A study on the form factor of light axial vector mesons

Mengchuan Du（杜蒙川）

In collaboration with Prof. Qiang Zhao（赵强）

2021年10月29日

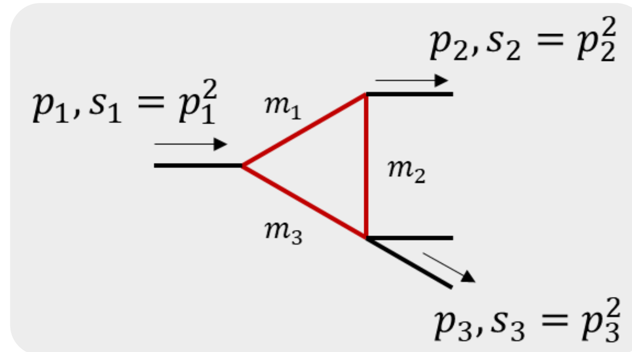
- **Triangle singularity**
- **Vertex correction by π exchange**
- **Implications for $\rho\pi$ and $\phi\pi$ channels**
- **Conclusion**

CONTENT

- **Triangle singularity**
- Vertex correction by π exchange
- Implications for $\rho\pi$ and $\phi\pi$ channels
- Conclusion

CONTENT

TRIANGLE SINGULARITY



- Non-resonant structure
- On-shell kinematics
- Kinematic region
- One type of Landau singularity

$$(m_1+m_3)^2 < s_1 < s_{1c} = (m_1+m_3)^2 + \frac{m_3}{m_2}\left[(m_1-m_2)^2 - s_2\right]$$

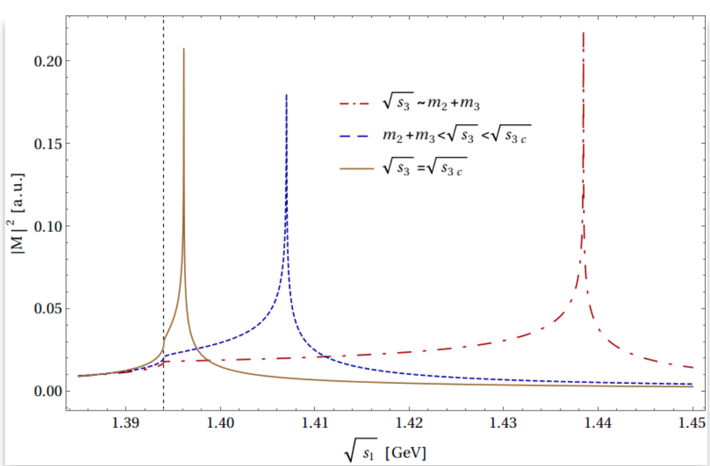
$$s_2 < (m_1-m_2)^2$$

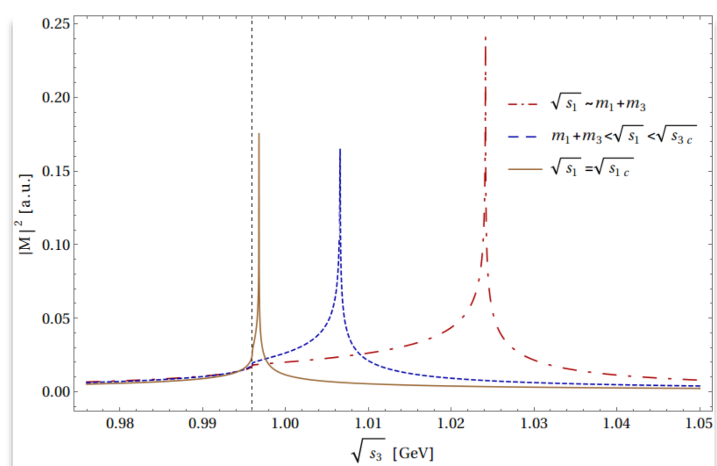
$$(m_2+m_3)^2 < s_3 < s_{3c} = (m_2+m_3)^2 + \frac{m_3}{m_1}\left[(m_1-m_2)^2 - s_2\right]$$

[1] L. D. Landau. Nucl. Phys., 1959, 13: 181
[2] Xiao-Hai Liu, Makoto Oka, Qiang Zhao, Phys. Lett. B 753 (2016) 297
[3] Feng-Kun Guo, Xiao-Hai Liu, S. Sakai, Prog.Part.Nucl.Phys. 112 (2020) 103757

- Typical behavior

$$m_1 = m_{K^*}, m_2 = m_K, m_3 = m_K$$





HADRONIC DECAY CHANNELS

STATES	$I^G J^{PC}$	CHANNELS
$f_1(1285), f_1(1420)$	0^+1^{++}	$a_0\pi, K^*\bar{K}, \kappa\bar{K}, \sigma\eta, [f_0\pi]$
$a_1(1260)$	1^-1^{++}	$\rho\pi, f_0\pi, K^*\bar{K}$
$h_1(1170), h_1(1415)$	0^-1^{+-}	$K^*\bar{K}, \kappa\bar{K}, [\phi\pi], [\omega\pi]$
$b_1(1235)$	1^+1^{+-}	$\omega\pi, K^*\bar{K}, \{\phi\pi\}$

- Promising decay channels → $f_1(f_1') \to a_0(f_0)\pi, a_1 \to f_0\pi\ , h_1(h_1') \to \phi\pi, \rho\pi, b_1 \to \phi\pi$

[1] C. Adolph et al. [COMPASS], PRL 115,082001(2015)
[2] M. G. Alexeev et al. [COMPASS], arXiv:2006.05342(2020)
[3] M. C. Du, Q. Zhao, Phys. Rev. D 104(2021)3,036008

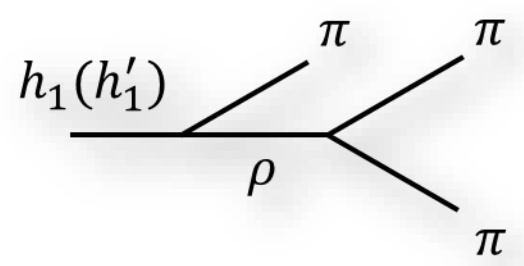
- Model dependence for a_0 and f_0

- Production processes → $\mathrm{B.R}\left(J/\psi \to \pi^\pm b_1^\mp\right) = (3 \pm 0.5) \times 10^{-3}$

$$\mathrm{B.R.}\left(J/\psi \to \eta' h_1' \to \eta' K^*\bar{K} + c.c.\right) = (2.16 \pm 0.31) \times 10^{-4}$$

