

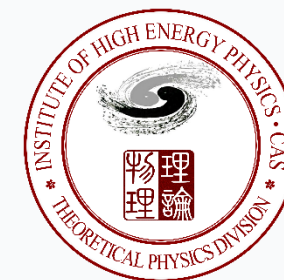
The study of X(6900): fully-heavy charmonium system

2021/9/10

理论室学术报告 “Weekly Theory Forum”

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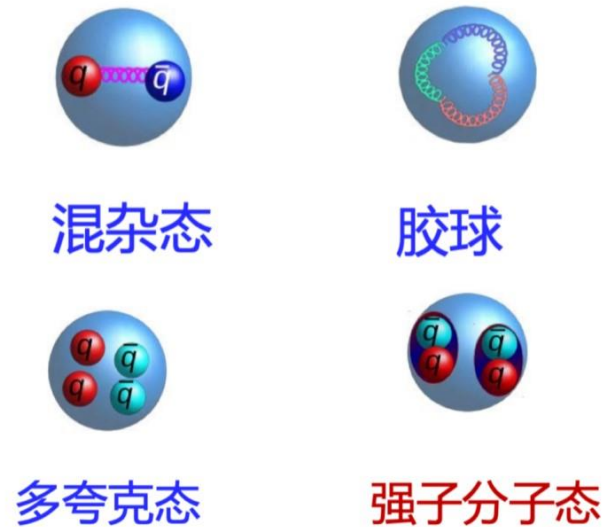


Outline

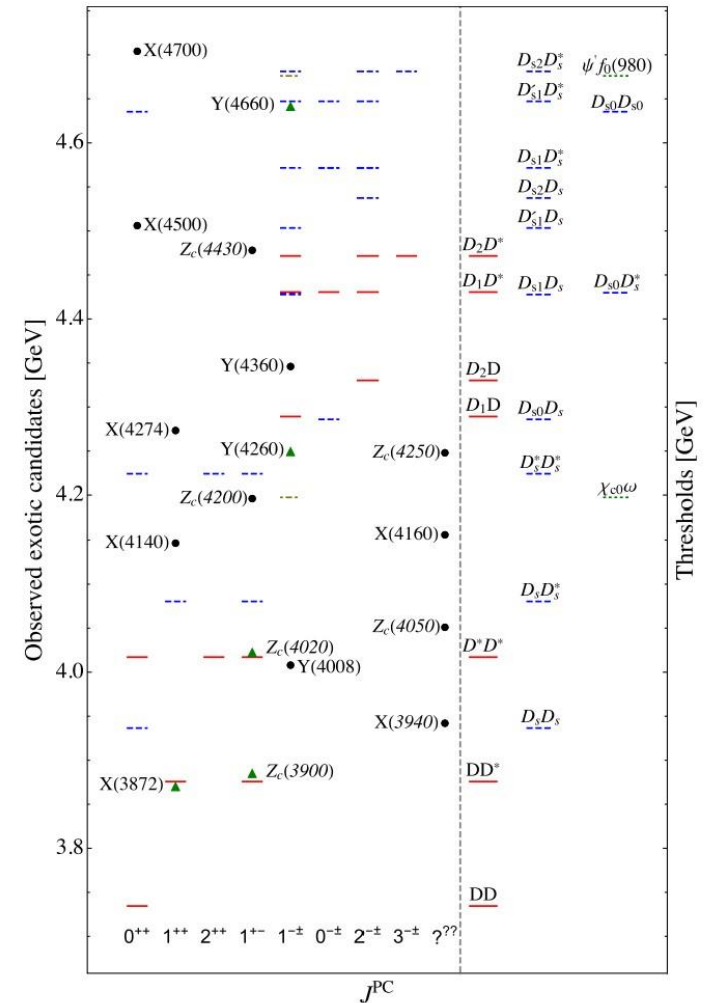
- The introduction to XYZ exotic states
- The observation of X(6900) state
- Theoretical review for the fully-heavy charmonium system
- The introduction to Pomeron
- Diffractive scattering between J/ψ - $\psi(2S)$ via the Pomeron exchange
- Summary

The introduction to XYZ exotic states

- 传统夸克模型：很好的描述了介子 ($q\bar{q}$)、重子 (qqq) 能谱
- 自2003年Belle实验合作组发现奇特态X(3872)以来，各实验组观测到了越来越多的新强子结构，包括混杂态、胶球、多夸克态等
- 许多奇特态都靠近一些强子态的阈值附近



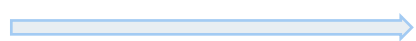
Guo et al., RMP90(2018)015004



The introduction to XYZ exotic states

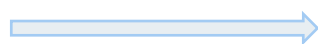
- 多夸克态(Multi-quarks): 传统夸克模型的延伸

介子: $3 \otimes \bar{3}$



四夸克态(Tetraquark): $3 \otimes 3 \otimes \bar{3} \otimes \bar{3}$

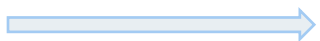
重子: $3 \otimes 3 \otimes 3$



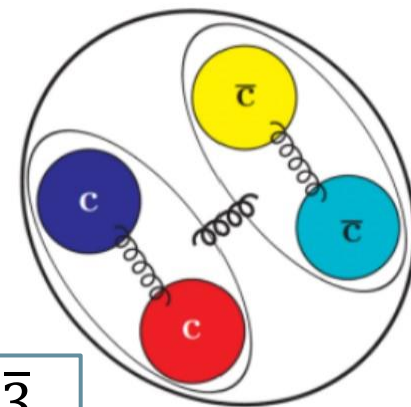
五夸克态(Pentaquarks): $3 \otimes 3 \otimes 3 \otimes 3 \otimes \bar{3}$

- 强子分子态(Hadronic molecules): 两体问题、三体问题...

