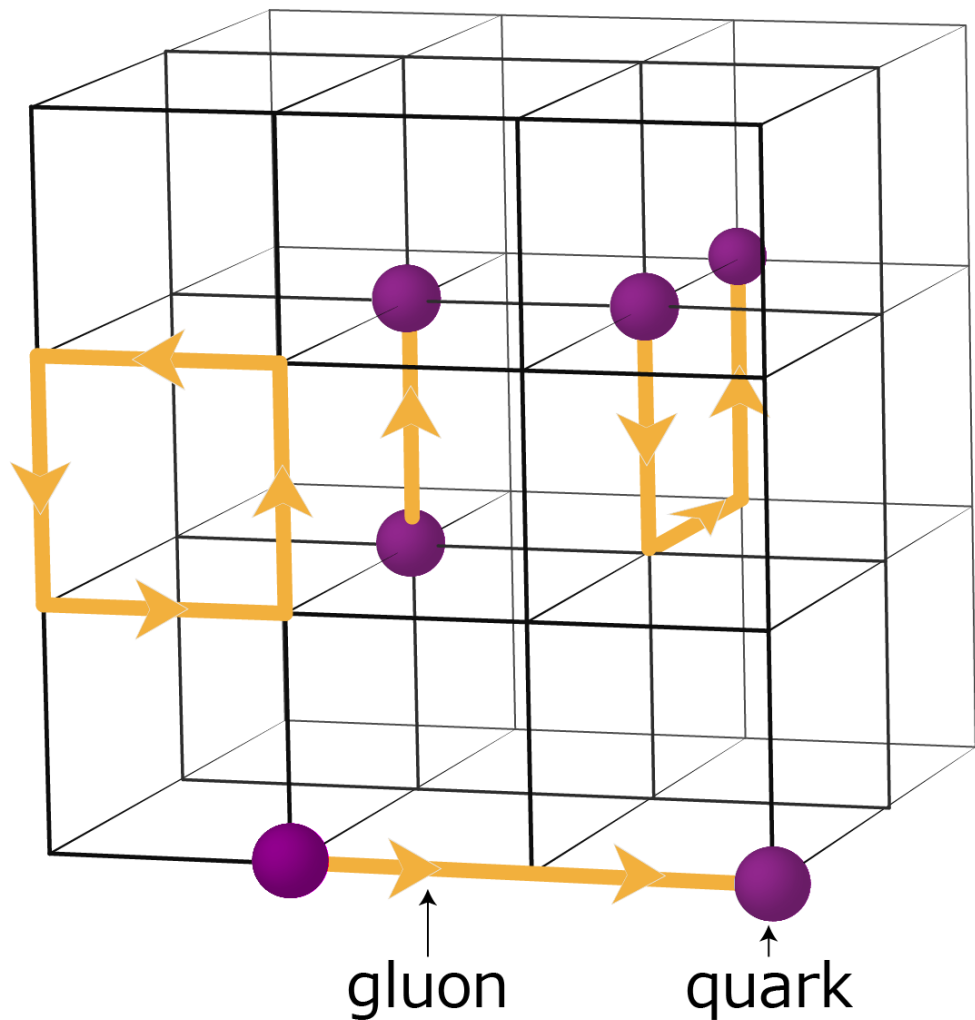


粲偶素相关性 质的格点研究

张仁强





1 蒸馏算法

胶球与赝标量粲偶
素的混合

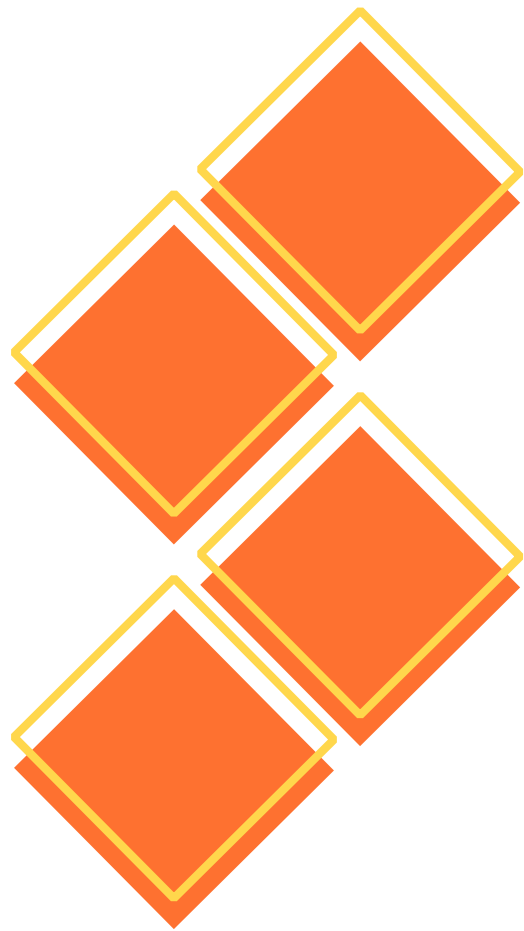
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3

非联通图对粲偶素质量的
贡献

END

4



1

蒸馏方法

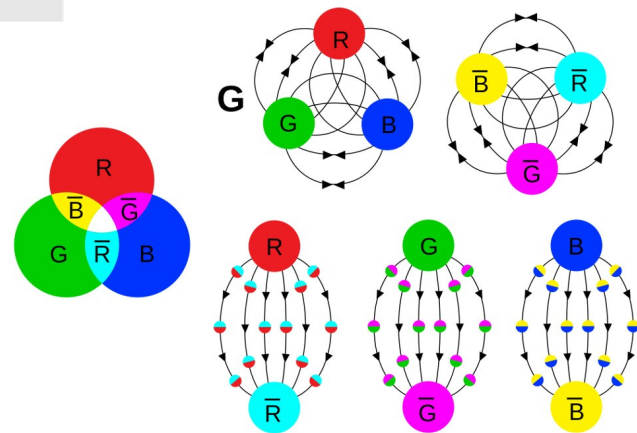
在格点上，计算传播子是非常耗费资源的。
传统方法极大的节省了资源消耗，但是信号
损失太多。为了计算全传播子，人们进行了
各种尝试



物理信息的提取依赖于关联函数

	能谱	共振态	Form Factor	PDF
2Pt	Y	Y	N	N
3Pt	N	N	Y	Y

- 关联函数的基本单元是传播子
- 传播子成为格点计算的热点



- 两点关联函数为例

$$\begin{aligned}\langle O(x)\bar{O}(y)\rangle &= -\langle \text{tr}[\Gamma D_u^{-1}(x,y)\Gamma D_d^{-1}(y,x)]\rangle \\ &= -\frac{1}{Z} \int \mathcal{D}[U] e^{-S_G[U]} \det[D_u] \det[D_d] \\ &\quad \times \text{tr}[\Gamma D_u^{-1}(x,y)\Gamma D_d^{-1}(y,x)]\end{aligned}$$

x,y 为时空指标

- 需要计算动量空间的关联函数，傅里叶变换

$$\langle O(\vec{p}, t_x)\bar{O}(\vec{p}, t_y)\rangle = \sum_{\vec{x}} \sum_{\vec{y}} e^{i\vec{p}(\vec{x}-\vec{y})} \dots$$