$\rho\pi$ & $\phi\pi$

Suppressed tree amplitudes

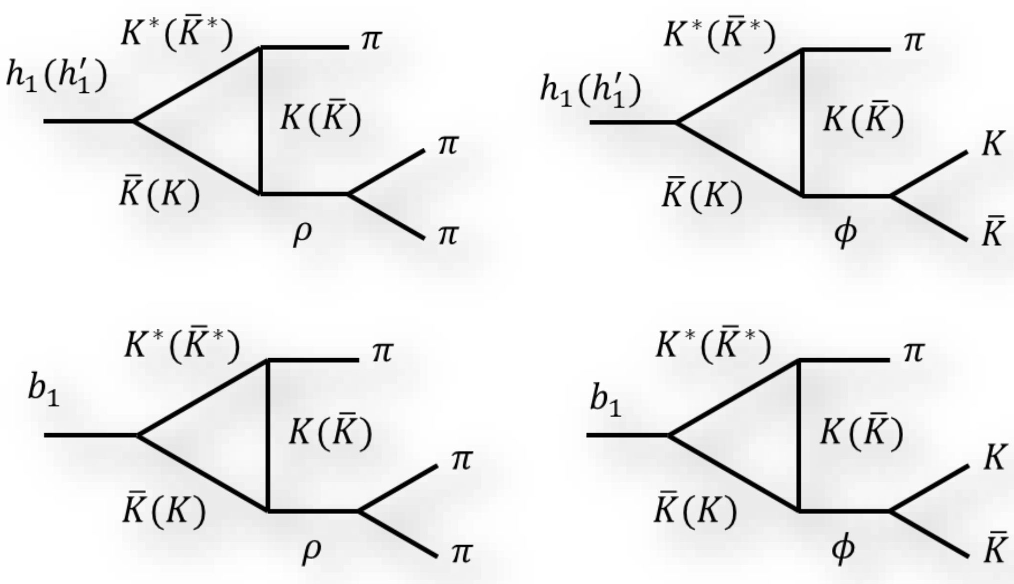


Triangle singularity enters

$$\boldsymbol{M^{tri}_{b_1\to\rho\pi} \sim \epsilon_{b_1\mu} I^{\mu\nu} \epsilon^{*}_{\rho\nu}}$$

$b_1 \to \rho\pi$

$b_1 \to \phi\pi$



$J/\psi \to \phi\pi^0\pi^0, \eta\phi\pi^0$:

PARAMETERS

- $V \to PP$ couplings
- $h_1, h_1', b_1 \to VP$

  $$L = g_{BVP}\langle B^{\mu}\{V_{\mu}, P\}\rangle$$

- $h_1 \to \rho\pi$ or $b_1 \to \omega\pi$ (S-wave)
- Monopole form factor

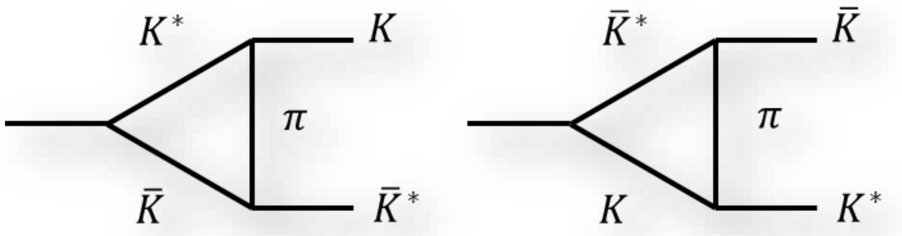
[1] Hao-Jie Jing, Feng-Kun Guo, Shuntaro Sakai, Bing-Song Zou, Phys. Rev. D 100(2019)11,114010

- **Triangle singularity**
- **Vertex correction by π exchange**
- **Implications for $\rho\pi$ and $\phi\pi$ channels**
- **Conclusion**

CONTENT

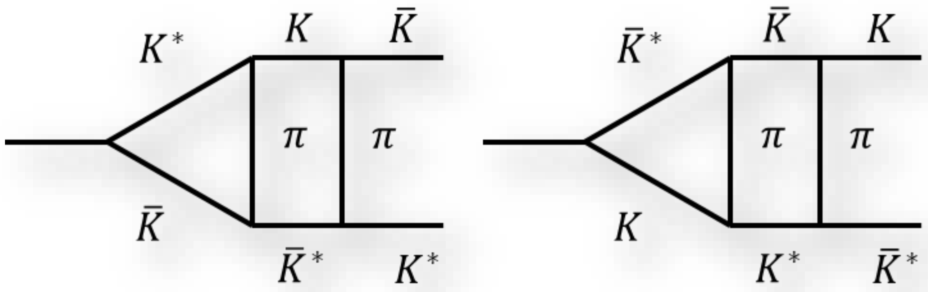
π EXCHANGE

One loop



$\sim O(v)$

Two loops



N loops

$\sim O(v^n)$

$$\boldsymbol{M^{tri}_{A\to K^*\bar{K}} \sim \epsilon_{A\mu} I^{\mu\nu} \epsilon^*_{K^*\nu}}$$

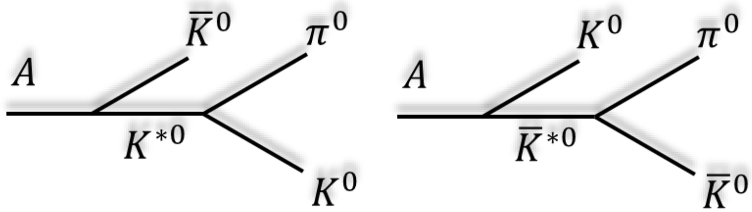
- B-S equation
- NR Power counting[1,2]
- Enhancement form complicated Landau singularities [3]

No leading singularity for N-loop diagrams ($N \geq 2$)