氦核: p-n



强子对 (hadron-hadron pairs)



The introduction to XYZ exotic states

- 理论研究方法

1. 唯象层面：组分夸克势能模型、介子交换模型、运动学效应（三角奇异性等）
2. 有效场论：手征对称+重夸克对称理论
3. QCD求和规则
4. Lattice QCD

- 重要问题

1. QCD非微扰能区，低能有效常数的抽取
2. 短程相互作用动力学机制
3. 耦合道效应

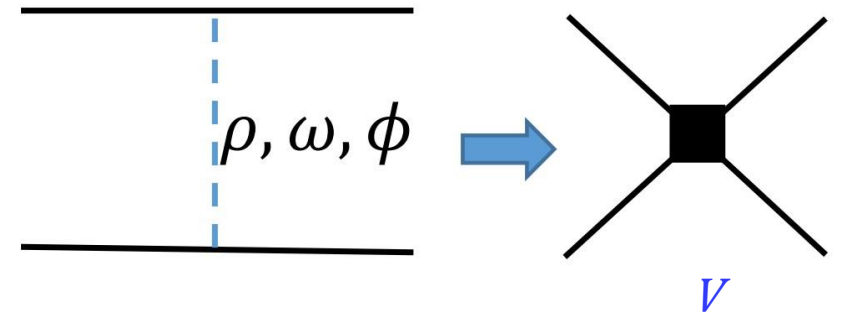
The introduction to XYZ exotic states

- 介子交换模型（单道single-channel）

▶ 两个强子之间通过交换轻矢量介子产生的接触项

▶ 对接触项进行重求和得到 T 矩阵表达式

$$T = \frac{V}{1 - VG}$$



G 是两点标量圈积分

$$G(E) = \frac{1}{16\pi^2} \left\{ a(\mu) + \log \frac{m_1^2}{\mu^2} + \frac{m_2^2 - m_1^2 + s}{2s} \log \frac{m_2^2}{m_1^2} + \frac{k}{E} \log \frac{(2kE + s)^2 - m_1^2 + m_2^2}{(2kE - s)^2 - m_1^2 + m_2^2} \right\}$$

$$G(E) = \int \frac{l^2 dl}{4\pi^2} \frac{\omega_1 + \omega_2}{\omega_1 \omega_2} \frac{e^{-2l^2/\Lambda^2}}{E^2 - (\omega_1 + \omega_2)^2 + i\epsilon} \quad \text{with } \omega_i = \sqrt{m_i^2 + l^2}$$

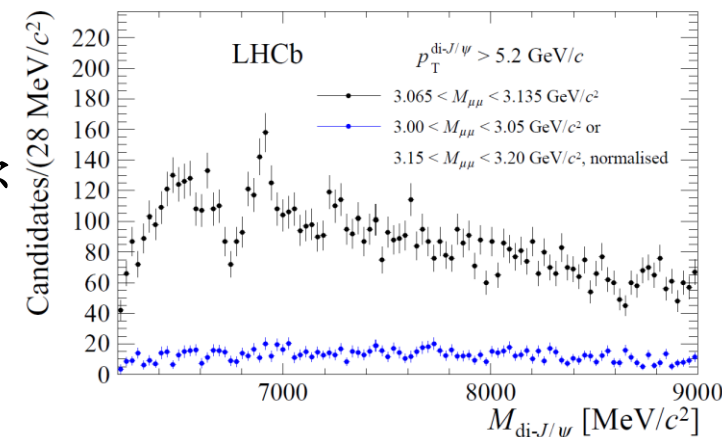
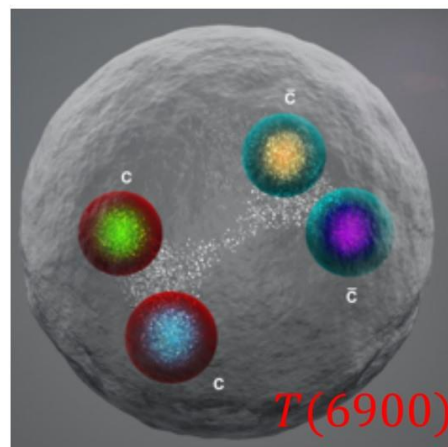
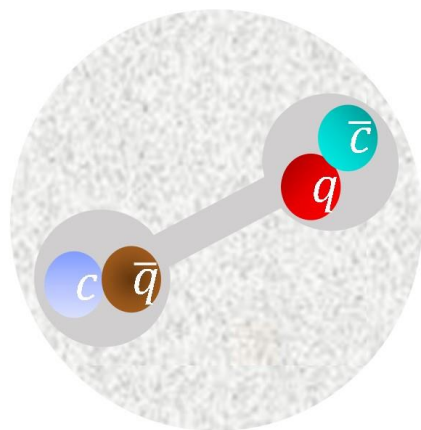
- 在介子交换模型框架下，X(3872)被认为 $D\bar{D}^*$ 耦合形成的束缚态结构；Y(4260)可以成功被解释为 $D_1\bar{D}$ 的分子态图像