- 传播子 $D^{-1}(x, y) = D^{-1}(\vec{x}, t_x; \vec{y}, t_y)$

是一个巨大的张量：

$$(T^*X^*Y^*Z^*Nc^*Ns)^2 \sim 10^{10}G$$

难计算 难存储

- 传统方法：将矩阵的求解变成列向量的求解

$$D_{ab}^{\alpha\beta}(\vec{z}, t_z; \vec{x}, t_x) D_{bc}^{\beta\gamma-1}(\vec{x}, t_x; \vec{y}, t_y) = \delta_{ac} \delta_{\alpha\gamma} \delta_{\vec{x}, \vec{y}} \delta_{t_x, t_y} \quad \begin{array}{l} t_0, y_0 \text{ 表示固定点} \\ \sim 10^{20}G \end{array}$$

→

$$D_{ab}^{\alpha\beta}(\vec{z}, t_z; \vec{x}, t_x) D_{bc}^{\beta\gamma-1}(\vec{x}, t_x; \vec{y}, t_0) S_{cd}^{\gamma\eta}(t_0, \vec{y}; t_0, \vec{y}_0) = \delta_{ac} \delta_{\alpha\gamma} \delta_{\vec{x}, \vec{y}} \delta_{t_x, t_0} S_{cd}^{\gamma\eta}(t_0, \vec{y}; t_0, \vec{y}_0)$$

- $D^{-1}(x, y) \rightarrow D^{-1}(x, y)S(y, y_0)$ 传播子的体积大幅减少

- 固定 y 不会影响动量

$$\begin{aligned} C_{\vec{p}}(t, t_0) &= \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \langle O(x) \bar{O}(\vec{y}_0, t_0) \rangle \\ &= \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \langle \text{tr}[\Gamma D_u^{-1}(x; \vec{y}_0, t_0) \Gamma D_d^{-1}(\vec{y}_0, t_0; x)] \rangle \end{aligned}$$

- 保证 Smear 波函数规范规范不变性和空间对称性，不会影响算符量子数

算符构造

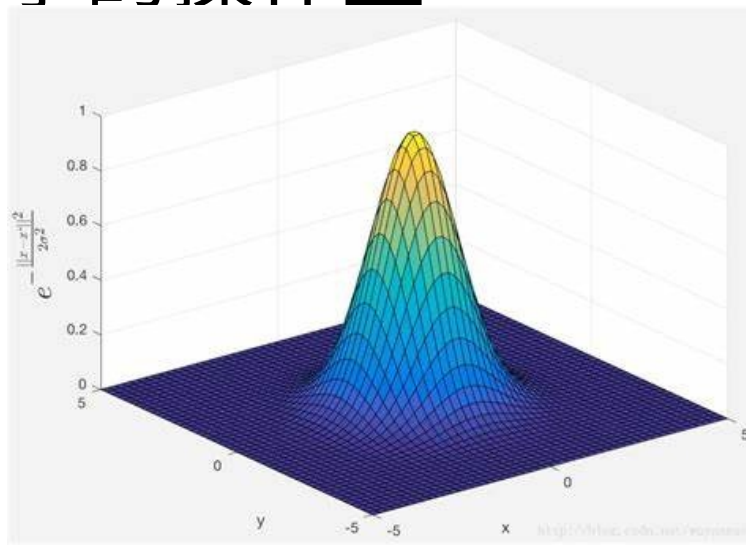
- 一般的算符构造

$$\bar{\psi}\Gamma(D)\psi(x)$$

- 在格点上，最终体现在对传播子的操作



算符构造上的困难



Disconnected Part

- 如果考虑算符为 $O(x) = \bar{c}\Gamma c(x)$

$$\begin{aligned}\langle O(\vec{p}, t_x) \bar{O}(\vec{p}, t_y) \rangle &= -\sum_{\vec{x}} \sum_{\vec{y}} e^{i\vec{p}(\vec{x}-\vec{y})} \langle \text{tr}[\Gamma D^{-1}(x, y) \Gamma D^{-1}(y, x)] \rangle \\ &\quad + \sum_{\vec{x}} \sum_{\vec{y}} e^{i\vec{p}(\vec{x}-\vec{y})} \langle \text{tr}[\Gamma D^{-1}(x, x)] \text{tr}[\Gamma D^{-1}(y, y)] \rangle\end{aligned}$$

- 非联通部分中间由胶子连接
- 通常不重要，但对 $\langle O_{cc}(x) O_G(y) \rangle$ 过程，只能通过非联通部分实现，没有联通部分

- 计算联通非部分关联函数

$$D^{-1}(x, x) \quad \text{Disconnected Loop}$$

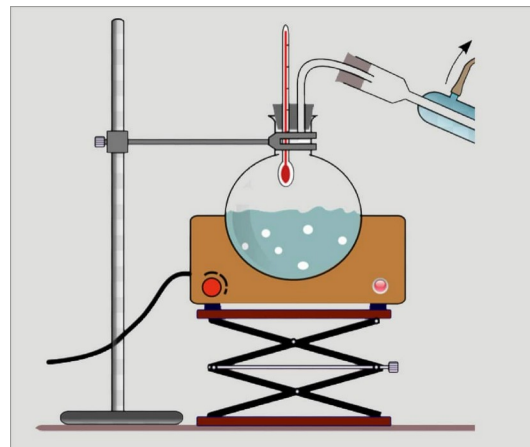
- Point Method 不能节省计算资源：传播子两端的时空点要相同
- 算符构造上的困难
- 信号极差

Point Method 的缺陷

1. 信号丢失
2. 复用性差
3. 难以计算 Disconnected loop
4. 算符构造不方便

蒸馏方法

$$\begin{aligned}
 J_{\sigma, n_{\sigma}}(t) &= \left(1 + \frac{\sigma \nabla^2(t)}{n_{\sigma}} \right)^{n_{\sigma}} \\
 &= \left(1 + \frac{\sigma V A V^{\dagger}(t)}{n_{\sigma}} \right)^{n_{\sigma}} \\
 &\approx V \left(1 + \frac{\sigma A}{n_{\sigma}} \right)^{n_{\sigma}} V^{\dagger}(t),
 \end{aligned}$$



$$\begin{aligned}
 \tau_{\alpha\beta}^{ab}(t, t') &= V_a^{\dagger}(\vec{\omega}, t) (M^{-1})_{\alpha\beta}(t, \vec{\omega}; t', \vec{x}') V_b(\vec{x}', t'), \\
 \Phi_{\alpha\beta}^{Aab}(t, \vec{y}, \vec{z}) &= V_a^{\dagger}(\vec{y}, t) \Gamma_{\alpha\beta}^A(t) V_b(t, \vec{z}).
 \end{aligned}$$

联通部分

$$\langle \text{tr} [\Phi_{\alpha\beta}^{Bab}(\vec{y}', t'; \vec{z}', t') \tau_{\beta\gamma}^{bc}(t', t) \Phi_{\gamma\kappa}^{Acd}(\vec{y}, \vec{z}, t) \tau_{\kappa\alpha}^{da}(t, t')] \rangle$$