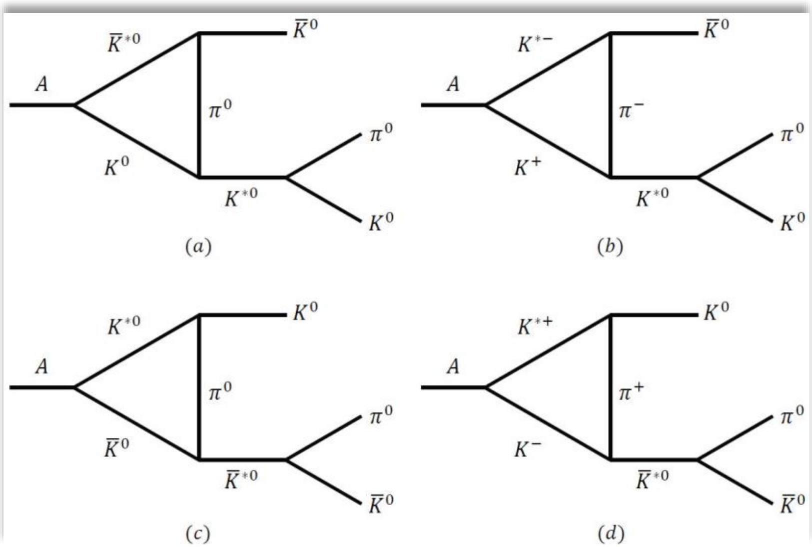
Sub-leading singularities are suppressed

[1] Guo F K, Hanhart C, Li G, et al. Phys. Rev. D, 2011, 83: 034013.
[2] Guo F K, Hanhart C, Meißner U G, et al. Rev. Mod. Phys., 2018, 90(1): 015004.
[3] L. B. Okun, A. P. Rudik, Nucl. Phys. 14,261.

Tree $\sim O(v^{-1})$



One loop $\sim O(v^{0})$

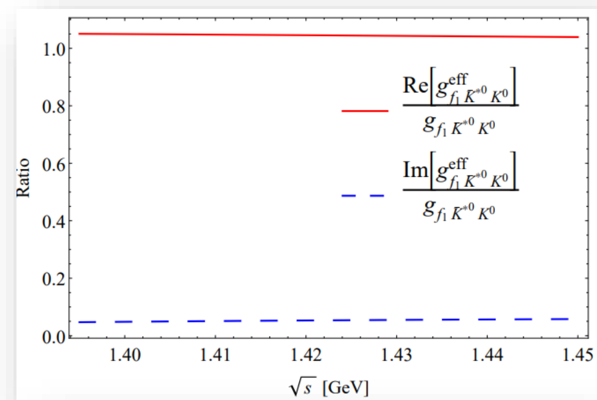


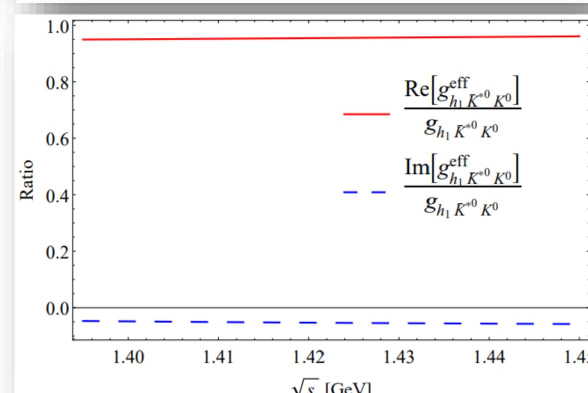
Constructive for 1^{++} Destructive for 1^{+-}

$$\frac{\delta\Gamma\big(f_1(1420)\to K^*\bar{K}\to K\bar{K}\pi\big)}{\Gamma\big(f_1(1420)\to K^*\bar{K}\to K\bar{K}\pi\big)} = 15\%$$

$$\frac{\delta\Gamma\big(h_1(1415)\to K^*\bar{K}\to K\bar{K}\pi\big)}{\Gamma\big(h_1(1415)\to K^*\bar{K}\to K\bar{K}\pi\big)} = -9.3\%$$

Effective couplings





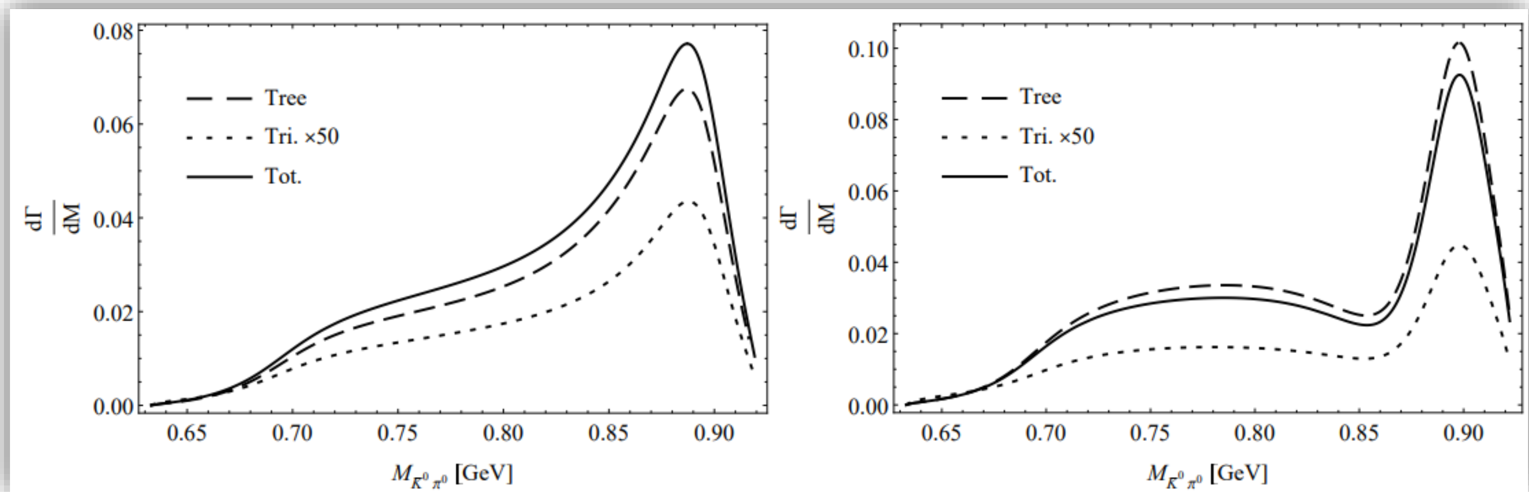
$$I^{\mu\alpha}_{c,d} = \int \frac{d^4q}{(2\pi)^4} \frac{\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{m_1^2}\right)(2p_b - q)_\nu (p_c + 2p_b - 2q)^\alpha F(q^2)}{(q^2 - m_1^2)[(q-p_b)^2 - m_2^2][(q-p_b-p_c)^2 - m_3^2]}$$