The observation of X(6900): fully-heavy charmonium system

- LHCb 在 $di\text{-}J/\psi$ 末态能谱中观测到了一些奇异结构
- 在 $J/\psi - \psi(2S)$ 阈值附近看到了一个较窄的共振结构，被称作 X(6900)，宽度大约为 80 MeV 左右，其内部夸克组分为 $cc\bar{c}\bar{c}$ ，属于全重粲味体系

$$\text{threshold } J/\psi + \psi(2S) = 6783 \text{ MeV}$$

- 这是实验上首次观测到可能的全重粲四夸克体系结构



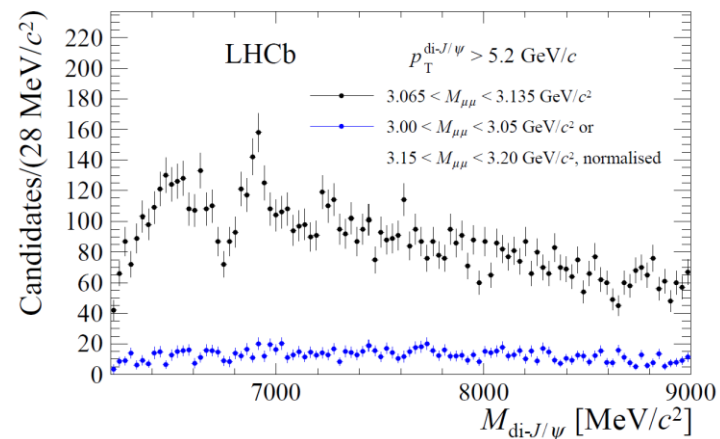
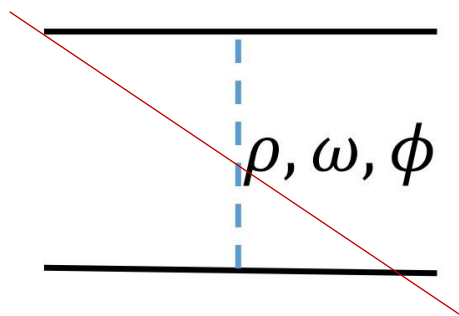
LHCb Sci.Bull. 65 (2020) 23, 1983-1993

$$M(X(6900)) = 6905 \pm 11 \pm 7 \text{ MeV}$$

$$\Gamma(X(6900)) = 80 \pm 19 \pm 33 \text{ MeV}$$

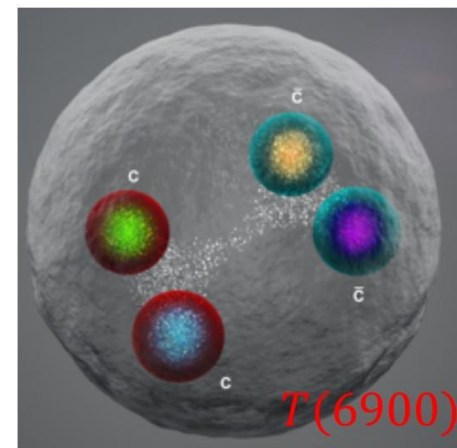
Theoretical review for the fully-heavy charmonium system

- 对于全重粲味的四夸克系统，长程的轻味介子交换在LO不贡献。而粲偶素的交换机制会受到传播子大质量压低以及 $c\bar{c}$ 产生的高能量标度压低



LHCb Sci.Bull. 65 (2020) 23, 1983-1993

- 从这个角度分析，X(6900)是一个好的Tetraquark候选者。有很多理论通过不同的势能模型在 $J/\psi - \psi(2S)$ 阈值附近预言了许多 $cc\bar{c}\bar{c}$ 的S波和P波结构，均可以衰变到di- J/ψ 末态



V. R. Debastiani and F. S. Navarra Chin.Phys.C 43,013105(2018)
 M. N. Anwar, J. Ferretti, F. K. Guo, E. Santopinto, and B. S. Zou, Eur. Phys. J. C 78, 647 (2018)
 M. S. liu, F. X. Liu, X. H. Zhong and Q. Zhao, [arXiv:2006.11952 [hep-ph]] (2020)
 Xin Jin, Yaoyao Xue, Hongxia Huang, Jialun Ping, Eur.Phys.J.C 80 (2020) 11, 1083(2020)
 Jacob Sonnenschein, Dorin Weissman(Okinawa Inst. Sci. Tech.) Eur.Phys.J.C 81 (2021) 1, 25

Theoretical review for the fully-heavy charmonium system

- 然而，在LHCb的实验结果中，在6.9GeV附近值观测到X(6900)一个共振结构
- 探究全重粲味系统的内部动力学耦合机制尤为重要，短程相互作用在束缚全重系统中发挥着重要作用， $cc\bar{c}\bar{c}$ 是研究非微扰QCD的良好场所。