非联通部分

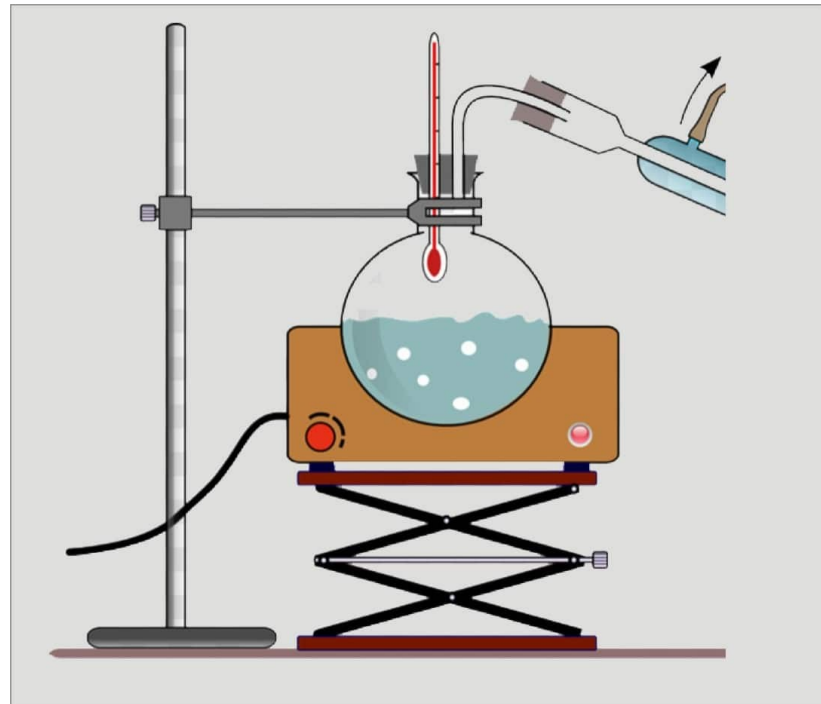
$$\langle \text{tr} [\Phi_{\gamma k}^{Acd}(\vec{y}, t; \vec{z}, t) \tau_{k\gamma}^{dc}(t, t)] \text{tr} [\Phi_{\alpha\beta}^{Bab}(\vec{y}', t'; \vec{z}', t') \tau_{\beta\alpha}^{ba}(t', t')] \rangle$$

优点：

- 算符构造方便
- 非联通部分计算方便
- 可重复利用
- 近似全传播子

缺点：

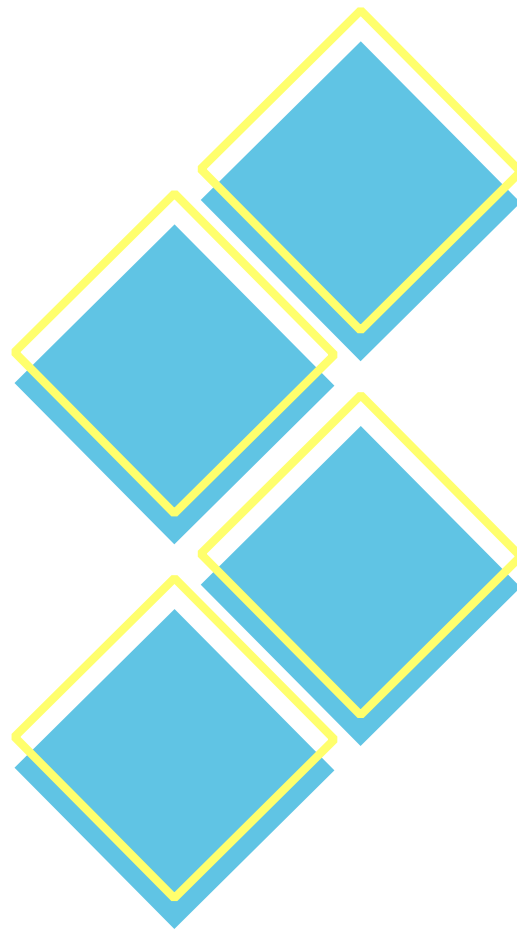
- 计算量大，存储空间大





2

胶球与赝标量 粲偶素的混合

从第一性原理出发探索胶球与介子的混合。



- 
- 在 QCD 中，胶球可能存在
 - 由于夸克和胶子相互作用，混合可能存在
 - 实验中，偏大，应当受到 OZI 规则压低
 - 如果考虑混合，可能可以解释变宽度较大的现象
- 

Lattice Setup

- 宫老师产生的海里面只有两味 charm 的组态

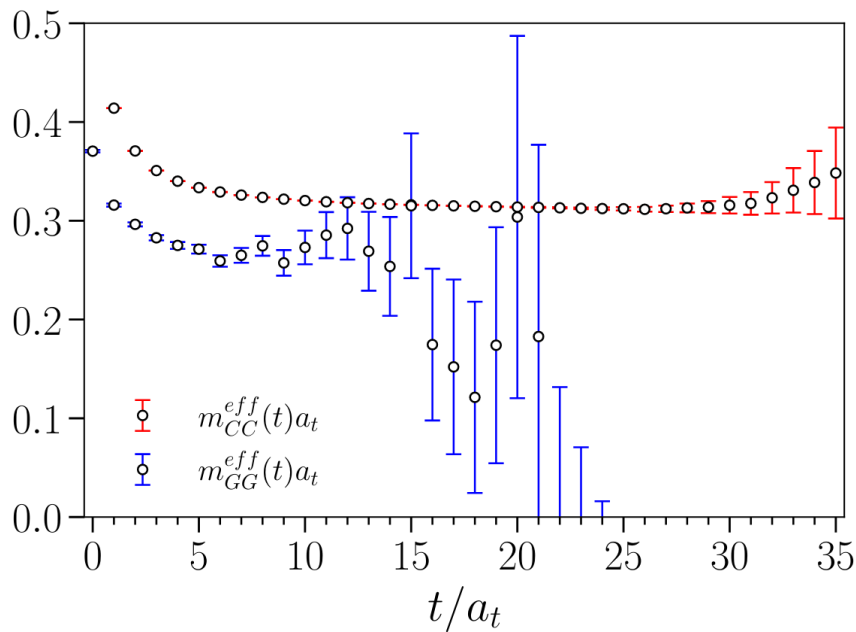
ensemble	$L^3 \times T$	β	$a_s(\text{fm})$	ξ	N_{cfdg}	$m_{J/\psi}(\text{MeV})$
I	$16^3 \times 128$	2.8	0.1026	5	~ 7000	2743
II	$16^3 \times 128$	2.8	0.1026	5	~ 6000	3068

- tadpole improved anisotropic clover fermion action and tadpole improved Morningstar-Pearson gauge action

• 关联函数矩阵

$$\begin{pmatrix} \langle O_{cc}(t)\bar{O}_{cc}(t_0) \rangle & \langle O_{cc}(t)\bar{O}_G(t_0) \rangle \\ \langle O_G(t)\bar{O}_{cc}(t_0) \rangle & \langle O_G(t)\bar{O}_G(t_0) \rangle \end{pmatrix}$$

