefining the energy-momentum tensor,

$$T_{\mu\nu} \to T_{\mu\nu} - \Lambda g_{\mu\nu}$$

  which is still conserved.

- The Einstein equation replaces the Newtonian gravity. From the Einstein equation one can obtain the familiar Newtonian potential equation $\nabla^2 \Phi = 4\pi\rho$, where ρ is the matter density and Φ is the gravitational potential. The latter is embedded in the metric tensor.
- Remark: I follow Weinberg's metric convention.

The interplay of geometry & dynamics

John Wheeler

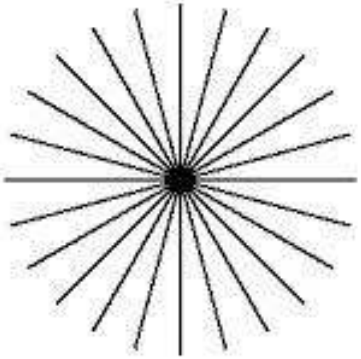
Geons, Black Holes, and Quantum Foam

- Spacetime tells matter how to move; matter tells spacetime how to curve.

III. The FLRW metric

- The Fridmann-Lamaitre-Robert-Watson "0th order" cosmology for a homogeneous and isotropic universe, satisfying the Copernican principle

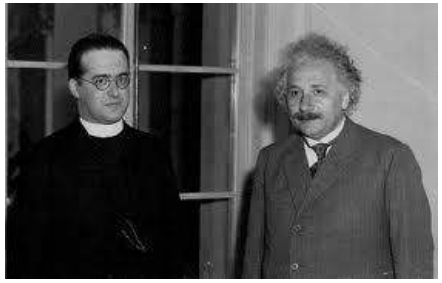
Homogeneous
Not isotropic



Isotropic
Not homogenous

Homogeneous
& Isotropic

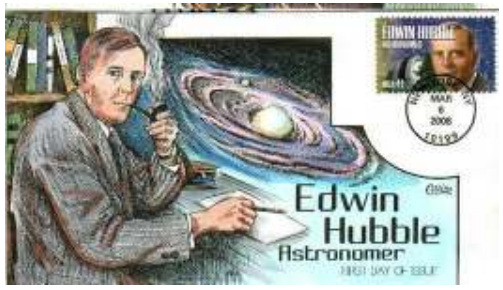
Friedmann
1924



Lamaitre
1927

Robertson
1938

Walker
1936



Hubble 1929
observation

The FLRW metric

- The FLRW metric is given by, in the comoving frame

$$g_{00} = -1 \quad g_{j0} = g_{0j} = 0, \quad g_{jk} = a^2(t)\left(\delta_{jk} + \kappa \frac{x^j x^k}{1-\kappa r^2}\right)$$

$$g^{00} = -1, \quad g^{0j} = g^{j0} = 0, \quad g^{jk} = a^{-2}(t)(\delta_{jk} - \kappa x^j x^k)$$

$a(t)$ is a scale factor, x^j is the *comoving coordinate*, r the *comoving radial coordinate*, and t the *cosmic time*.

- κ is the curvature constant. Since it always appears together with $x^j x^k$, κ can be normalized to:

  +1 (closed spherical), 0 (flat, Euclidean), or
  -1 (open hyperspherical).

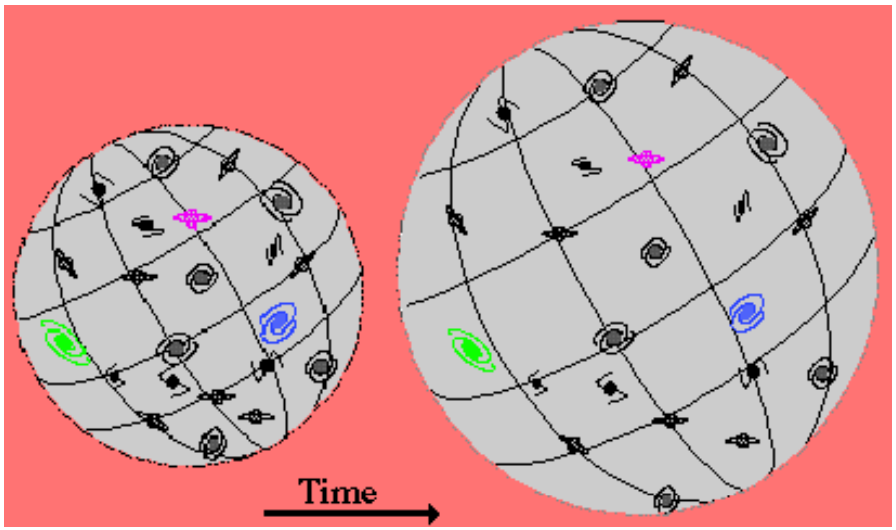
- The FLRW line element:

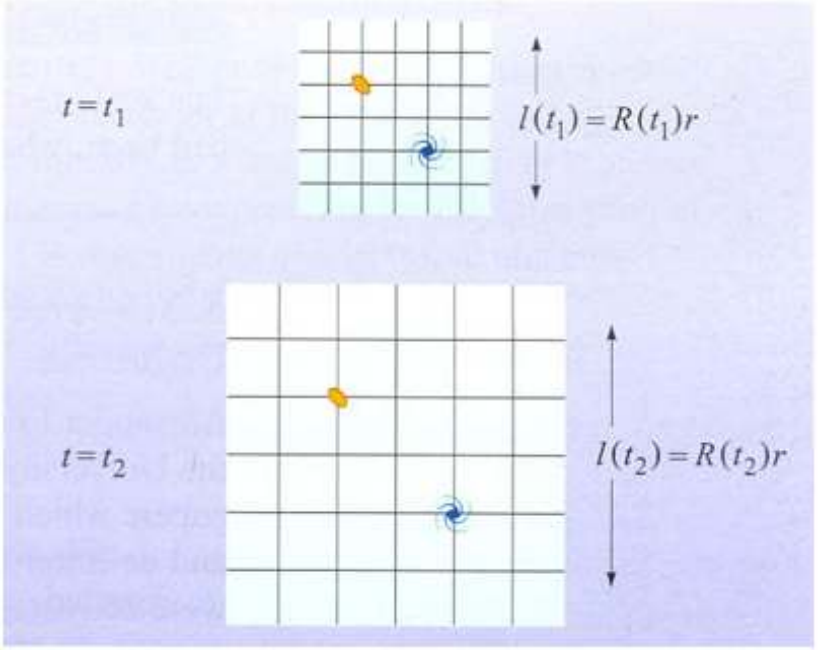
  Rectangular coordinates: $$ds^2 = -g_{\mu\nu}dx^\mu dx^\nu = dt^2 - a^2(t)\left((d\vec{r})^2 + \kappa \frac{(\vec{r}\cdot d\vec{r})^2}{1-\kappa r^2}\right)$$

  Spherical coordinates: $$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-\kappa r^2} + r^2(d\theta^2 + \sin^2\theta d\phi)\right)$$

$$g_{00} = -1, \quad g_{rr} = \frac{a^2(t)}{1-\kappa r^2}, \quad g_{\theta\theta} = a^2(t)r^2, \quad g_{\phi\phi} = a^2(t)r^2\sin^2\theta, \quad other\ g_{\mu\nu} = 0$$

Comoving frame and the expanding universe





- Galaxies and clusters are fixed in the comoving coordinates x^j *or* r, θ, ϕ which are time independent when the peculiar velocity is neglected.
- The expansion of the universe is through the scale factor $a(t)$. To determine the scale factor is an important job in cosmology.