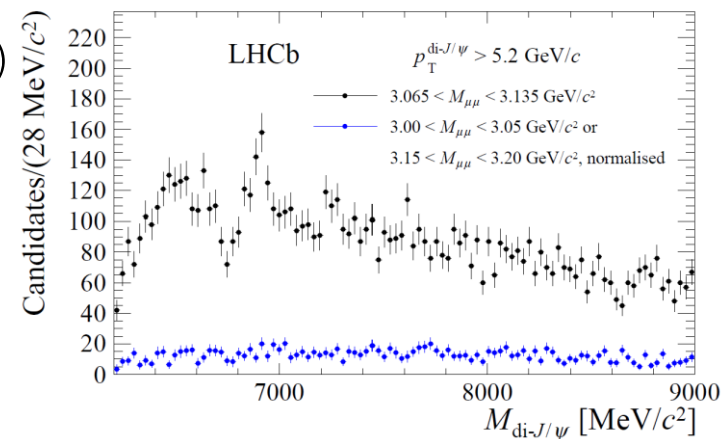
soft gluons



short-distance interaction

- 目前，X(6900)全重系统中软胶子传递强相互作用的理论工作主要有

- ✓ **Hybrid interpretation of X(6900):** Bing-Dong Wan, Cong-Feng Qiao(GUCAS) Phys.Lett.B 817 (2021) 136339
- ✓ **Two pions interaction:** Xiang-Kun Dong, Vadim Baru, Feng-Kun Guo, Christoph Hanhart, Alexey Nefediev, and Bing-Song Zou [arXiv: 2107.03946 [hep-ph]]



LHCb Sci.Bull. 65 (2020) 23, 1983-1993

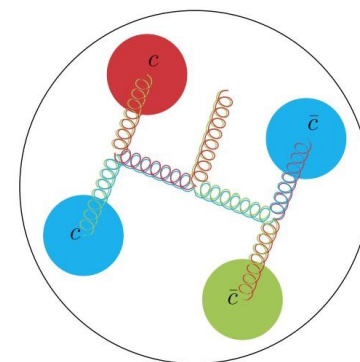
Theoretical review for the fully-heavy charmonium system

- X(6900): 混杂态解释

Bing-Dong Wan, Cong-Feng Qiao(GUCAS) Phys.Lett.B 817 (2021) 136339

$$[\bar{3}]_{cc} \otimes [8]_G \otimes [3]_{\bar{c}\bar{c}}$$

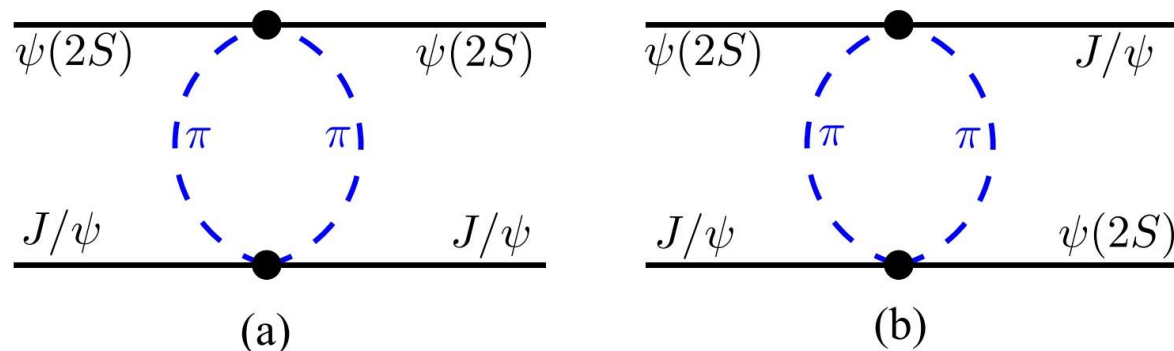
- 该工作给出X(6900)的量子数为 $J^{PC} = 0^{++}$
- 利用QCD求和规则计算得到 $m_{0^{++}} = (6.92 \pm 0.14)\text{GeV}$
- 预言在7.2GeV附近存在一个 $J^{PC} = 0^{-+}$ 混杂态



Theoretical review for the fully-heavy charmonium system

- 双- π 交换相互作用机制

Xiang-Kun Dong, et. al [arXiv: 2107.03946 [hep-ph]]



- 从 $\psi(2S) \rightarrow J/\psi\pi\pi$ 过程中抽取出 π 介子和矢量介子的相互作用强度，中性双 π 和带电双 π 道是 $\psi(2S)$ 的主要衰变道，其衰变分支比分别为 34.86% 和 18.24%
- 考虑双 π 末态相互作用，以及和 KK 道的耦合道效应，作者认为双 π 相互作用有可能可以提供 di- J/ψ 系统的吸引势