- 介子的计算非常高的精度
- 胶球的信号非常差，持续时间短
- 靠前时间片，激发态的效应明显
- 非对角部分的关联函数，信号会远
比胶球的信号差



蒸馏算法 --> critical

中科院
先导一号

2000DCU

24h / 组态
8000 个组态
4 day

高能所
GPU 集群

100GPU

12h / 组态
8000 个组态
1 Month

假设:

- 混合程度很小, 且只考虑质量相近态的混合
- 格点上计算的是纯的胶球态和 CC 态

$$H = \begin{pmatrix} m_{G_1} & x_1 \\ x_1 & m_{(cc)_1} \end{pmatrix} \oplus \begin{pmatrix} m_{G_2} & x_2 \\ x_2 & m_{(cc)_2} \end{pmatrix} \oplus \dots$$

$$H|\eta_i\rangle = m_{\eta_i}|\eta_i\rangle, \quad H|g_i\rangle = m_{g_i}|g_i\rangle$$

$$\begin{bmatrix} |g_i\rangle \\ |\eta_i\rangle \end{bmatrix} = \begin{bmatrix} \cos \theta_i & -\sin \theta_i \\ \sin \theta_i & \cos \theta_i \end{bmatrix} \begin{bmatrix} |G_i\rangle \\ |(c\bar{c})_i\rangle \end{bmatrix}$$

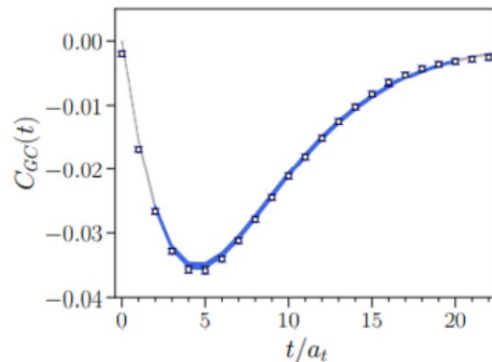
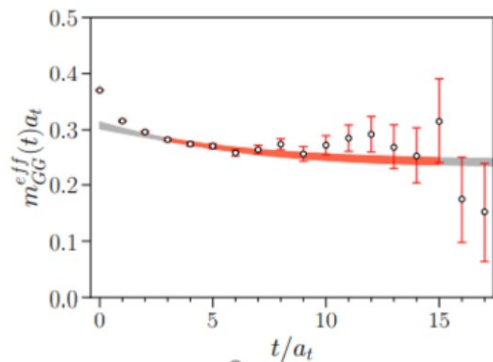
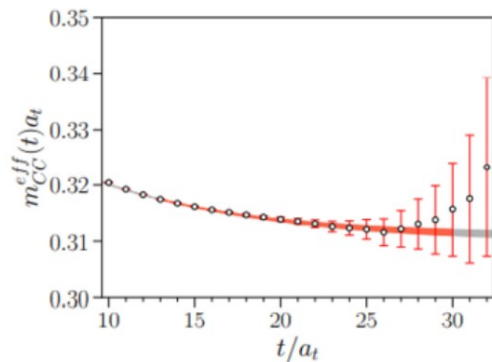
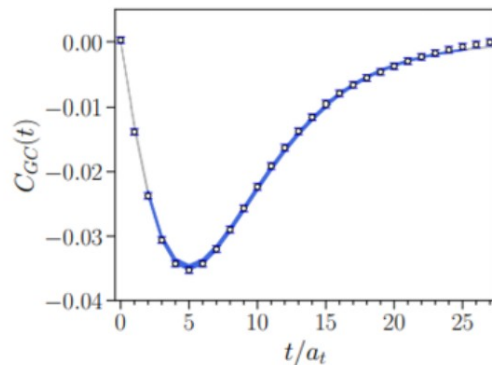
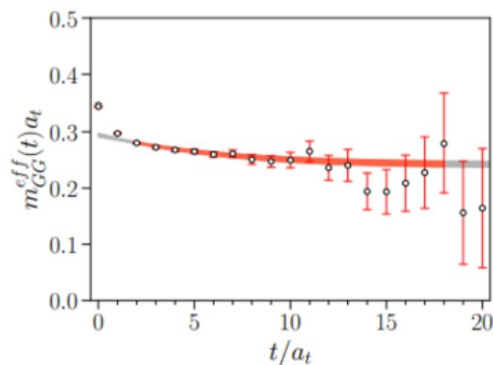
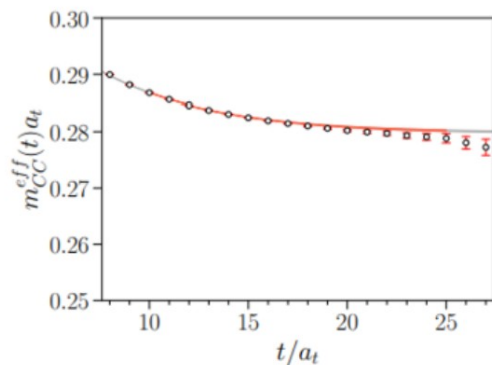
$$\sin \theta_i \approx \pm \frac{x_i}{\Delta_i}, \quad \text{where } \Delta_i = m_{(cc)_i} - m_{G_i}$$

$$C_{GC}(t) = - \sum_i \sqrt{Z_{G_i} Z_{(\gamma_5),i}} \cos \theta_i \sin \theta_i \left(e^{-m_{g_i} t} - e^{-m_{g_i}(T-t)} - (e^{-m_{\eta_i} t} - e^{-m_{\eta_i}(T-t)}) \right)$$