Energy momentum tensor

- Taken as a perfect fluid, the energy-momentum tensor, with a cosmological constant included, is determined by the *energy and momentum densities,* ρ and P ,

$$T_{\mu\nu} = Pg_{\mu\nu} + (P+\rho)u_\mu u_\nu - \frac{\Lambda}{8\pi G_N} g_{\mu\nu}, \qquad u_0 = -u^0 = -1, \quad \mathrm{u}_j = \mathrm{u}^j = 0$$

- Observations indicates that the universe is close to being flat, $\kappa = 0$, which we will assume. This significantly simplifies many calculations.
- Energy-momentum conservation, $T^{\mu\nu}_{;\nu} = 0$, determines the time evolution of the energy and pressure densities as functions of the scale factor:

$$\dot{\rho}(t) + 3\frac{\dot{a}(t)}{a(t)}(\rho(t) + P(t)) = 0, \qquad \dot{P}(t) - \frac{1}{a^3(t)}\frac{\partial}{\partial t}\left(a^3(t)(\rho(t) + P(t))\right) = 0$$

The Hubble expansion rate

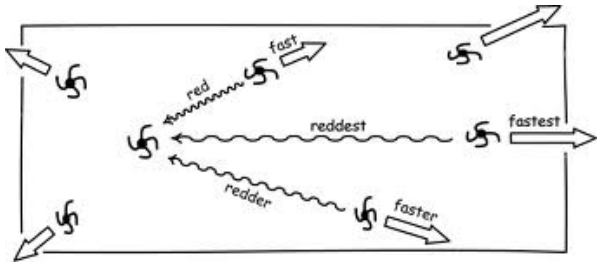
- The Einstein equation gives rise to an *acceleration equation* and a *velocity equation* of the scale factor

$$\frac{\ddot{a}(t)}{a(t)} = -\frac{4\pi G_N}{3}\left(\rho(t) + 3P(t)\right) + \frac{\Lambda}{3}$$

$$H^2(t) \equiv \frac{\dot{a}^2(t)}{a^2(t)} = \frac{8\pi G_N}{3}\left(\rho(t) + \rho_\Lambda\right); \qquad \rho_\Lambda = \frac{\Lambda}{8\pi G_N}$$

$$\rho(t) = \sum_j \rho_j(t) = \rho_R(t) + \rho_M(t) + \rho_\kappa(t), \quad \textit{flat univers} : \rho_\kappa = 0$$

- The velocity equation defined by $H(t)$ is the *Hubble expansion rate*, an ubiquitous quantity in cosmology.
- The acceleration equation shows that if the Einstein constant dominates in the cosmos' energy densities we will have an accelerating universe.
- Hubble expansion: Receding velocity proportional to the distance.



Equation of state

- A thermodynamic system is often characterized by relationships among its variables, call *equations of state*. An example: A system of dilute ideal gas is characterize by the ideal gas equation which relates its volume, pressure. and temperature, $PV = nRT$.
- For the j-th energy-matter density component:

$$P_j = w_j \rho_j; \quad w_j, \text{ equation of state, constant}$$

- Relations among energy density, scale factor, and time

$$\rho_j(t) \sim (\mathrm{a(t)})^{-3(1+w_j)}, \quad a(t) \sim t^{\frac{2}{3(1+w_j)}}$$

$$w_R = 1/3, \quad \rho_R \sim a^{-4}(t), \quad a \sim t^{1/2}$$

$$w_M = 0, \quad \rho_M \sim a^{-3}(t), \quad a \sim t^{2/3}$$

$$w_\Lambda = -1, \quad \rho_\Lambda \sim a^{0}(t), \quad a \sim e^{Ht}$$

w_j	Particle species j
1	Free massless scalar fields
$\frac{1}{3}$	radiation and relativistic matter
0	Dust, i.e., non-relativitic matter
$-\frac{1}{3}$	curvature energy
$-\frac{2}{3}$	Domain walls
-1	cosmological constant

Cosmic redshift

- Distant galaxies are receding from earth with a velocity proportional to the galaxy-earth distance. The light emitted from the galaxies and received by an observer is redshifted with a *redshift value z*.
- As the cosmos' expansion is due to a(t), $a_0 \equiv a(t_0)$

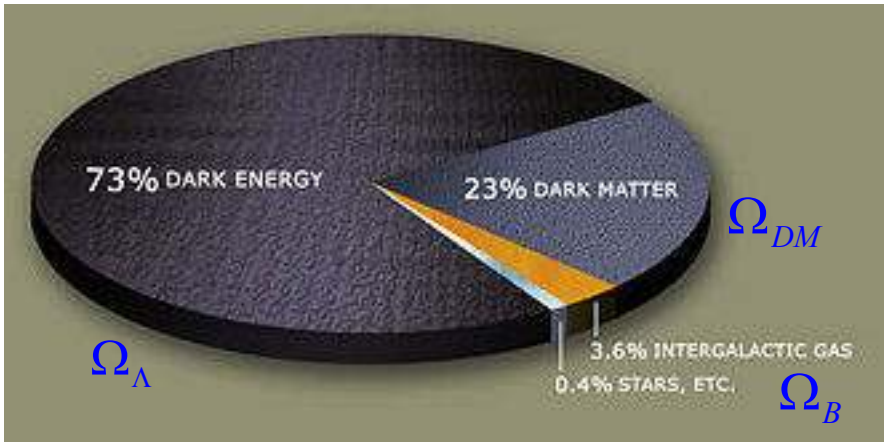
$$a(t) = \frac{a_0}{1+z}, \qquad t_0 = time\ at\ the\ present\ epoch, \quad z_0 = 0$$

$$\nu \to \nu/(1+z), \quad \lambda \to \lambda(1+z)$$

- So we have the cosmic time, the scale factor, and the redshift are all associated with one another.
- The cosmic redshift is not the Doppler redshift defined in special relativity. For galaxies of small Hubble velocity, not far away from earth, the cosmic redshift looks like the Doppler redshift: $z_{Dop} \approx z \approx v_{line\ of\ sight}$

IV. ΛCDM & Cosmic composition

- ΛCDM, the established model of cosmology, is based on the Big Bang and inflationary scenario to jump start the universe and seeding the eventually complicate structure of the presently observed universe.
- The cosmic composition is quite simple:



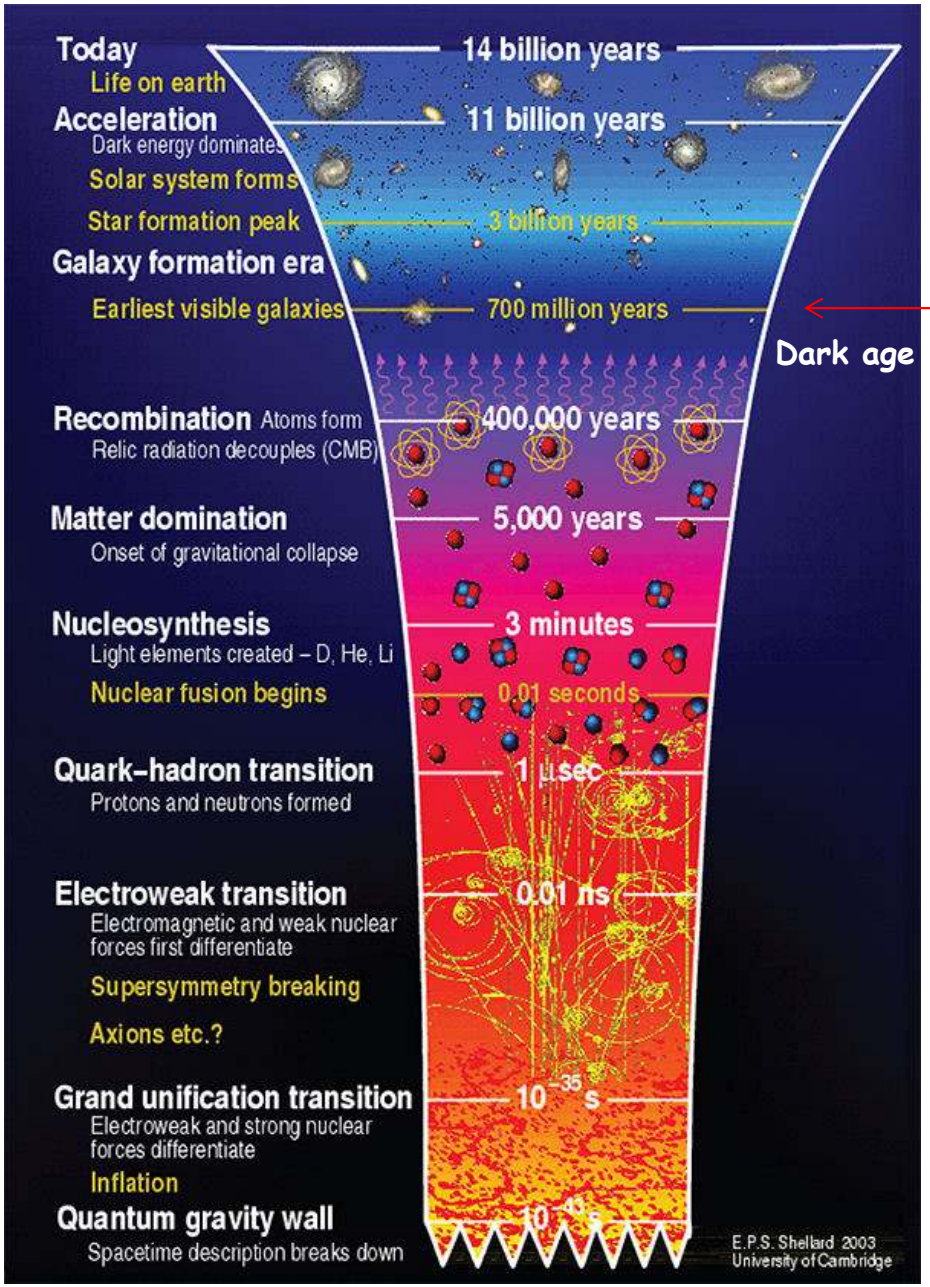
$$\Omega_R \approx 5\times10^{-5}, \quad h = 0.71, \quad \tau_U = 13.75\,\mathrm{Gyr}$$