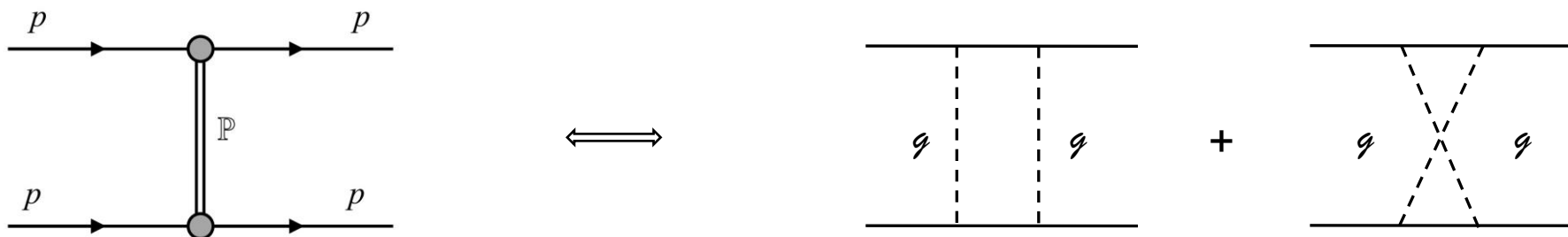
Theoretical review for the fully-heavy charmonium system

- 双- π 交换相互作用机制

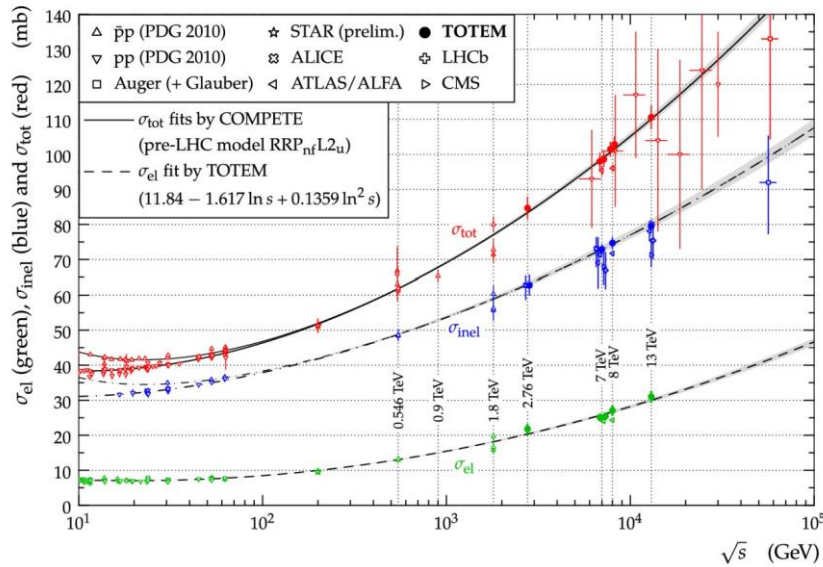
Xiang-Kun Dong, et. al [arXiv: 2107.03946 [hep-ph]]

➤ $\psi(2S) \rightarrow J/\psi\pi\pi$ 衰变过程中，主要由软胶子refusion到双 π 的道贡献

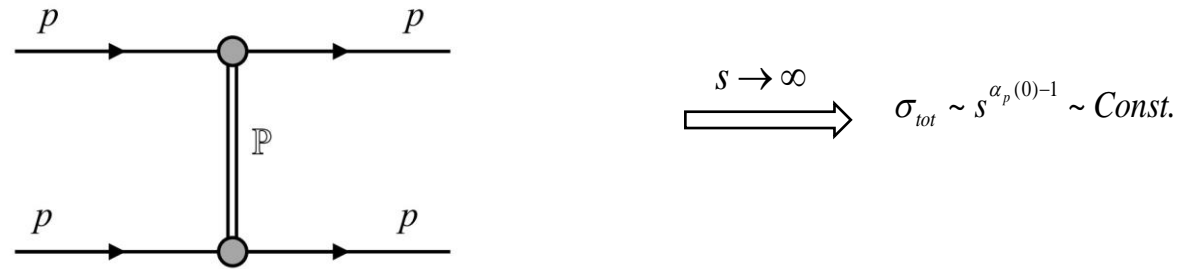
- 因此短程的软胶子相互作用在全重粲味系统中起着非常重要的作用
- 在我们的工作中，提出矢量介子之间交换Pomeron的相互作用机制



The Introduction To Pomeron



- 衍射过程是一个很很好的研究软强子-强子相互作用动力学的平台，在 高能碰撞过程中，总散射截面表现出一个渐进常数的行为



P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)
 P. D. B. Collins, An Introduction to Regge Theory and High-Energy Physics
 A. Donnachie and P. V. Landshoff, Nucl. Phys. B231, 189 (1984)

- 光学定理:

$$s\sigma_{tot} = \text{Im}T_{el}(s, t=0)$$

- 在Regge理论中，pp弹性散射截面表示为:

$$T_{el}(s, t) \propto s^{\alpha_P(t)} \Rightarrow \sigma_{tot} \propto s^{\alpha_P(0)-1}$$

- Pomeron trajectory: $\alpha_P(t=0) = 1$
- 理论认为Pomeron交换可以唯象为一对非微扰胶子的相互作用行为

A. Donnachie and P. V. Landshoff, Nucl. Phys. B311, 509 (1988)

The Introduction To Pomeron

- Donnachie和Landshoff在Regge理论框架下，将质子-质子弹性振幅描述为：

$$\frac{d\sigma}{dt} = \frac{1}{4\pi} (3\beta_0 F_1(t))^2 \left(\frac{s}{s_0}\right)^{2\alpha_p(t)-2}$$