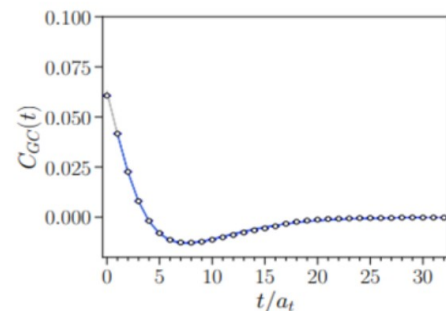
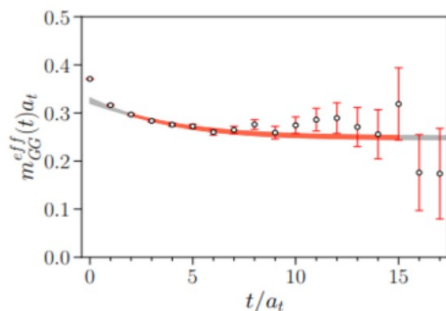
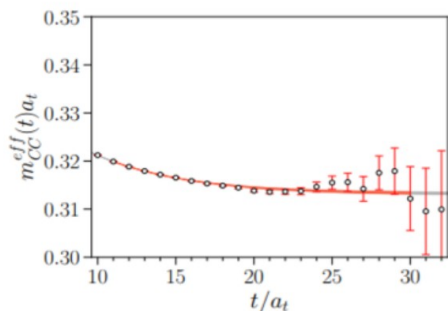
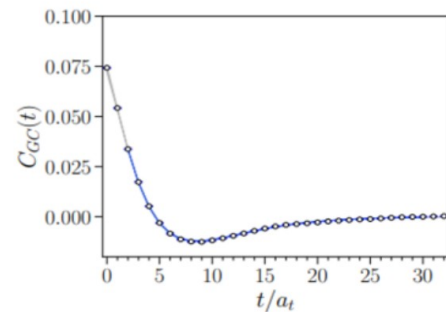
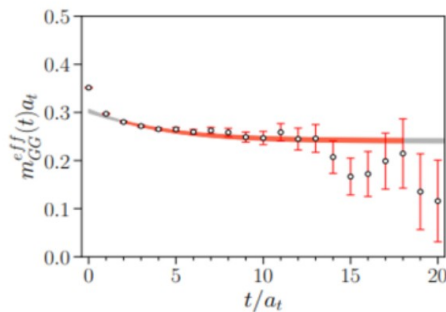
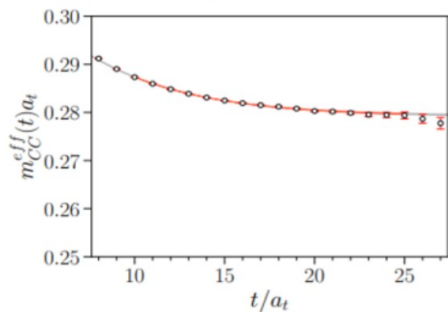
● $O_{CC} = \bar{c} \gamma_5 c$


$$C_{GG}(t) = \sum_i Z_{G_i} \left[\cos^2 \theta_i \left(e^{-m_{g_i} t} + e^{-m_{g_i}(T-t)} \right) + \sin^2 \theta_i \left(e^{-m_{\eta_i} t} + e^{-m_{\eta_i}(T-t)} \right) \right]$$

$$C_{CC}(t) = \sum_i Z_{(\gamma_5),i} \left[\cos^2 \theta_i \left(e^{-m_{\eta_i} t} + e^{-m_{\eta_i}(T-t)} \right) + \sin^2 \theta_i \left(e^{-m_{g_i} t} + e^{-m_{g_i}(T-t)} \right) \right]$$




$\bullet O_{cc} = \bar{c}\gamma_4\gamma_5c \quad C_{GC}(t) \approx \sqrt{Z_{G_1}} \langle 0 | O_{\gamma_5\gamma_4} | G_1 \rangle \cos^2 \theta_1 \left(e^{-m_{g_1}t} + e^{-m_{g_1}(T-t)} \right)$
 $- \sum_{i=1}^2 \sqrt{Z_{G_i} Z_{(\gamma_5\gamma_4),i}} \cos \theta_i \sin \theta_i \left(e^{-m_{g_i}t} + e^{-m_{g_i}(T-t)} - (e^{-m_{\eta_i}t} + e^{-m_{\eta_i}(T-t)}) \right)$





ensemble	Γ	m_{η_1} (MeV)	m_{g_1} (MeV)	θ_1	x_1 (MeV)
I	γ_5	2691(2)	2317(51)	$7.7(1.1)^\circ$	50(10)
	$\gamma_5\gamma_4$	2685(1)	2317(43)	$6.8(8)^\circ$	44(7)
	avg.	2686(1)	2317(46)	$7.1(9)^\circ$	46(8)
II	γ_5	2987(9)	2308(63)	$4.9(6)^\circ$	58(9)
	$\gamma_5\gamma_4$	3013(3)	2385(40)	$4.2(3)^\circ$	46(4)
	avg.	3010(4)	2363(47)	$4.3(4)^\circ$	49(6)