Note: The recent Planck observation provided a slight different set of parameters:

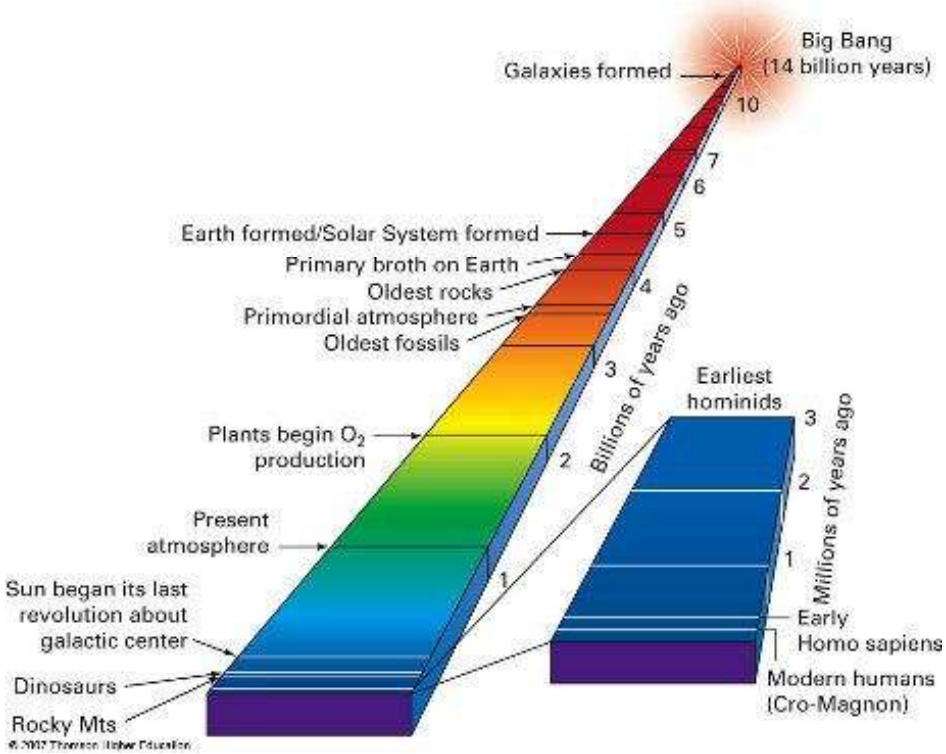
$$\Omega_{\Lambda} = 0.682, \Omega \approx 0.318, \mathrm{h} \approx 0.670$$

$$\tau_U \approx 13.81\,\mathrm{Gyr}$$

Outline of Cosmic History



Farthest galaxies confirmed at 13.1 Gyr away z=7.51 (10/2013).



V. Distance and time measures

- Because of expansion, distances between galaxies are ever increasing. Hence distance measures are complicated and can be confusing, although the flat geometry reduces the complication.
- Time and distance measures in cosmology are defined in terms of electromagnetic signals, following a null path along constant angular variables ($d\Omega = 0$) relates time with distance: ($\kappa = 0$)

$$ds^2 = 0 = c^2 dt^2 - a^2(t) dr^2, \qquad \frac{cdt}{a(t)} = dr \,(\text{conformal time})$$

- Since comoving coordinates are time independent, the 2nd expression defines the time measure in terms of a distance measure, and vice versa.

Time, distance, and redshift

- A useful relationship: relating the time variable to the scale factor and the Hubble expansion rate:

$$dt = \frac{da}{\dot{a}} = \frac{da}{aH} = \frac{da}{aH_0(H/H_0)}$$

- H_0 : the present Hubble expansion rate, a measurable quantity, defined by the normalized Hubble constant "h"

$$H_0 = h(9.77775Gyr)^{-1} = 100\,h\,km\,s^{-1}\,\mathrm{Mpc}^{-1} = 3.3356h\times10^{-4}\,\mathrm{Mpc}^{-1} = 3.2408h\times10^{-18}\,s^{-1}$$

- Related to redshift $(\kappa = 0)$

$$\left(\frac{H(t)}{H_0}\right)^2 = \frac{1}{\rho_0}\left(\rho_\Lambda(t)+\rho_M(t)+\rho_R(t)\right), \qquad H_0{}^2 = \frac{8\pi G_N}{3}\rho_0$$

$$= \Omega_\Lambda + \Omega_M(1+z)^3 + \Omega_R(1+z)^4, \qquad \frac{\rho_j(t)}{\rho_0} \equiv \Omega_j(1+z)^{3(1+w_j)}$$

- Change variable to redshift $(\kappa = 0)$

$$dt = \frac{\tau_H}{1+z}\frac{dz}{\tilde{E}(z)}, \qquad \tilde{E}(z) = \sqrt{\Omega_\Lambda + \Omega_M(1+z)^3 + \Omega_R(1+z)^4} \qquad \tau_H = \frac{1}{H_0}$$

$$dr = \frac{cdt}{a(t)} = \frac{D_H}{a_0}\frac{dz}{\tilde{E}(z)}, \qquad \mathrm{a(t)}=\frac{a_0}{1+z}, \qquad D_H = \frac{c}{H_0}$$

Time and distance measures

- Time measures: *Hubble time*, *lookback time*, and *age of the universe*.
- Distance measures: *Hubble length*, *proper distance*, *comoving distance*, *particle horizon*, *angular diameter distance*, *comoving angular diameter distance*, *luminosity distance*, ***and cosmic event horizon***.
- For a flat universe some of these distance measures are the same.
- Some relevant historical milestones:
  - Radiation-matter equality, at 50,000 yr, z=3500
  - Last scattering surface, at 0.38 Myr, z=1100
  - Star and galaxy formation, ~a few hundred Myr, z~O(10)
  - Λ–radiation equality, at 540 Myr, z=9
  - Λ–matter equality, at 9.2 Gyr, z=0.42