其中 $F_1(t)$ 是质子的电磁形状因子（Pomeron相互作用在 高能弹性散射过程中体现出类光子行为）

β_0 是Pomeron和轻味夸克的耦合常数

Pomeron reggeon trajectory: $\alpha_p(t) = 1 + 0.08 + 0.25t$

M. Diehl, Z. Phys. C 66, 181-193 (1995)

A. Donnachie, J. Gravelis, and G. Shaw, Phys. Rev. D. 63, 114013(2001)

The Introduction To Pomeron

- Pomeron 交换模型很好地描述了高能强子碰撞程中小t区域的散射行为和衍射行为

- pp弹性散射的振幅为

$$M_{pp} = (3\beta_0 F_1(t))^2 (\bar{u} \gamma_\mu u) (\bar{u} \gamma^\mu u) \left(\frac{s}{s_0}\right)^{\alpha(t)-1}$$

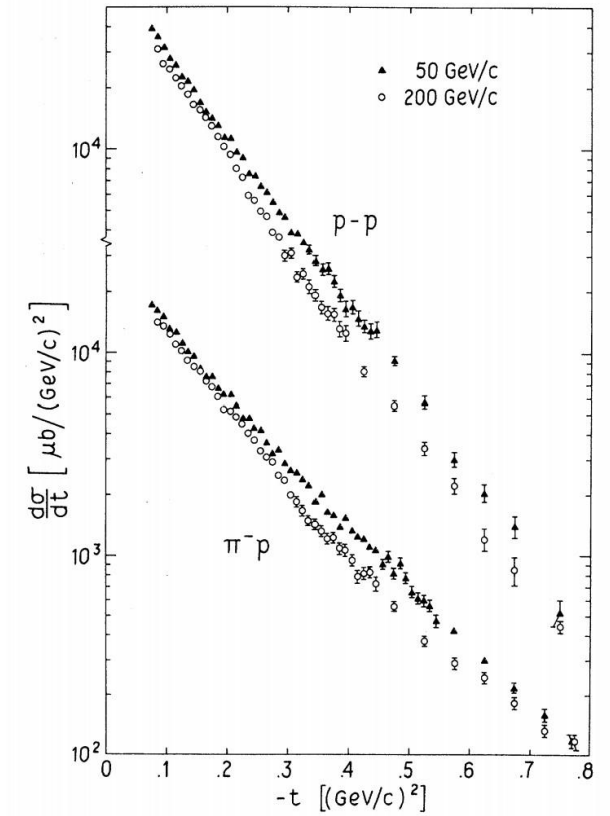
G. A. Jaroszkiewicz and P. V. Landshoff, Phys. Rev. D10, 170 (1974)

- Pomeron 交换贡献到介子-质子弹性散射过程的微分截面大小为

$$\frac{d\sigma}{dt} = \frac{(2\beta_0 F_\pi(t))^2 (3\beta_0 F_1(t))^2}{4\pi} \left(\frac{s}{s_0}\right)^{2\alpha(t)-2}$$

$F_\pi(t)$ 是 π 的弹性散射形状因子，拟合实验上 πp 弹性散射实验结果，得到

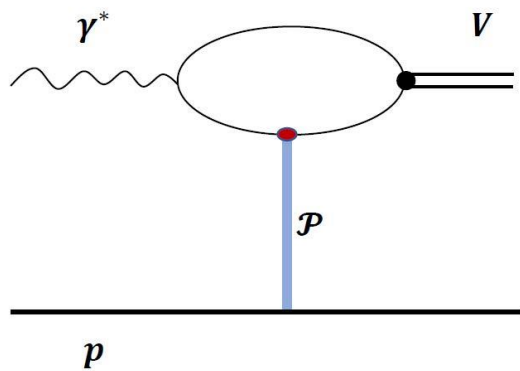
$$F_\pi(t) = \frac{1}{1-t/\lambda_0^2} \quad (\lambda_0^2 = 0.5\text{GeV}^2)$$



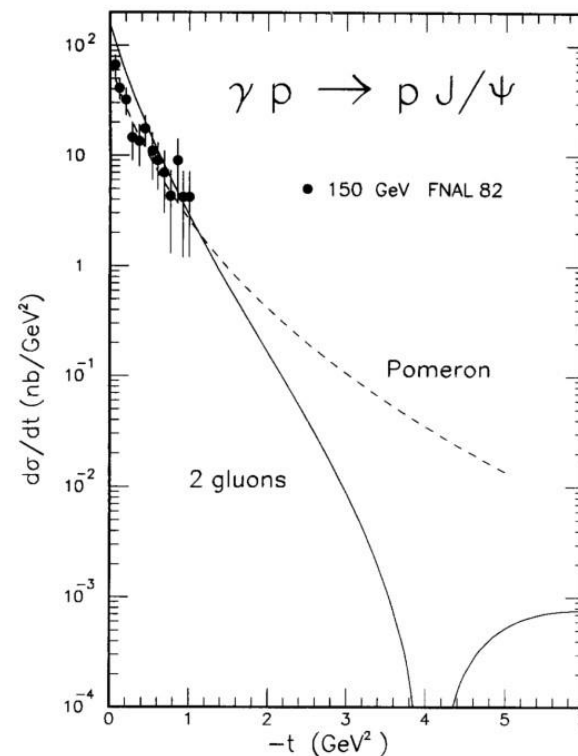
C.W. Akerlof et. Phys.Rev.D 14 (1976) 2864