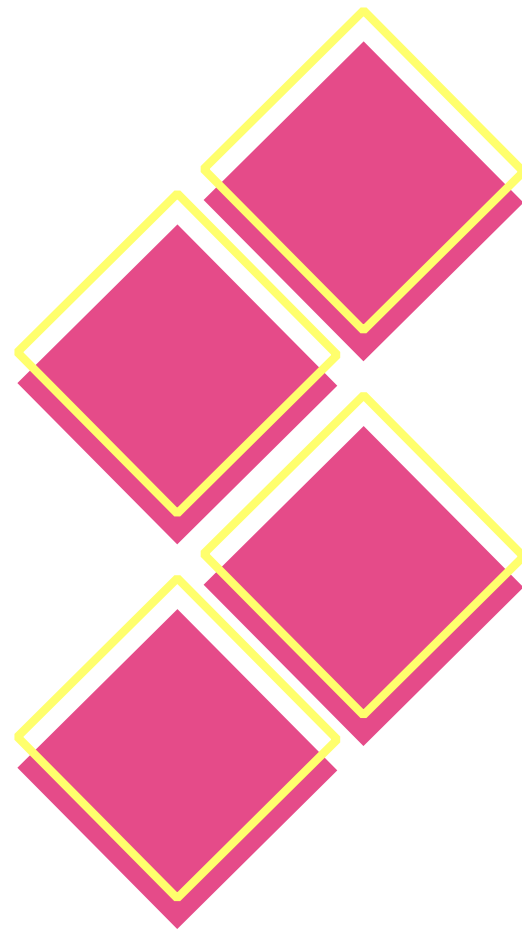


混合角非零

3

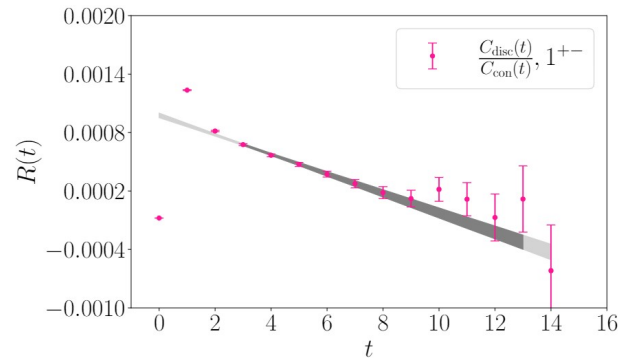
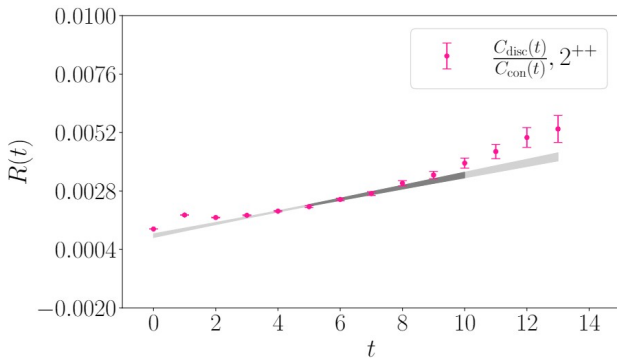
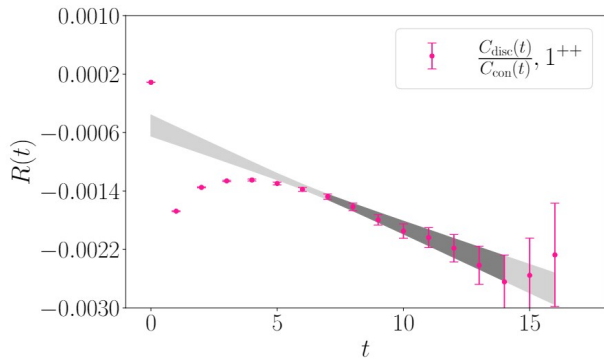
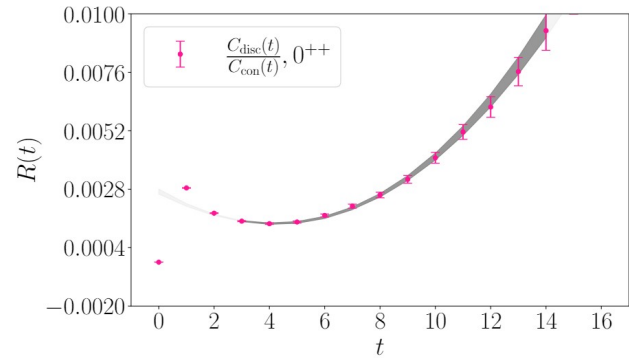
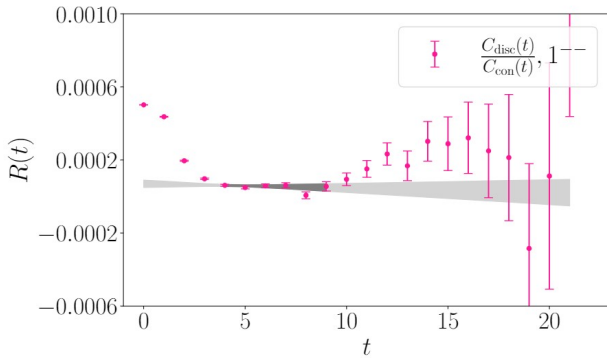
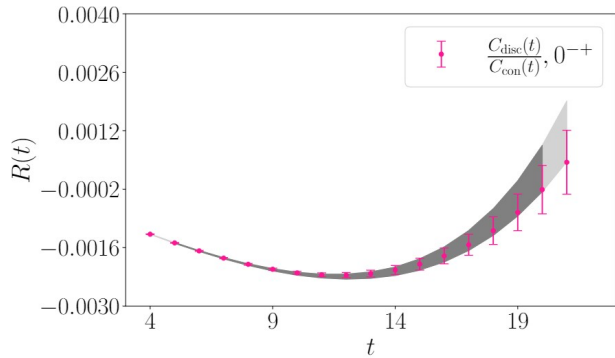
非联通图对 簇偶素质量 的贡献

利用大统计量和蒸馏算法，精确计算非联通图质量贡献。



-
- Hyperfine splitting $\Delta M_{hyp} = M_{J/\psi} - M_{\eta_c}$ 在实验上测得113.0 (0.5) Mev, 在格点上的计算是120.3(1.1)Mev, 差距7.3 (1.2) Mev
 - 湮灭图可能对粲偶素的质量有贡献从而导致了这几个Mev的差别
 - 顺便计算了Center of Gravity Mass of P-wave Charmonium的关系
-