Summary of time & distance measurements

Distance/Time		Expession (Current value)[a]	Flat universe $\kappa = 0$
Hubble time	$\tau_H = \frac{1}{H_0}$	(13.77 Byr/14.53 Byr)[b]	
Hubble distance	$D_H = \frac{c}{H_0} = c\tau_H$	(4.222 Gpc/4.455 Gpc)[b]	
Hubble expansion rate	$H(z) = H_0\tilde{E}_z$	$H_0\sqrt{\Omega_\Lambda + \Omega_R z_+^4 + \Omega_M z_+^3 + \Omega_\kappa z_+^2}$[c]	$H_0\tilde{E}_z$
Comoving radial coordinate	$\chi_1 = \frac{1}{\sqrt{\kappa}}\sin^{-1}(\sqrt{\kappa}r_1)$[d]	$\int_0^{r_1}\left(dr/(1-\kappa r^2)\right) = c\int^{t_1}(dt/a(t))$ $= \frac{D_H}{a_0}\int_0^{z_1}(dz'/\tilde{E}_{z'})$	$\frac{D_H}{a_0}\int_0^{z_1}(dz'/\tilde{E}_{z'})$
	$r_1 = \frac{1}{\sqrt{\kappa}}\sin(\sqrt{\kappa}\chi_1)$	$\frac{1}{\sqrt{\kappa}}\sin\left(\sqrt{\kappa}\frac{D_H}{a_0}\int_0^{z_1}(dz'/\tilde{E}_{z'})\right)$	$\frac{D_H}{a_0}\int_0^{z_1}(dz'/\tilde{E}_{z'})$
Look back time	$\Delta t_{\rm lb} = \int_{t_1}^{t_0} dt$	$\tau_H\int_0^{z_1}(dz'/((1+z')\tilde{E}'_z))$	$\tau_H\int_0^{z_1}(dz'/((1+z')\tilde{E}'_z))$
Age of universe	$\tau_U = \Delta_{\rm lb}\vert_{t_1=0}$	$\tau_H\int_0^{\infty}(dz'/((1+z')\tilde{E}_{z'}))$	$(0.992\tau_H/0.948\tau_H)$ (13.66 Byr/13.83 Byr)
Proper distance	$d_{\rm p}(t,r_1) = a(t)\chi_1$	$a(t)\frac{1}{\sqrt{\kappa}}\sin^{-1}(\sqrt{\kappa}r_1) = \frac{D_H}{1+z}\int_0^{z_1}(dz'/\tilde{E}_{z'})$	$\frac{D_H}{1+z_1}\int_0^{z_1}(dz'/\tilde{E}_{z'})$
Comoving distance	$d_{\rm c}(r_1) = a(t_0)\chi_1$ $= d_{\rm p}(t_0,r_1) = c\Delta t_{\rm lb}$	$D_H\int_0^{z_1}(dz/\tilde{E}(z))$	$D_H\int_0^{z_1}(dz'/\tilde{E}'_z)$
Particle horizon	$d_{\rm ph} = d_{\rm c}(r_{\rm max})$	$D_H\int_0^{\infty}(dz'/\tilde{E}_{z'})$	$(3.383D_H/3.171D_H)$ (14.28/14.19) Gpc
Angular diameter distance	$d_{\rm A}(r_1) \equiv \frac{d_{\rm tr}}{\delta\theta} = a(t_1)r_1$	$\frac{a_0}{1+z_1}\frac{1}{\sqrt{\kappa}}\sin\left(\sqrt{\kappa}\frac{D_H}{a_0}\int_0^{z_1}(dz'/\tilde{E}_{z'})\right)$	$\frac{D_H}{1+z_1}\int_0^{z_1}(dz'/\tilde{E}_{z'})$
Comoving angular diameter	$d_{\rm cA}(r_1) = a_0 r_1$	$(1+z_1)d_{\rm A}(r_1)$	$D_H\int_0^{z_1}(dz'/\tilde{E}_{z'})$
Luminosity distance	$d_{\rm L}(r_1) = (1+z_1)d_A(r_1)$	$(1+z_1)d_{cA}(r_1) = (1+z_1)^2 d_{\rm A}(r_1)$	$(1+z_1)D_H\int_0^{z_1}(dz'/\tilde{E}_{z'})$
Cosmic event horizon			

Hubble time τ_H and *Hubble length* D_H

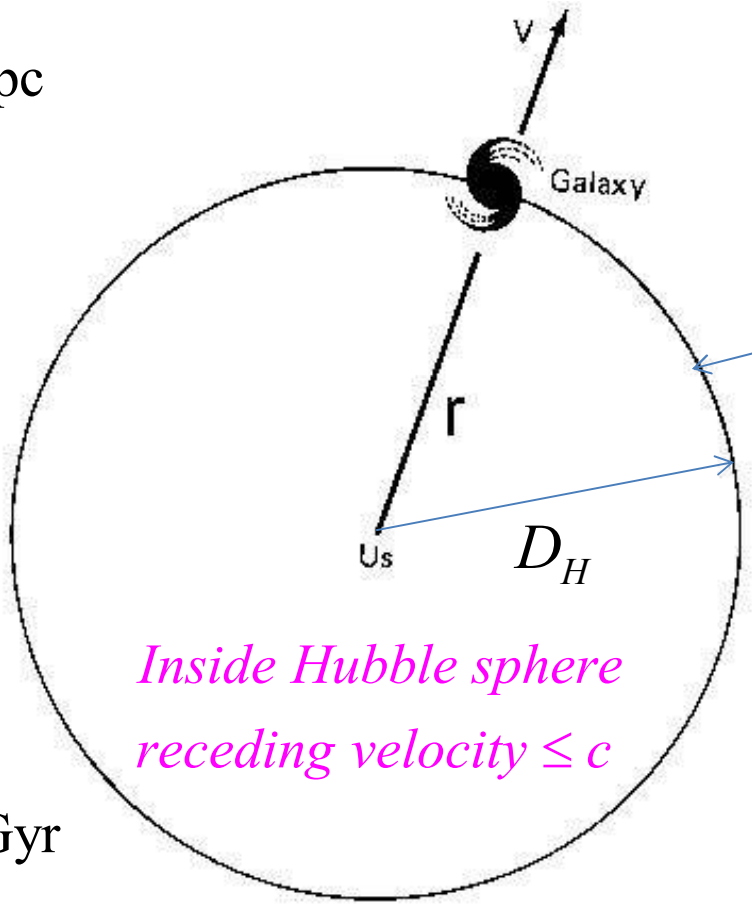
- A natural time scale is given by the present Hubble expansion rate $\tau_H = H_0^{-1} = 13.77\,\mathrm{Gyr}, \quad \text{for h=0.71}$.
- A natural distance scale is given by the present Hubble length $D_H \equiv c\tau_H = 4.22\,\mathrm{Gpc}, \text{ for h=0.71,}.\ \ 1\mathrm{pc}=3.086\times10^{16}\ \mathrm{m}$
- Meaning of the Hubble length: Galaxies at a proper distance D_H from an observer on earth will have receding velocity equals to c. If a sphere of radius D_H is drawn around an observer, all galaxies outside the sphere will have receding velocity $\geq c$, while galaxies inside have receding velocity $\leq c$.
- Objects with receding velocity $\geq c$ does not violate the general principle of special relativity, since this velocity due to the scale factor is part of the geometry of the space-time of general relativity.

Cartoon of Hubble length

$D_H \equiv c\tau_H = 4.22\,\mathrm{Gpc}$

for h=0.71

$1\mathrm{pc}=3.086\times10^{16}\,\mathrm{m}$



The Hubble sphere: Having a radius equaling to the Hubble length

Inside Hubble sphere

receding velocity $\leq c$

Outside Hubble sphere

receding velocity $\geq c$

$\tau_H = H_0^{-1} = 13.77\,\mathrm{Gyr}$

for h=0.71

Lookback time & age of the universe

- Consider an EM signal emitted at t_1 from a star located at the comoving radial coordinate r_1 of redshift z_1, and reaches the observer (earth) at the present epoch t_0. The time interval $\Delta t_1 = t_0 - t_1$ is known as the lookback time defined by:

$$\Delta t_1 \equiv t_0 - t_1 = \int_{t_1}^{t_0} dt = \tau_H \int_0^{z_1} \frac{1}{1+z} \frac{dz}{\tilde{E}(z)}$$

- The age of the universe is obtained from the lookback time for $t_1 = 0 \ or \ z_1 \to \infty$:

$$\tau_U = t_0 = \tau_H \int_0^{\infty} \frac{dz}{\tilde{E}(z)} = 0.992\tau_H \approx 13.67 \ Gyr,$$

using the cosmic parameters: $\Omega_\Lambda = 0.73, \ \Omega_M = 0.27, \ \Omega_R \approx 0$ and $h = 0.71$. The PDB gives $\tau_U^{(obs)} = 13.69 \pm 0.13 \ Gyr.$