Diffraction scattering between $J/\psi - \psi(2S)$ via the Pomeron exchange

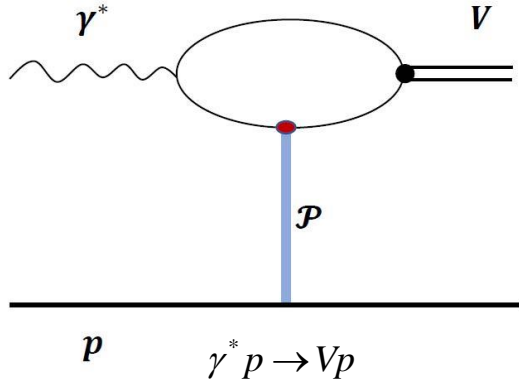
- 类比质子-质子、介子-质子的情形，我们可以写出介子-介子之间通过交换 pomeron 的相互作用行为
- 目前，还没有关于 J/ψ - p 的弹性散射实验数据，不过 Pomeron 相互作用仅依赖于味道，我们可以从 J/ψ 的光产生过程中提取出 charm 夸克和 Pomeron 相互作用的耦合系数 β_c



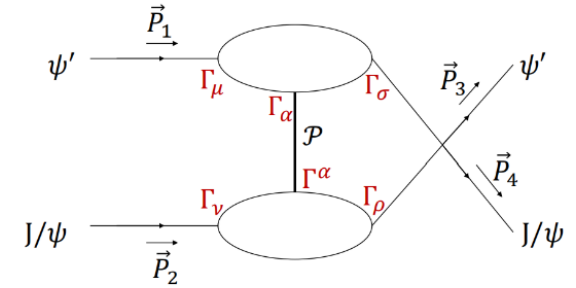
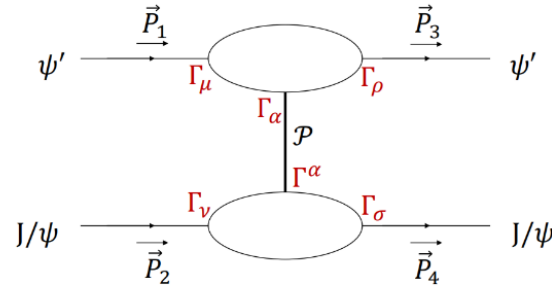
$$\gamma^* p \rightarrow V p$$



Diffractive scattering between $J/\psi - \psi(2S)$ via the Pomeron exchange



β_c



- t道和u道的散射振幅可以写为：

$$T_t = (2\beta_c)^2 \left(\frac{s}{s_0}\right)^{\alpha(t)-1} \exp\left(\frac{t}{2\lambda_{J/\psi}^2}\right) \exp\left(\frac{t}{2\lambda_\psi^2}\right) \Gamma_\alpha^{\mu\rho} \Gamma^{\nu\alpha\sigma}$$

$$T_u = (2\beta_c)^2 \left(\frac{s}{s_0}\right)^{\alpha(u)-1} \exp\left(\frac{u}{2} + \frac{m_\psi^2}{4} - \frac{m_{J/\psi}^2}{4}\right) \exp\left(\frac{u}{2} + \frac{m_\psi^2}{4} - \frac{m_{J/\psi}^2}{4}\right) \Gamma_\alpha^{\mu\sigma} \Gamma^{\nu\alpha\rho}$$

$$\Gamma^{\mu\alpha\rho} = (p_1 + p_3)^\alpha g^{\mu\rho} - 2p_1^\rho g^{\alpha\mu}$$

Q. Zhao, J. P. Didelez, M. Guidal, and B. Saghai, Nucl. Phys. A660, 323 (1996)

Alexander I. Titov, Yong-seok Oh, Shin Nan Yang, Tosiuyuki Morii, Phys.Rev.C 58 (1998) 2429-2449

Diffraction scattering between $J/\psi - \psi(2S)$ via the Pomeron exchange

- 两个矢量介子的S波耦合量子数可能为：

$$J^{PC} = 0^{++}, 1^{++}, 2^{++}$$

- 我们发现Pomeron交换势是t依赖的，在近域处，我们只关注S波的贡献，因此我们将角度依赖的部分积分掉得到一个contact顶点

$$V(s) = \frac{1}{2} \int V(s, t) d(\cos \theta)$$

- 这个顶点中包含了三种量子数的贡献，我们采用下面的投影算子将contact顶点进行投影

$$\mathcal{P}^{(0)} = \frac{1}{4} \epsilon_\mu(p_1) \epsilon^\mu(p_2) \epsilon_\nu(p_3) \epsilon^\nu(p_4),$$