$$I^{\mu\alpha}_{a,b} = I^{\mu\alpha}_{c,d}\Big|_{p_b\to p_a, p_a\to p_b}$$

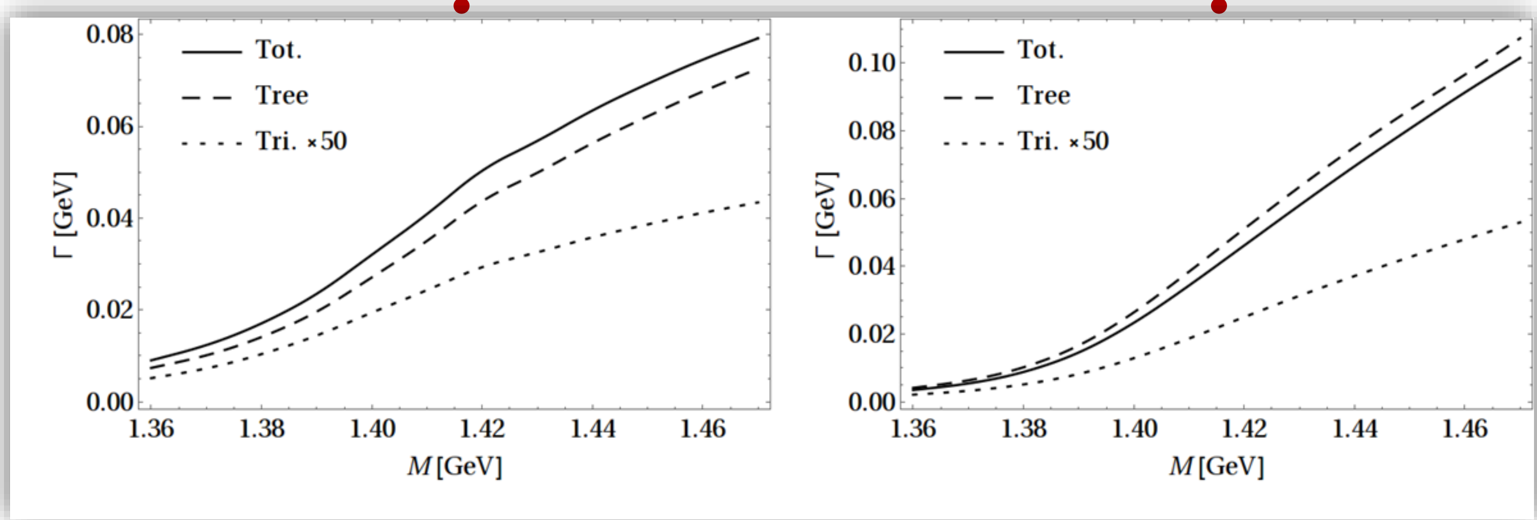
$$F(q^2) = \prod \frac{m_i^2 - \Lambda_i^2}{q_i^2 - \Lambda_i^2}$$