Some relevant distance measures

- *Proper distance*: The distance between a galaxy and the observer measured along a null path at a constant cosmic time t (of reshift z). The observer is at the origin of the comoving frame and the signal emitter at the comoving radial coordinate r_1. The measurement has $dt = 0$. The FLRW line element becomes $ds_{//} = \sqrt{g_{rr}}dr = a(t)dr$. Then

$$d_p(t, r_1) = \int ds_{//} = a(t)\int_0^{r_1} dr = a(t)r_1 = \frac{D_H}{1+z}\int_0^{z_1} \frac{dz'}{\tilde{E}(z')}$$

- This measurement cannot be made physically.
- We can obtain the Hubble's law: $\dot{d}_p(t_0, r_1) = H(t_0)d_p(t_0, r_1)$
- Let the receding velocity be the light speed: $c = H(t_0)d_p(t_0, r_1')$,

$$d_p(t_0, r_1') = \frac{c}{H_0} = D_H$$

- The *particle horizon*, the maximal visible cosmos is given by

$$d_{ph} = d_p(t_0, r_{max}) = D_H\int_0^{\infty} \frac{dz}{\tilde{E}(z)} = 3.38D_H = 14.28\,Gpc = 4.41\times10^{26}\,m$$

Luminosity distance & apparent flux

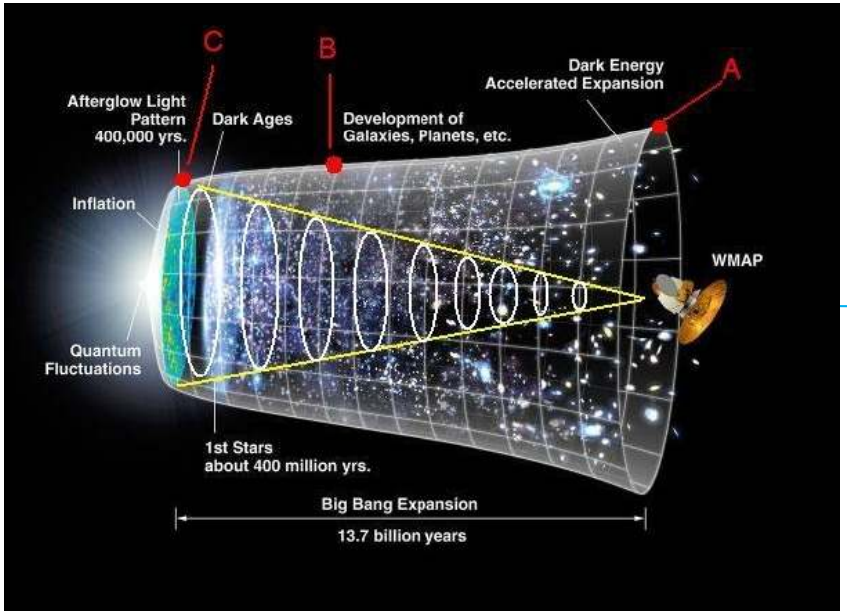
- Let the true luminosity a galaxy of reshift z_1 be L . The flux received by an observer is called the apparent flux defined by: $f_a = L/(4\pi d_L^2), \quad d_L$ luminosity distance
- Three factors influence the flux received by an observer:
  - Hubble expansion stretches the light wavelength, and therefore reduces the energy, by a factor $1+z_1$.
  - Since flux is energy per unit time, the stretched time also suppress the flux by a factor $1+z_1$.
  - The Distance between the galaxy and the observer.
- The luminosity distance is the distance entering the regular flux equation increased by a factor $1+z$. The apparently flux is the regular flux reduced by $(1+z_1)^{-2}$.

VI. Cosmic event horizon
Looking into the future

- Since the age of 9.2 Gyr or about 4.6 Gyr ago, when dark energy began to dominate over matter and radiation, the universe re-entered a phase of accelerated expansion. The scale factor a(t) becomes more and more de Sitter like, in the form of exponential expansion $a(t) \sim e^{Ht}$.
- This has a simple consequence: galaxies and clusters which are not gravity bound will separate faster and faster as time evolves.
- Presently the particle horizon grows as increasing parts of the universe become visible and in causal contact with earth observers. In the meantime, distant objects also get farther and farther apart. If you are an astronomer you may ask "What will happen to my visible universe for research, eventually?"

The competition:
Enlarged light cone vs increased separation

- A cartoon illustration of the problem:
- Time axis: the past light cone stretches to the left. Spatial axis: $a(t)r$.



t

$a(t)r$

t

- The observer's past light-cone is enlarged with increasing time. The spatial axis is enlarged by the scale factor a(t).
- Who wins eventually?

The *cosmic event horizon* (CEH)

- The answer is in the *cosmological event horizon*: the largest distance from which light signal emitted can ever reach an (earth) observer. Outside the event horizon, no causal contact with an earth observer is ever possible.
- Let the observer be the origin of the comoving frame, the radial coord. of the signal emitter be r_1 , and the signal be emitted at t_{em} and received at t_{ob} . The cosmic event horizon for a given t_{em} , denoted by $d_{eh}(t_{em})$, is obtained by taking $t_{ob} \to \infty$.
- From the null path of the EM signal, $ds^2 = 0 = c^2 dt^2 - a^2(t) dr^2$, we have

$$r_1(t_{ob}, t_{em}) = c\int_{t_{em}}^{t_{ob}} \frac{dt}{a(t)} \quad \Rightarrow \quad d_{eh}(t_{em}) = a_0 r_1(t_{em}, t_{ob} \to \infty,) = a_0 c\int_{t_{em}}^{\infty} \frac{dt}{a(t)}$$

Finite or infinite CEH?

- If the integral diverges, $d_{eh}(t_{emi}) \to \infty$, all parts of the universe can be reached by the (earth) observer. The event horizon is infinite.
- If the integral converges, $d_{eh}(t_{emi})$ is finite and all objects lying beyond it can never be in causal contact with the observer. Summarizing the conditions:

$$\frac{a(t)}{t}\Big|_{t\to\infty} \begin{cases} = 0 & \text{integral diverges, event horizon infinite} \\ \neq 0 & \text{integral converges, event horizon finity} \end{cases}$$

- Whether or not there is a finite event horizon depends on the cosmological model. If the universe is matter dominated $a(t) \sim t^{2/3}$, or radiation dominated $a(t) \sim t^{1/2}$, all parts of the cosmos can in principle become observable from earth. For dark energy domination, there is an finite event horizon. The observable cosmos is limited!

Calculation of CEH

- Look into the future, based on measures at the present time $t = t_0$. An explicit expression for a(t) can be derived, using an approximation valid after the dark age when observations on galaxies can be made. We define a rescaled scale factor $\tilde{a} \equiv a/a_0$. Either t or $\tilde{a}$ can be used as the independent variable.
- From $\rho_j = \rho_{j0}\tilde{a}^{-3(1+w_j)}$ ($\kappa = 0$).

$$H^2 / H_0^2 = \Omega_\Lambda + \Omega_M \tilde{a}^{-3} + \Omega_R \tilde{a}^{-4}, \qquad \text{dropping the vrery small } \Omega_R$$

$$dt = \frac{da}{\dot{a}} = \frac{da}{aH} = \frac{da}{aH_0(H/H_0)} \approx \tau_H \frac{\tilde{a}d\tilde{a}}{\sqrt{\Omega_\Lambda \tilde{a}^4 + \Omega_M \tilde{a}}}$$

- This is integrable and the result is valid for $t > t_0$

$$\tilde{a}(t) = \left(\frac{\Omega_M}{\Omega_\Lambda}\right)^{1/3} \sinh^{2/3}\left(\frac{3\sqrt{\Omega_\Lambda}}{2}\frac{t}{\tau_H}\right), \qquad \Omega_\Lambda + \Omega_M = 1$$

Check the solution

- Initial condition: $\tilde{a}(t=0)=0$, OK.
- Age of the universe: $\tilde{a}(t=t_0)=1$,

$$t_0 = \frac{2}{3\sqrt{\Omega_\Lambda}}\tau_H \sinh^{-1}\left(\sqrt{\frac{\Omega_\Lambda}{\Omega_M}}\right)$$

giving $t_0 = 13.67 (=\tau_U)$ Gyr, for $\Omega_\Lambda = 0.73$, h=0.71 , OK.