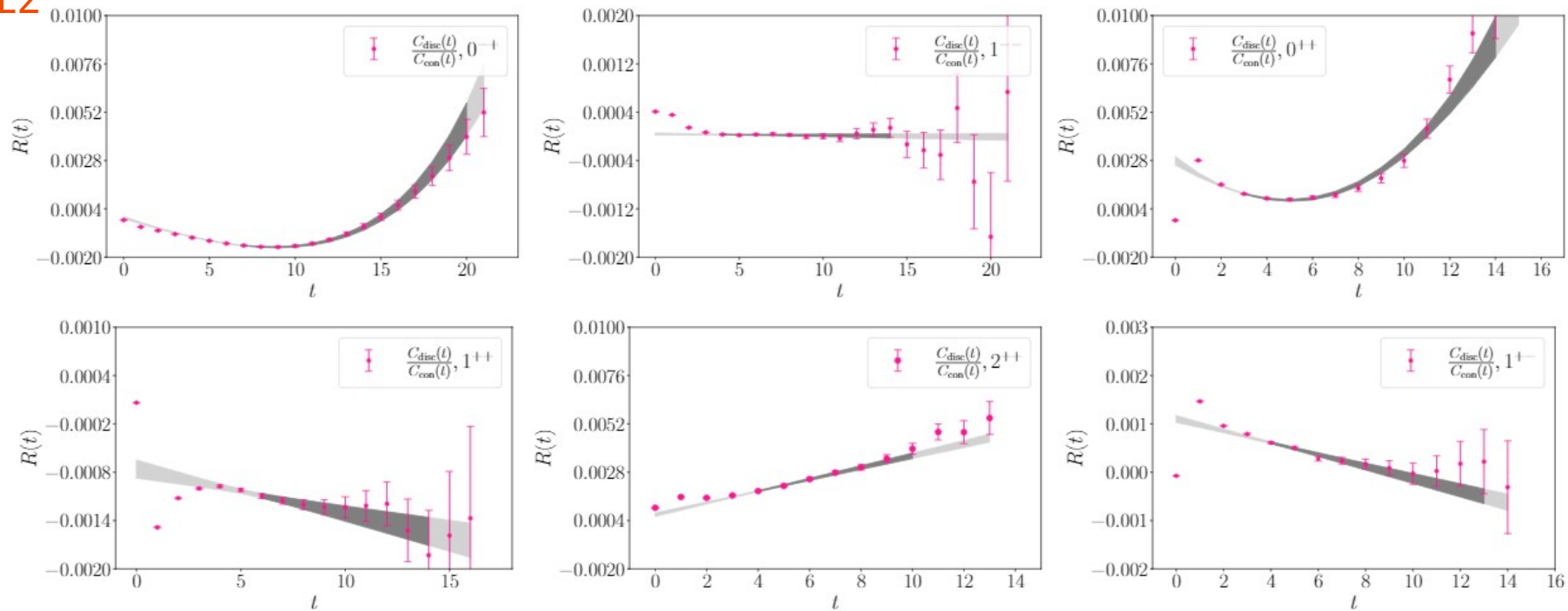
E1



拟合公式 $R'(t) = \delta m t + \epsilon + W e^{\Delta t}$

$$R^\Gamma(t) \equiv \frac{2D_{11}^\Gamma(t)}{C_{11}^\Gamma(t)},$$

E2

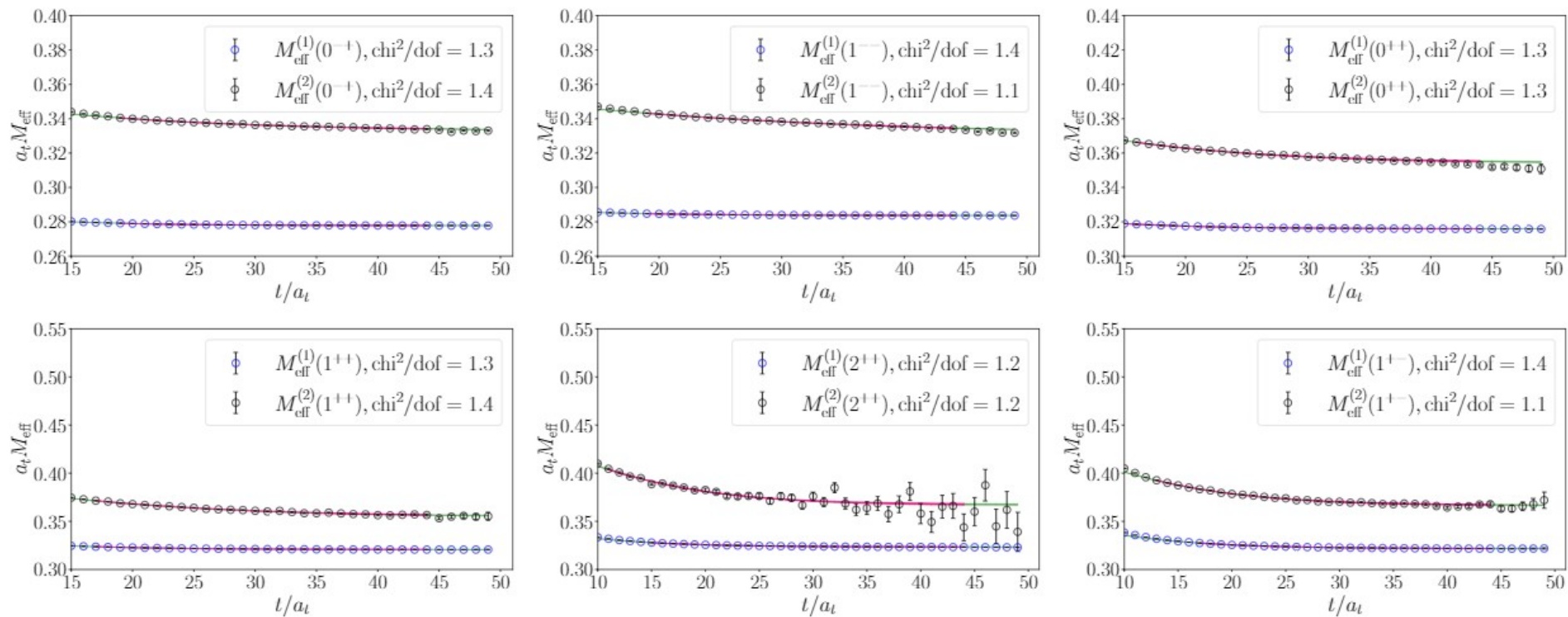


	$\eta_c(0^{-+})$	$J/\psi(1^{--})$	$\chi_{c0}(0^{++})$	$\chi_{c1}(1^{++})$	$\chi_{c2}(2^{++})$	$h_c(1^{+-})$
$[t_{\min}, t_{\max}]$	[5, 20]	[4, 9]	[3, 14]	[7, 14]	[5, 10]	[3, 13]
$\delta m_1^\Gamma a_t$ (E1)	0.00031(1)	$\sim 10^{-6}$	0.007(5)	0.00014(2)	-0.00031(3)	0.00010(1)
χ^2/dof	1.5	0.78	0.62	0.59	0.82	0.71
$[t_{\min}, t_{\max}]$	[6, 20]	[4, 14]	[3, 14]	[6, 14]	[4, 10]	[4, 13]
$\delta m_1^\Gamma a_t$ (E2)	0.00032(2)	$\sim 10^{-6}$	0.003(4)	0.00006(2)	-0.00029(2)	0.00012(2)
χ^2/dof	1.4	0.59	1.2	0.64	0.71	1.3

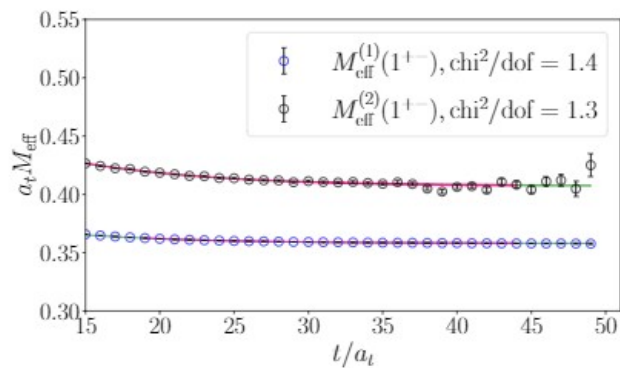
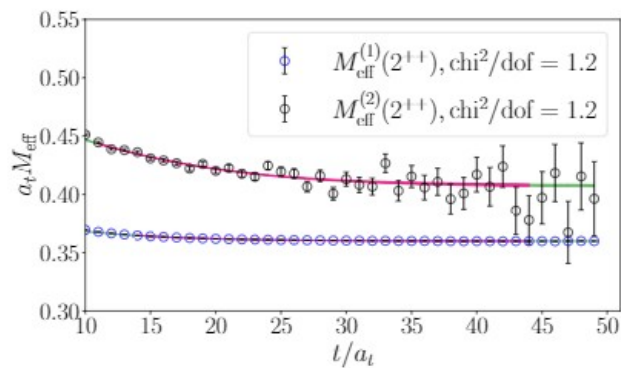
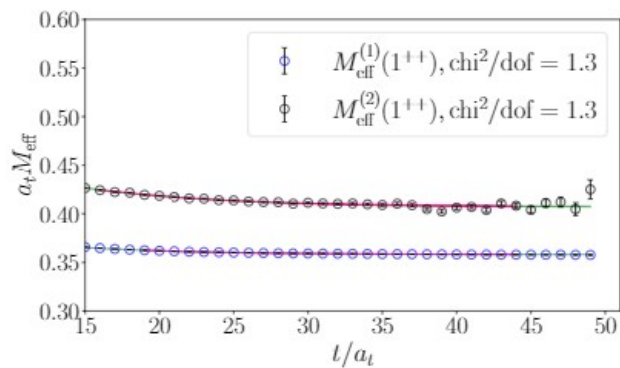
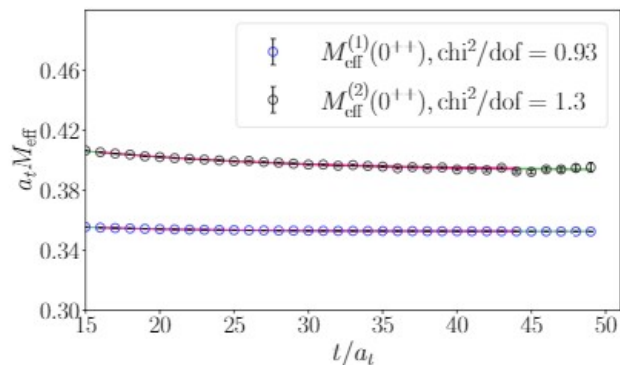
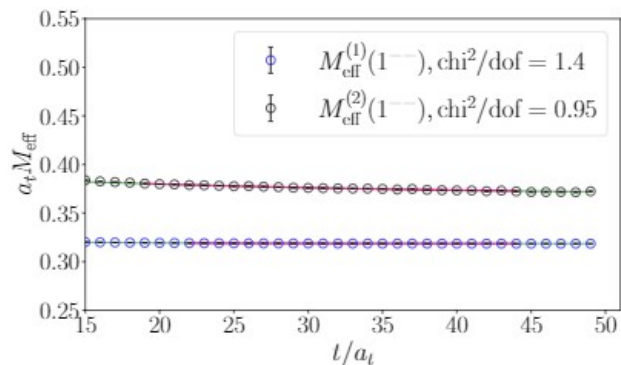
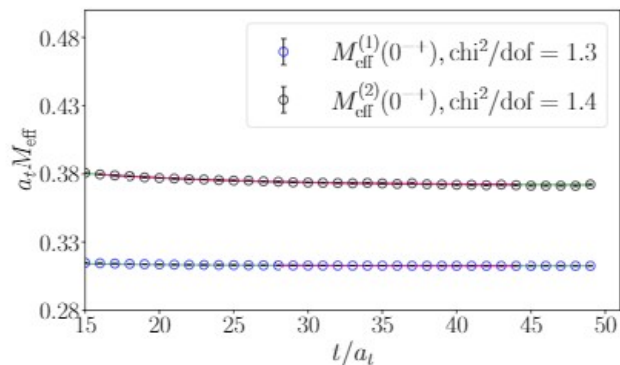
E1 : $\delta m_1(0^{-+}) = 3.0(1)$ MeV
 $\delta m_1(1^{++}) = 1.3(2)$ MeV
 $\delta m_1(2^{++}) = -3.0(3)$ MeV
 $\delta m_1(1^{+-}) = 1.0(1)$ MeV
 E2 : $\delta m_1(0^{-+}) = 3.1(2)$ MeV
 $\delta m_1(1^{++}) = 0.6(2)$ MeV
 $\delta m_1(2^{++}) = -2.8(2)$ MeV
 $\delta m_1(1^{+-}) = 1.2(2)$ MeV.