$$\Lambda_i = m_i + \beta\Lambda_{QCD}$$

Spectra

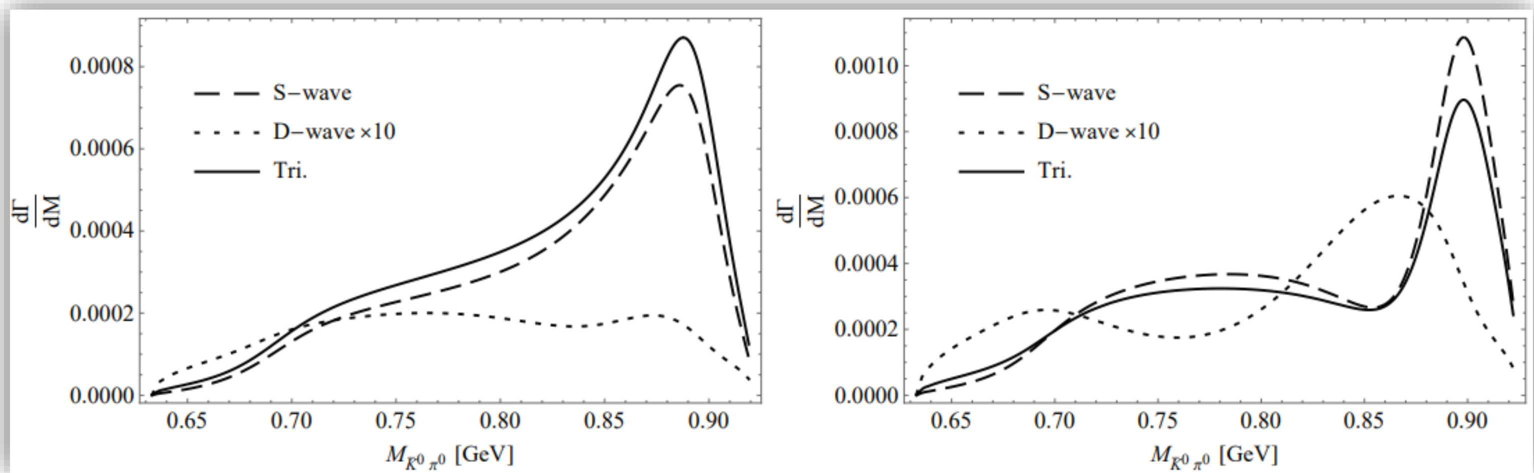


$f_1(1420)$ $h_1(1415)$



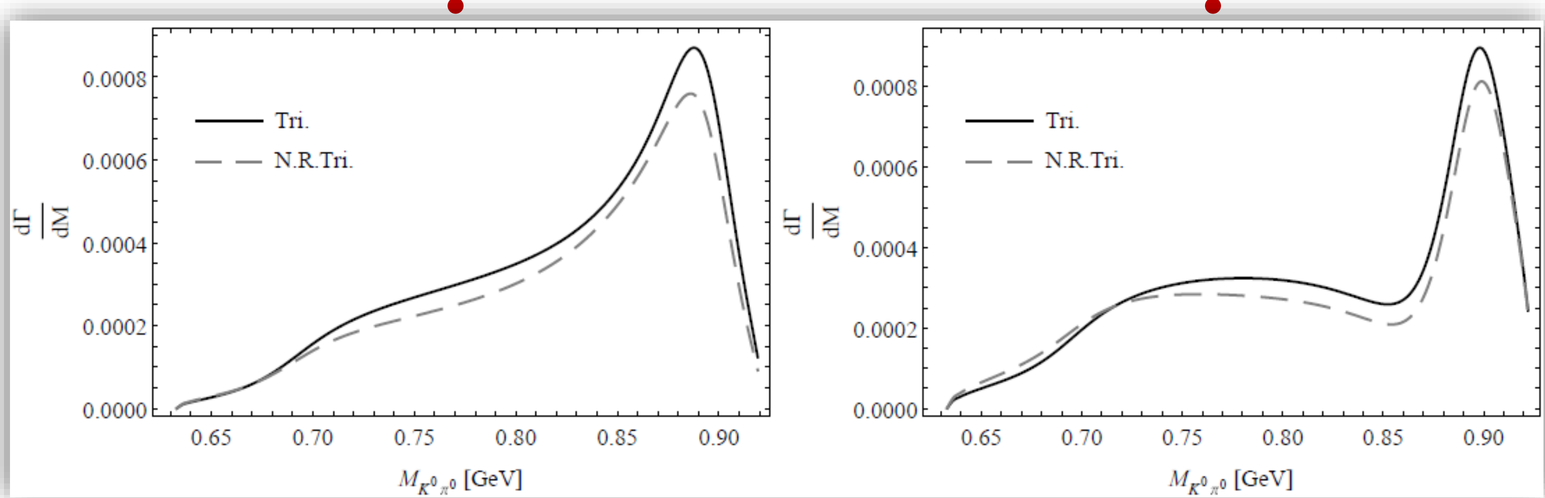
? Where is the triangle singularity

$$I^{\mu\alpha} = I_S^{\mu\alpha} + I_D^{\mu\alpha}$$



$f_1(1420)$

$h_1(1415)$



NR approximation: Num.→ $4p_b^\mu p_b^\alpha - 4p_b^\mu q^\alpha - 2q^\mu p_b^\alpha + 2q^\mu q^\alpha$

- Triangle singularity
- Vertex correction by π exchange
- **Implications for $\rho\pi$ and $\phi\pi$ channels**
- Conclusion

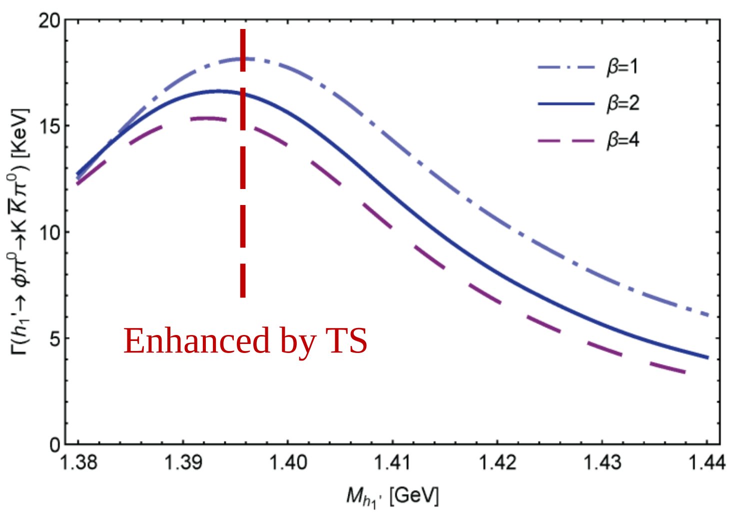
CONTENT

- The significance of triangular singularity depends on dynamics

Initial state	**Final state**	**Phase space [MeV]**
$h_1(1415)$	$K^*\bar{K}$	21
$h_1(1415)$	$\phi\pi$	260
$h_1(1415)$	$\rho\pi$	510
$b_1(1235)$	$\phi\pi$	80
$b_1(1235)$	$\rho\pi$	330