- The form of $\tilde{a}(t)$ extrapolates from the past not too close to the Big Bang (say, z < 20) to the distant future. We can write the time in the past ($t \le t_0$) in terms of z

$$t(\le t_0) = \frac{2}{3\sqrt{\Omega_\Lambda}}\tau_H \sinh^{-1}\left(\sqrt{\frac{\Omega_\Lambda}{\Omega_M}}(1+z)^{-3/2}\right), \quad \text{for z} \le \text{a few tens}$$

- For large t, $t \gg \tau_H$, giving an exponential expansion:

$$\tilde{a}(t) = \left(\frac{\Omega_M}{\Omega_\Lambda}\right)^{1/3} \cdot \exp\left(\frac{3}{2}\sqrt{\Omega_\Lambda}\,\frac{t}{\tau_H}\right)$$

CEH

- Rewrite CEH for a signal emitted at $t_{\rm em} \le t_0$:

$$d_{\rm eh}(t_{\rm em}) = c\int_{t_{\rm em}}^{\infty} \frac{dt}{\tilde{a}(t)} = \frac{2}{3}\left(\frac{1}{\Omega_\Lambda \Omega_M^2}\right)^{1/6} \int_{y_{\rm em}}^{\infty} \frac{dy}{(\sinh(y))^{2/3}}$$

$$y_{\rm em} \equiv \frac{3}{2}\sqrt{\Omega_\Lambda}\,\frac{t_{\rm em}}{\tau_H} = \sinh^{-1}\left(\sqrt{\frac{\Omega_\Lambda}{\Omega_M (1+z_{\rm em})^3}}\right)$$

- The overall event horizon is obtained for $t_{\rm em} = 0$,

$$d_{\rm eh}(0) \approx \frac{2}{3} D_H (\Omega_\Lambda \Omega_M^2)^{-1/6} \int_0^{\infty} \frac{dy}{\sinh^{2/3}(y)} = 4.62 D_H = 19.5\ \text{Gpc}$$

$$= 63.6\ Gyr = 1.42 \times 10^{26}\ m,$$

This is approximate, integrand incorrect near the lower limit of the integration. But the error is expected to be small. So the maximal size of the portion of the cosmos causally related to us is about 20 Gpc and the latest light we receive is emitted when the universe is about 64 Myr.

Fate of all objects currently visible

- Question: If we set up a program to monitor all the presently visible galaxies, how long can we study each of the galaxies? Or, what will eventually happen to the night sky?
- Each visible galaxy has a redshift called z_1. For visible stars z_1 is less than a few tens, so our approximate expression of the Hubble scale factor is valid.
- The cosmic age when the light is emitted is then

$$\tau_{z_1}(\le t_0) = \frac{2}{3\sqrt{\Omega_\Lambda}}\tau_H \sinh^{-1}\left(\sqrt{\frac{\Omega_\Lambda}{\Omega_M}}(1+z_1)^{-3/2}\right), \quad \text{for z} \le \text{a few tens}$$

- The comoving radial coord. of the galaxy is

$$r_1 = c\int_{t_1=\tau_1}^{t_0}\frac{dt}{a(t)} = \frac{c}{a_0 H_0}\int_0^{z_1}\frac{dz}{\tilde{E}(z)}$$

Solve for CEH

- We can calculate the latest time of light emission that can reach an earth observer. Call it t_{Lem}. Define $\hat{t} = t / \tau_H$

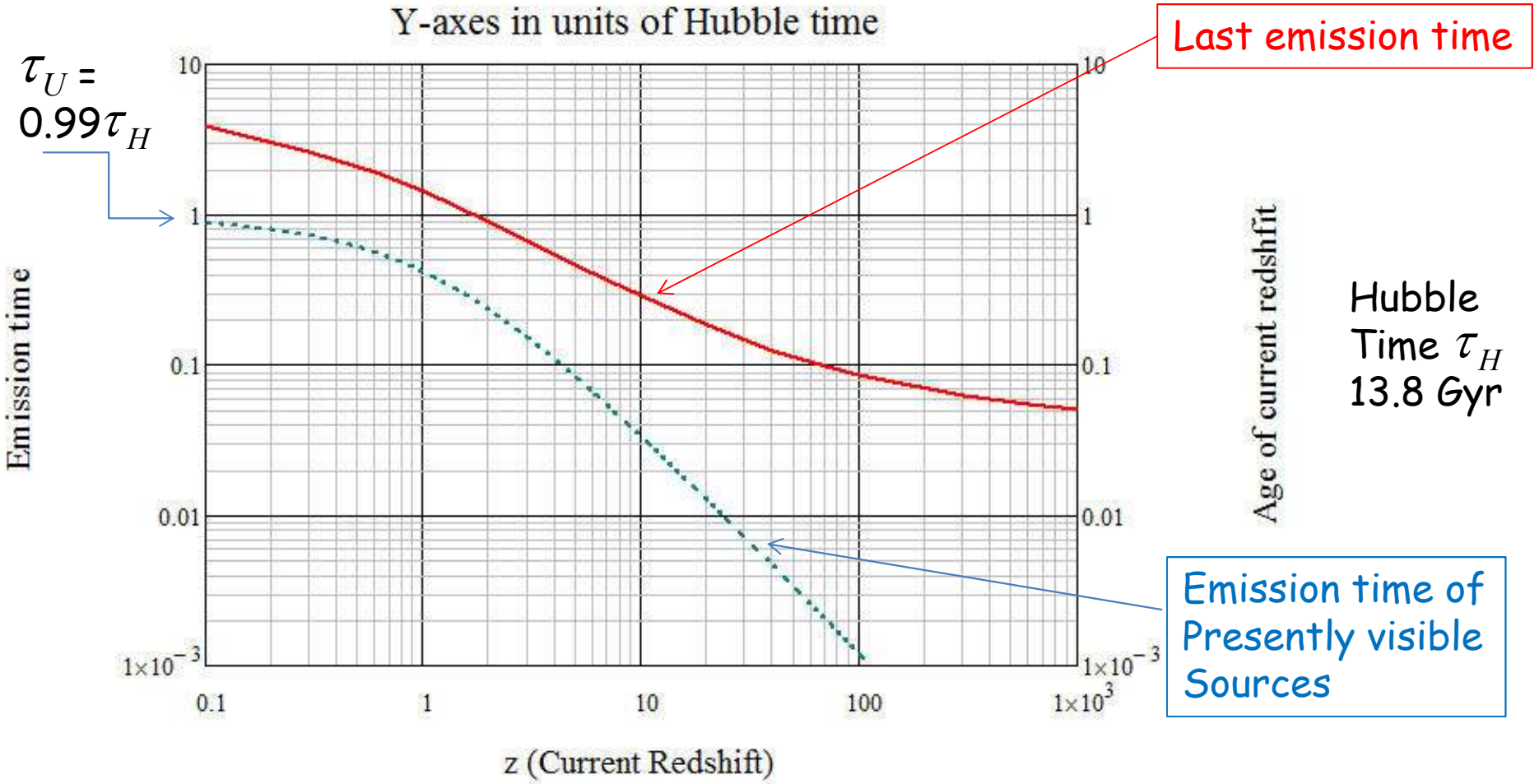
$$\int_0^{z_1} \frac{dz}{\tilde{E}(z)} = \int_{\hat{t}_{\text{Lem}}(z_1)}^{\infty} \frac{d\hat{t}}{\tilde{a}(\hat{t})} = \left(\frac{\Omega_\Lambda}{\Omega_M}\right)^{1/3} \int_{\hat{t}_{\text{Lem}}(z_1)}^{\infty} \frac{d\hat{t}}{(\sinh((3\sqrt{\Omega_\Lambda}/2)\hat{t})^{2/3}}$$

- Given z_1 we can solve for $\hat{t}_{\text{Lem}}$ and $t_{\text{Lem}} = \hat{t}_{\text{Lem}} \tau_H$. Some results