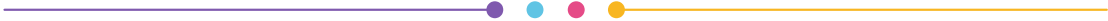
-
- COG relation $M_{\text{COG}}(nP) = \frac{1}{9} [5M(n^3P_2) + 3M(n^3P_1) + M(n^3P_0)]$
 - $m_{1+-} = \frac{1}{9} (m_{0++} + 3 * m_{1++} + 5 * m_{2++})$
 - 3*3 变分得到基态和第一激发态
 - 分别拟合关联函数
 - 验证 COG relation
-

E1

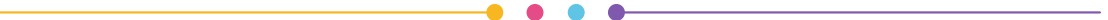


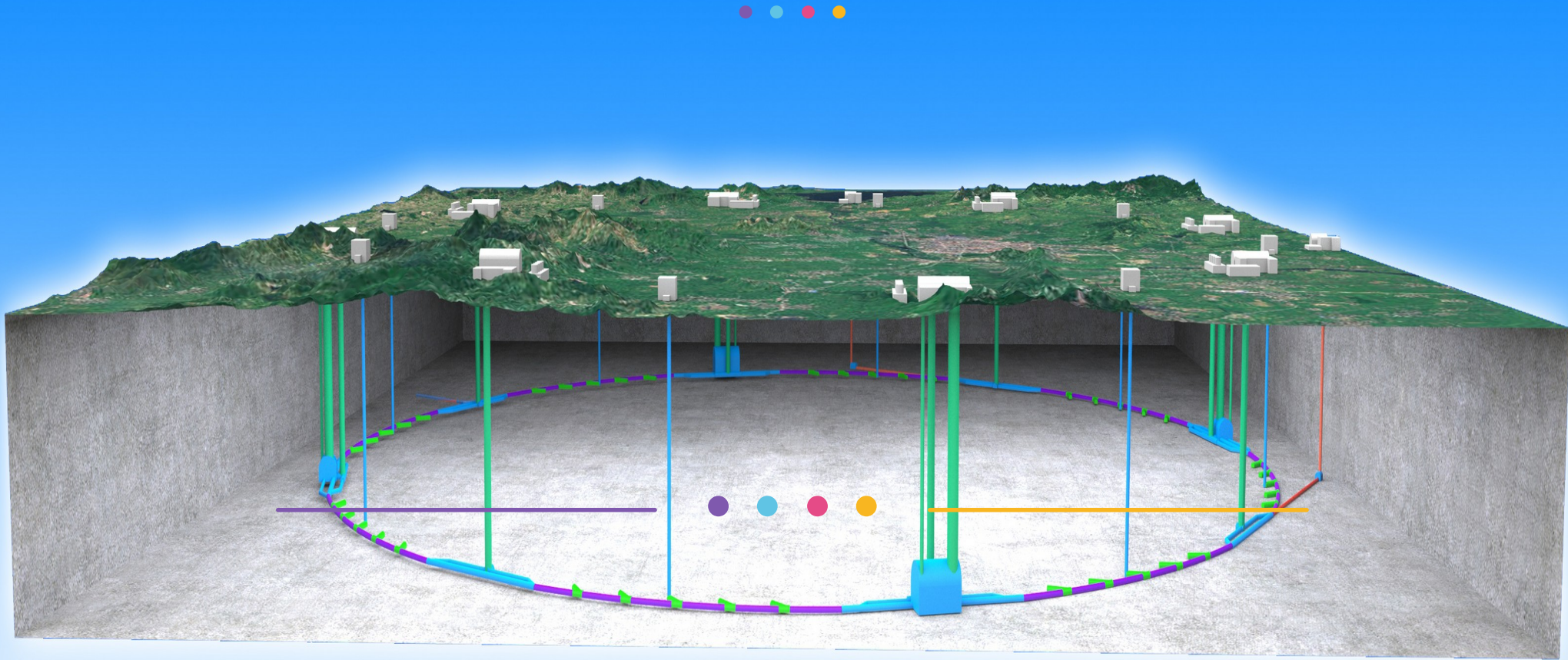
E2





	J^{PC}	$\chi_{c0}^{(')}(0^{++})$	$\chi_{c1}^{(')}(1^{++})$	$\chi_{c2}^{(')}(2^{++})$	$h_c^{(')}(1^{+-})$	m_{COG}
E1	$M(1P)$ (MeV)	3039.1(4)	3083.8(6)	3109.7(6)	3094.0(7)	3093.2(4)
	$M(2P)$ (MeV)	3410(4)	3412(7)	3531(8)	3526(5)	3478(5)
E2	$M(1P)$ (MeV)	3391.8(6)	3433.8(9)	3461.8(7)	3443.8(9)	3444.7(5)
	$M(2P)$ (MeV)	3785(6)	3792(9)	3916(11)	3917(6)	3860(6)
Exp.	$M(1P)$ (MeV)	3414.7(3)	3510.7(1)	3556.2(1)	3525.4(1)	3525.31(7)
	$M(2P)$ (MeV)	3862(51)	3871.7(1)	3922(1)		3898(6)





THANK YOU