Same tensor structure

① $h_1' \to \phi\pi \to K\bar{K}\pi$

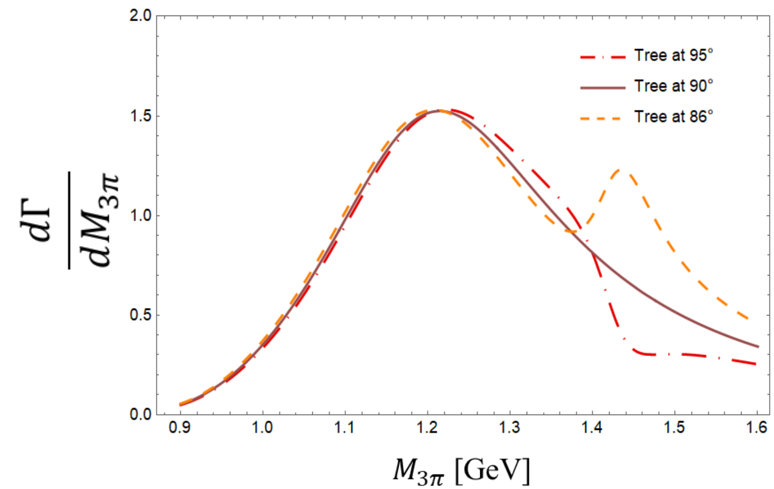


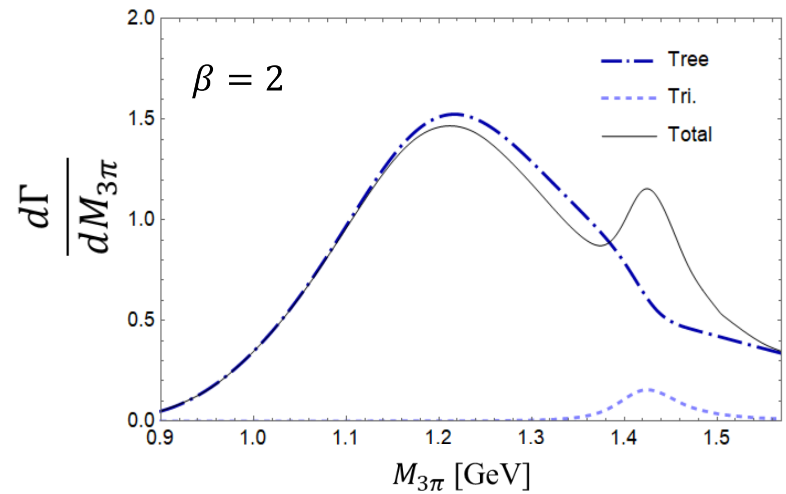
$\text{B.R.}\,(J/\psi \to \eta' h_1' \to \eta' K^*\bar{K} + c.c.) = (2.16 \pm 0.31) \times 10^{-4}$

$\text{B.R.}\,(J/\psi \to \eta h_1' \to \eta\phi\pi) \sim 6.3 \times 10^{-8}$

② $h_1' \to \rho\pi$

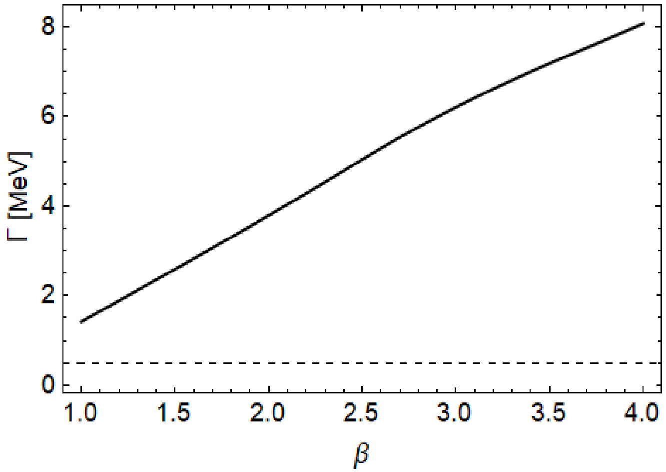
$J/\psi \to \eta(h_1 + h_1') \to \eta(\rho\pi)^0 \to \eta\pi^+\pi^-\pi^0$