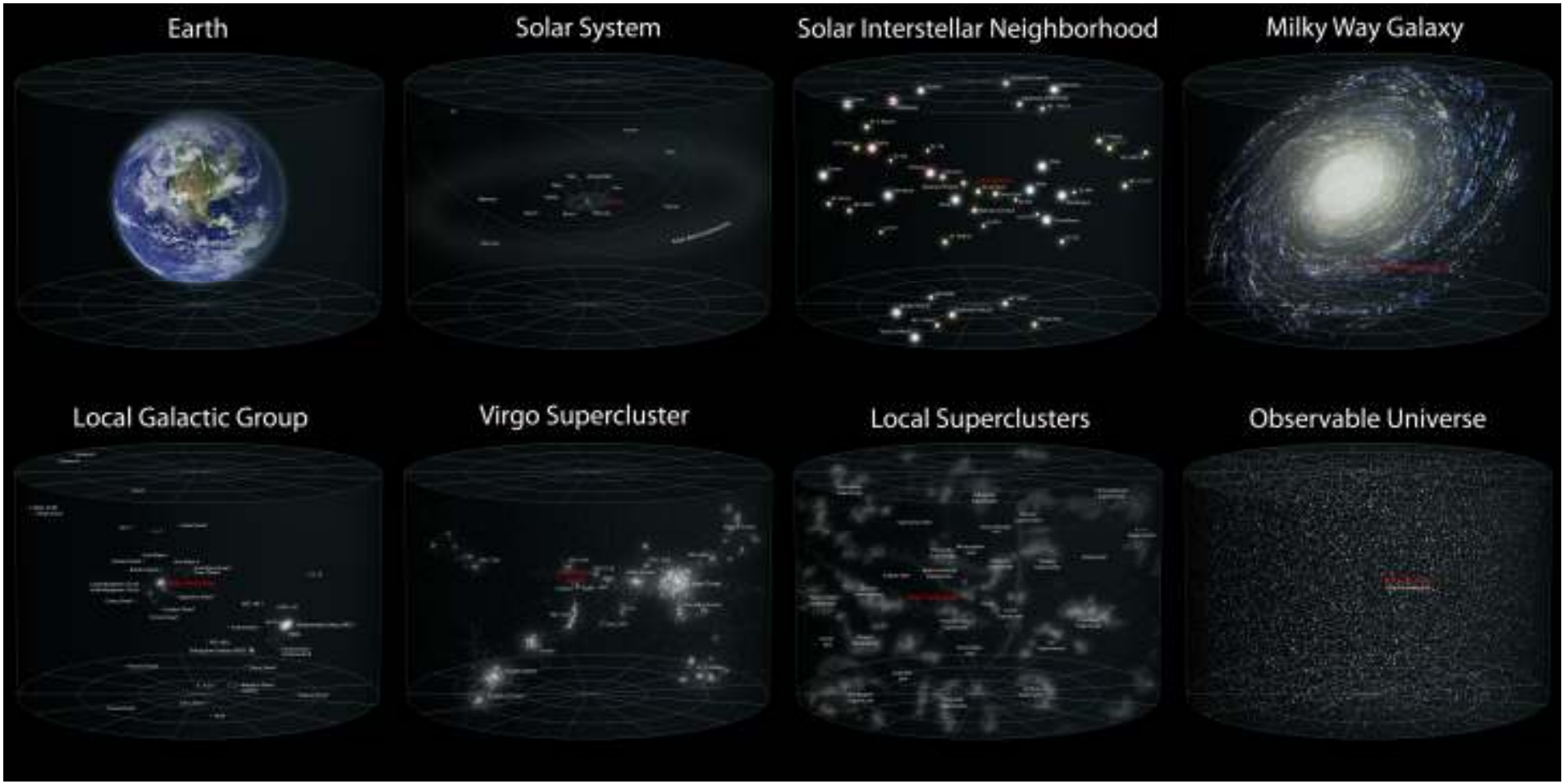
z_1	0.1	1	5	7.51	10	11.9
$t_1 = \tau_1$	$12\,Gyr$	$5.5\,Gyr$	$1.1\,Gyr$	$700\,Myr$	$450\,Myr$	$340\,Myr$
t_{Lem}	$55\,Gyr$	$17\,Gyr$	6.4	$4.8\,Gyr$	$4.1\,Gyr$	$3.5\,Gyr$

- Meaning: Take z=7.51, the recently confirmed oldest galaxy. The light seen by us was emitted when the cosmos was 700 Myr old. We can only receive its light signals being emitted when the universe is no more thaan 4.8 Gyr old. Lights emitted afterward will never reach us. We will not have any knowledge of it beyond that age.

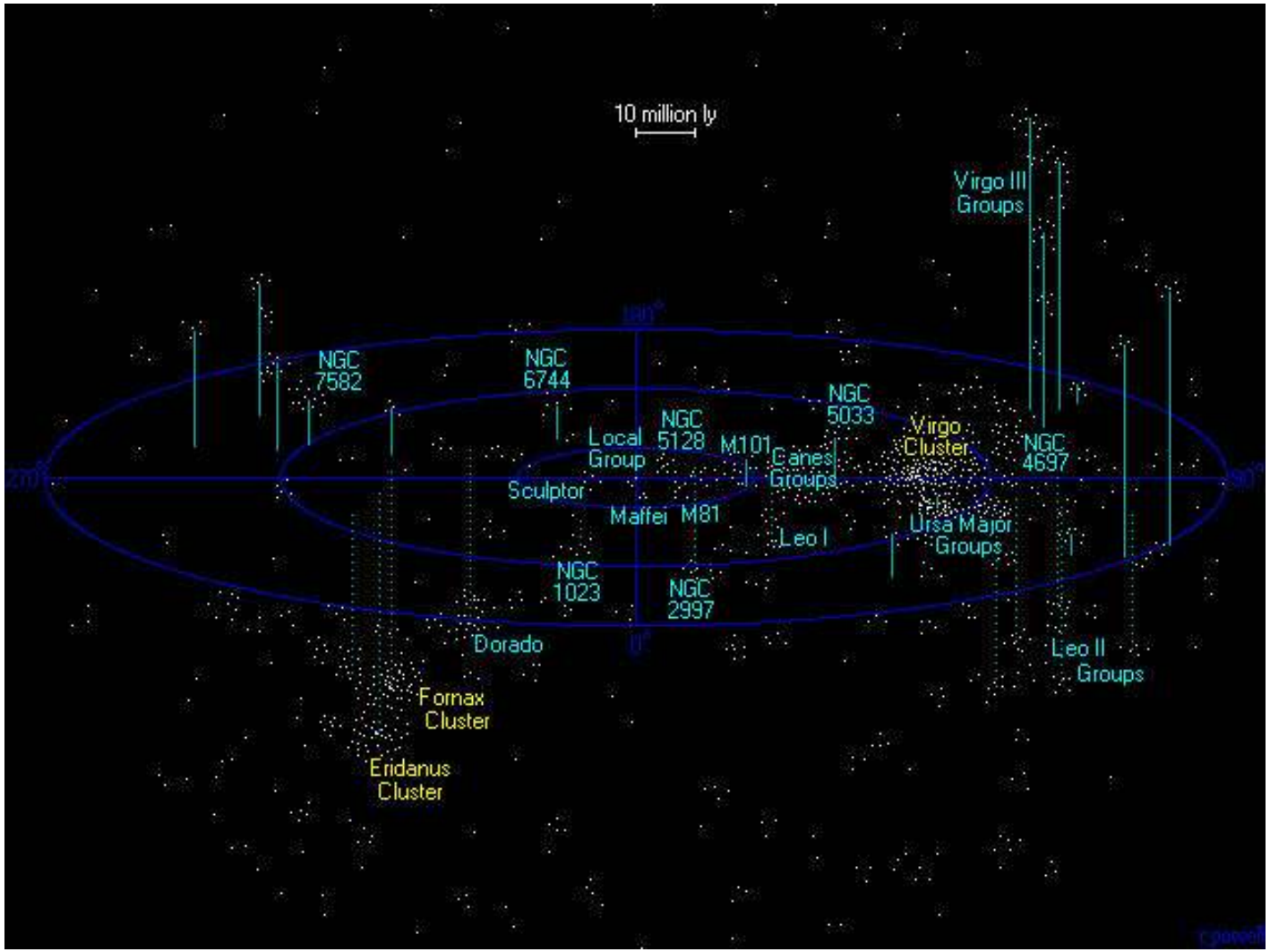
Emission time of presently visible light sources and their last emissions that can reach an earth observer



Eventually visible stars-gravitationally bound to us: parts of the local superclusters (Loeb (2002)). The presently visible universe: millions of superclusters

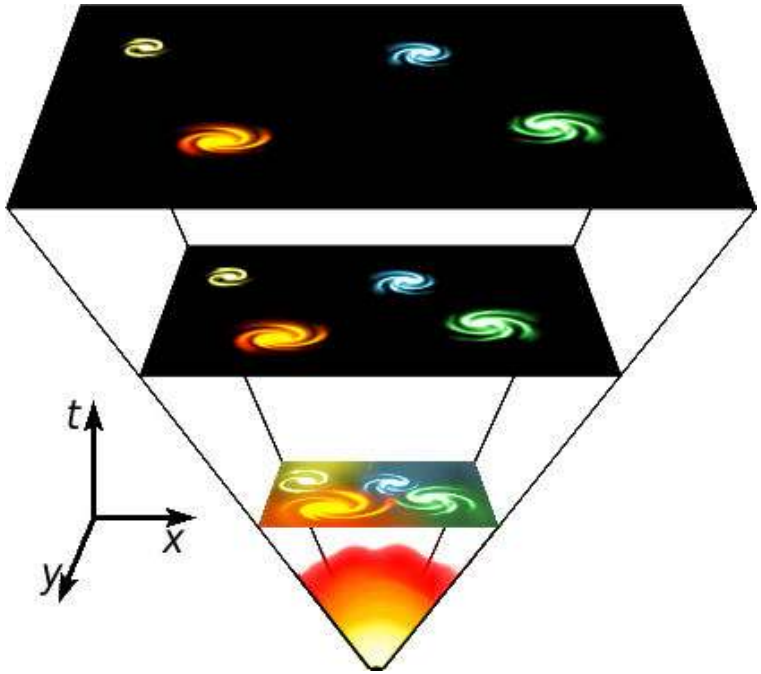


Virgo supercluster

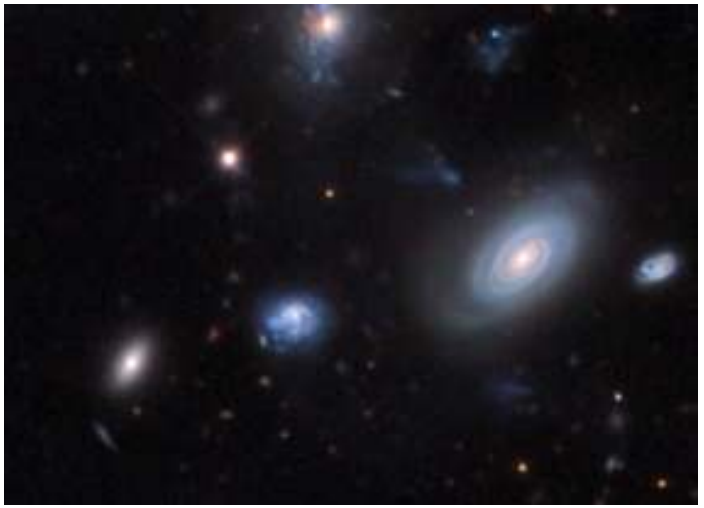


The Virgo Supercluster, 1 millionth of the visible cosmos, is made of about 1000 members, which is the only Part of the universe eventually visible to us.

Exponential expansion



- From a busy cosmic galactic neighborhood to a lone supercluster



VII. Summary & comments

Summary

- There is a finite region of the universe, a sphere of about 64 Gyr in radius, that is ever visible to an observer located at any part of the cosmos.
- Most of the visible galaxies presently will move out of our light cone after they grow to a certain age, in the sense that the signal they emitted after that specific age will never be able to reach us.
- In principle we will continue to receive from them lights emitted before that age. These lights will be redshifted in arriving time, wavelength, and intensity. But their angular size in the sky will be frozen to a constant value.
- If the Einstein constant does not change in about ~100 Gyr, eventually we will only be able to see galaxies in our own supercluster.

Comments

- The scenario of the long term view of the universe presented here depends critically on the assumption that the dark energy is in the form of an Einstein constant. If Λ vanishes soon enough, say within tens Gyr, the exponential expansion phase can be avoided afterward. This can happen in the inflationary scenario of quintessence of rolling scalar field.
- Predictions so far into the future, billions years down the road, carries no warrantee. In fact, past records of very long term predictions of anything, particularly things in basic science, have not been good. Surprises likely show up unexpectedly.
- But one thing which may be certain is the continuing philosophical discussions:

Who creates the universe?

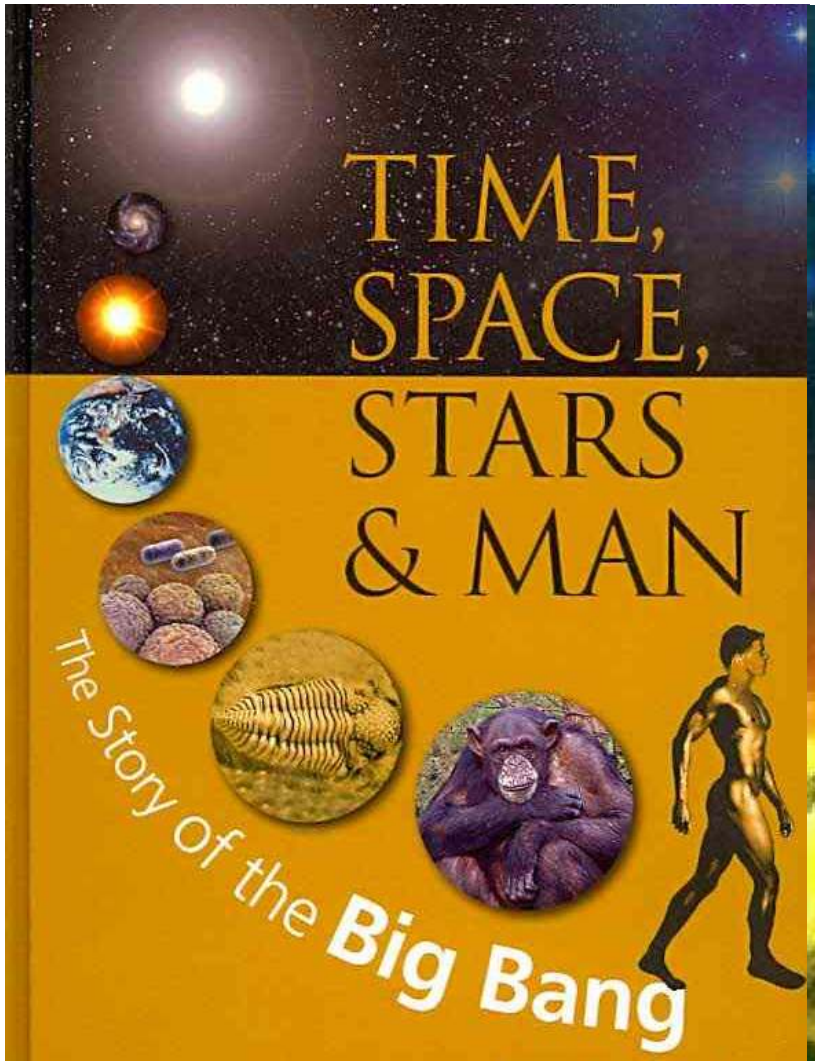
God did not create the universe, gravity did, says Stephen Hawking.

"***The Grand Design***" co-authored by Stephen Hawking & Leonard Mlodinow

- *"Because there is a law such as gravity, the universe can and will create itself from nothing. Spontaneous creation is the reason there is something rather than nothing, why the universe exists, why we exist"*

Pope Benedict XVI, 2008 at the Pontifical Academy

Time, space, stars, man & the future



Some sample references

- Loeb, astro-ph/0107568, nice discussions but very little details.
- Krauss & Starkman, astro-ph/9902189, on humanities in an Einstein constant dominated universe.
- Starobinsky, astro-ph/9912054, warning about the uncertainties in long term predictions.
- Riedman, Kaiser, & Gallicchio, astro-ph/1305.3943, more sophistic discussions on long term astronomy.

谢谢