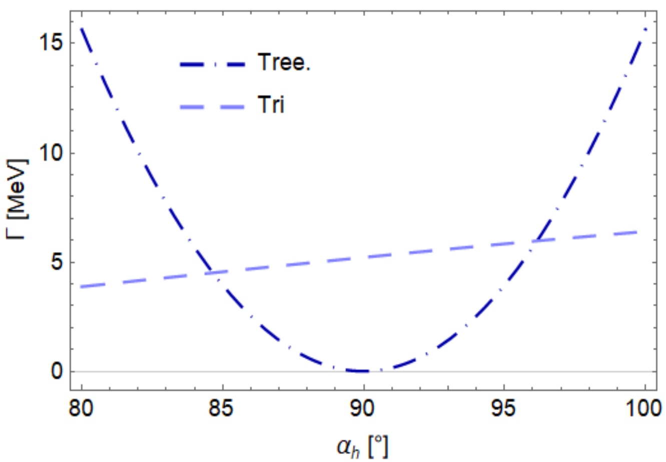


$\alpha_h = 92°, \delta\alpha_h \sim 6°$

Cut-off dependence of
$h_1' \to \rho\pi$

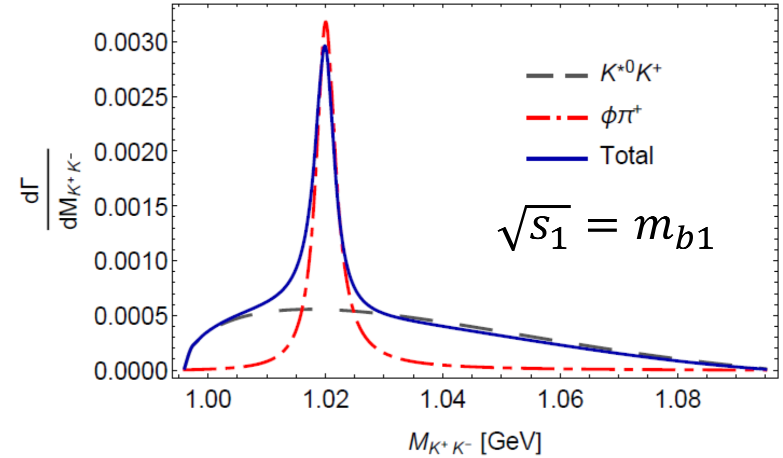


Mixing angle dependence of
$h_1' \to \rho\pi$

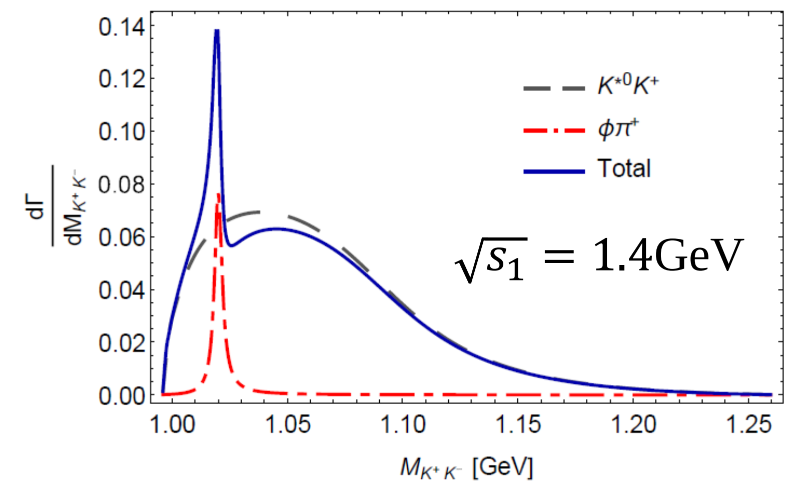


- Triangle diagrams are important for a large range of α_h

③ $b_1 \to \phi\pi$



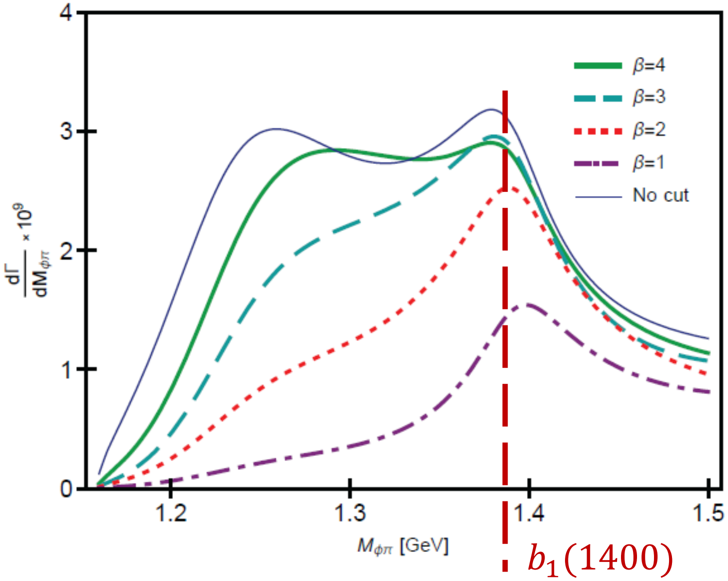
$b_1 \to \phi\pi \to K\bar{K}\pi$
$b_1 \to K^*\bar{K} + c.c. \to K\bar{K}\pi$



- The signal of ϕ is significant
- TS effect is suppressed by sharp ϕ

③ $b_1 \to \phi\pi$

$J/\psi \to \pi b_1 \to \phi\pi\pi$

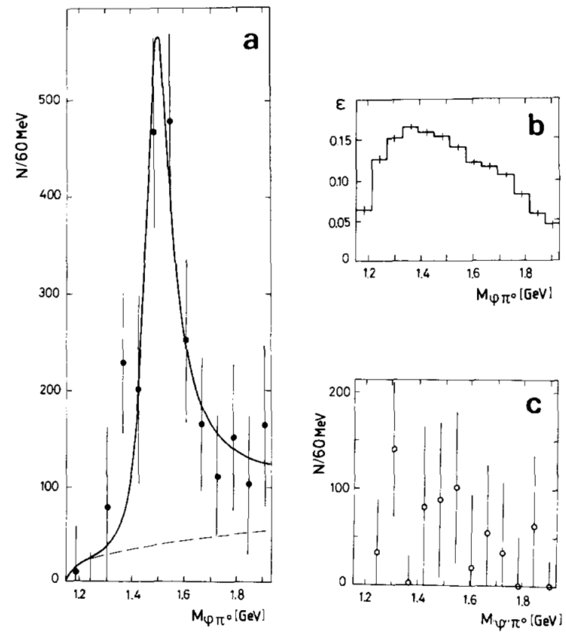


On-shell:

$$\frac{\mathrm{B.R.}\,(b_1 \to \phi\pi \to K\bar{K}\pi)}{\mathrm{B.R.}\,(b_1 \to \omega\pi)(S - wave)} = 1.8 \times 10^{-4}$$

After integration:

$$\frac{\mathrm{B.R.}\left(J/\psi \to b_1^{\pm}\pi^{\mp} \to \phi\pi^+\pi^-\right)}{\mathrm{B.R}\left(J/\psi \to \pi^{\pm} b_1^{\mp}\right)} = \frac{1.0 \times 10^{-5}}{(3 \pm 0.5) \times 10^{-3}} \sim 3 \times 10^{-3}$$

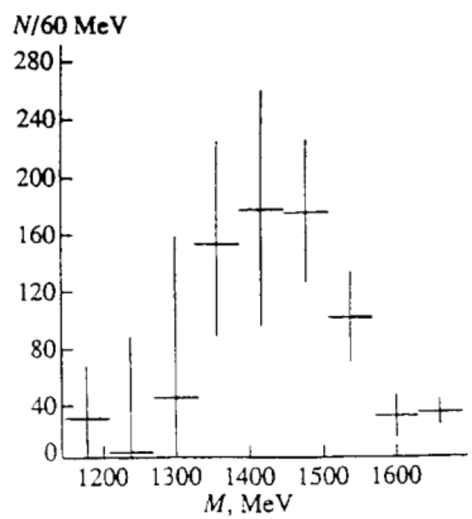


(a) The acceptance-corrected $\phi\pi^0$ mass spectrum in $\pi^- p \to \phi\pi^0 n$.

(b) The acceptance of the Lepton-F spectrometer for $\phi\pi^0$ events.

(c) The false $\phi\pi^0$ mass spectrum *[1]*.

"C(1480)" is reported with $I = 1\ J^{PC} = 1^{--}$, by OPE. $M = 1480 \pm 40$ MeV, $\Gamma = 130 \pm 60$ MeV [1]

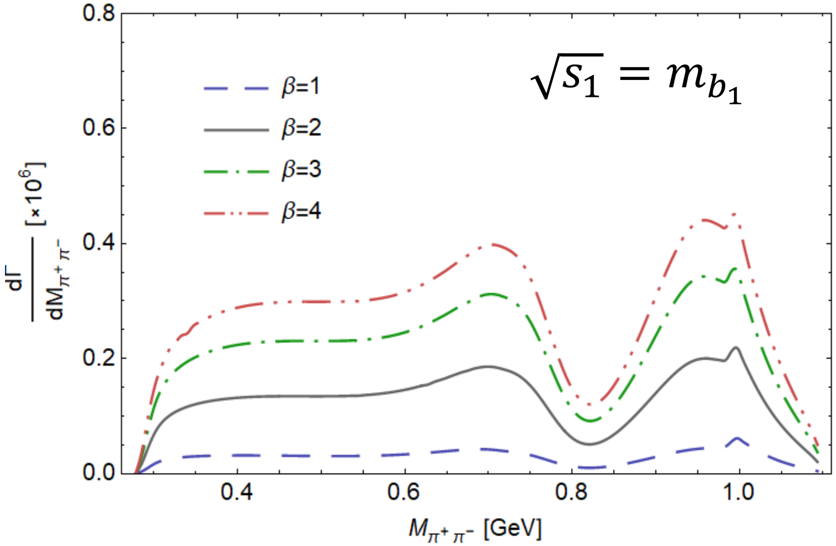


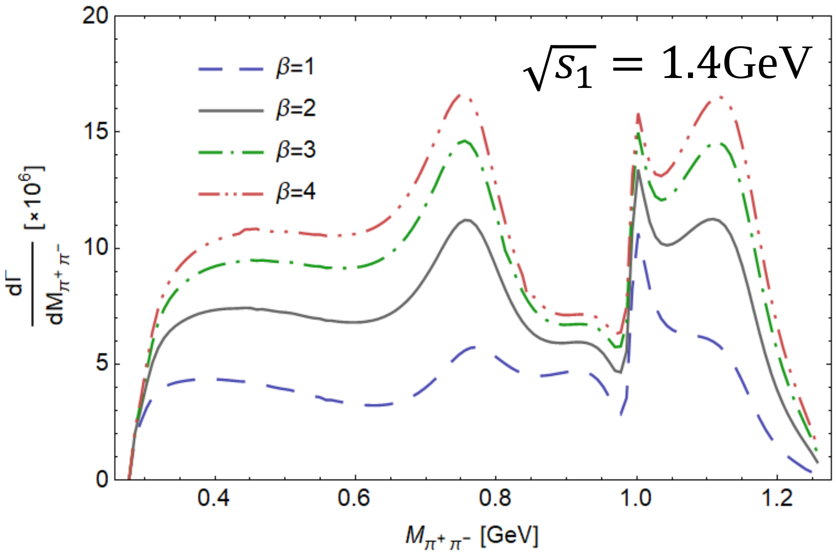
Effective mass spectrum of the $\phi\pi^0$ system in the reaction $\pi^- p \to \phi\pi^0 n$ (the results are weighted with detector efficiency) [2,3]. The "anti-OPE" selection $|t'| > 0.1\ \mathrm{GeV}^2$ is applied, which affects only slightly the efficiency for $\pi^- p \to b_1 n$ (not proceeding via OPE exchange), but which reduces the background from the OPE-mediated reaction $\pi^- p \to C(1480) n$ by a factor of 5 [2,3].

PDG: $< 4 \times 10^{-3}$[2]

[1] Bityukov S.I. et al. Phys. Lett. B 118,383 (1987); [2] V. A. Viktorov et al. Phys. Atom. Nucl. 59,1184 (1996);[3] S.V. Golovkin et al. Z. Phys. A 359,435 (1997)

④ $b_1 \to \rho\pi \to \pi^+\pi^-\pi^0$

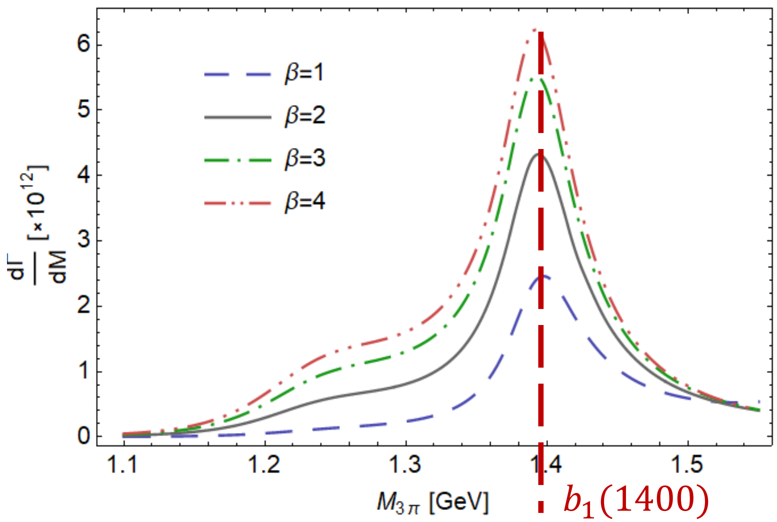




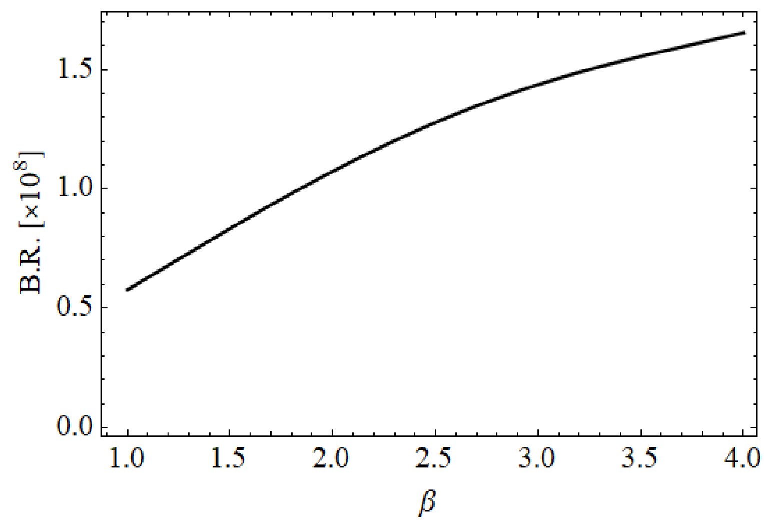
- Significant peak near the $K\bar{K}$ threshold

$J/\psi \to \pi^0 b_1^0 \to \pi^0\rho\pi \to \pi^+\pi^-\pi^0\pi^0$

The first π^0 is distinguishable



Cut-off dependence



- More statistics of J/ψ is required

- **Triangle singularity**
- **Vertex correction by π exchange**
- **Implications for $\rho\pi$ and $\phi\pi$ channels**
- **Conclusion**

CONTENT

CONCLUSION

- The vertex correction of $A \to K^*\bar{K} + c.c.$ due to the pion exchange is not expected to be strong. In particular, the triangle singularity in the one-loop diagram does not manifest itself due to the insufficient phase space.
- However, in other hadronic decay channels, i.e. $\phi\pi$ and $\rho\pi$ for $h_1(h_1')$ and b_1, the similar dynamics can produce non-trivial structures in the spectra.
- The condition of the triangle singularity is process-independent, but the significance is dependent on the dynamics.

THANK YOU