$$\mathcal{P}^{(1)} = \frac{1}{2} (\epsilon_\mu(p_1) \epsilon_\nu(p_2) \epsilon^\mu(p_3) \epsilon^\nu(p_4) - \epsilon_\mu(p_1) \epsilon_\nu(p_2) \epsilon^\nu(p_3) \epsilon^\mu(p_4)),$$

$$\mathcal{P}^{(2)} = \frac{1}{2} (\epsilon_\mu(p_1) \epsilon_\nu(p_2) \epsilon^\mu(p_3) \epsilon^\nu(p_4) + \epsilon_\mu(p_1) \epsilon_\nu(p_2) \epsilon^\nu(p_3) \epsilon^\mu(p_4)) - \frac{1}{4} \epsilon_\mu(p_1) \epsilon^\mu(p_2) \epsilon_\nu(p_3) \epsilon^\nu(p_4)$$

R.Molina, D. Nicmorus, and E. Oset Phys. Rev. D. 78, 114018 (2008)

Diffractive scattering between $J/\psi - \psi(2S)$ via the Pomeron exchange

- 对于矢量介子圈的重求和，由于圈上粒子矢量求和导致张量结构较为复杂，无法直接进行计算。但是，粲偶素介子的大质量可以提供一个好的近似，圈积分的三动量在近域范围内属于小量，在这种近似下，利用LS方程进行重求和

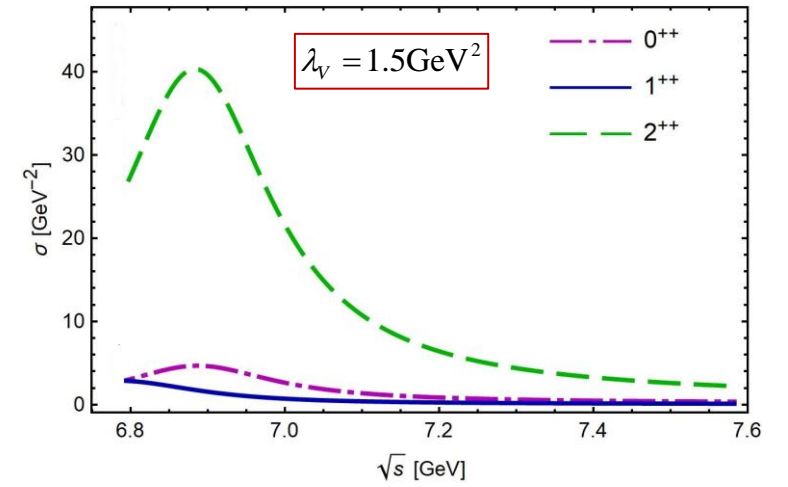
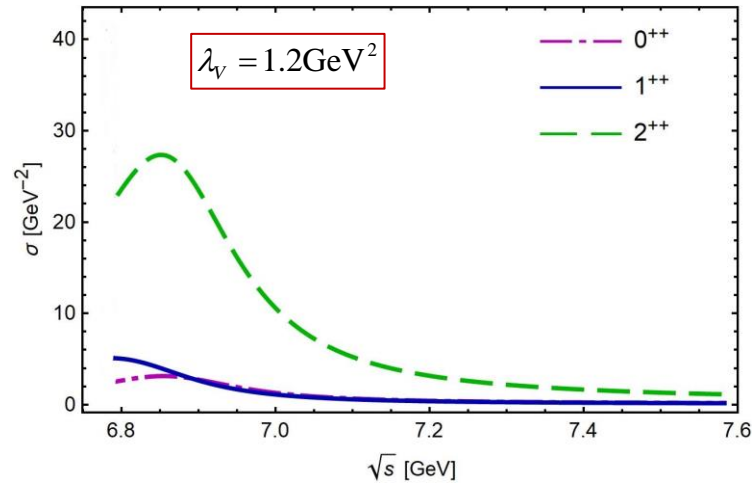
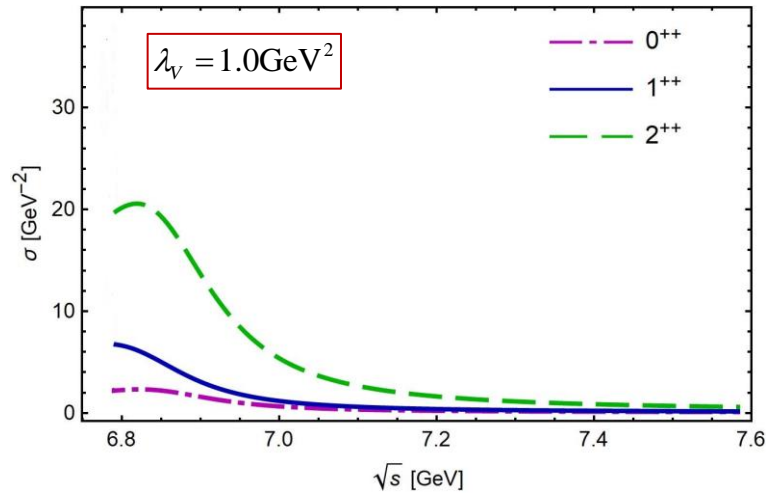


$$T = \frac{V}{1 - VG}$$

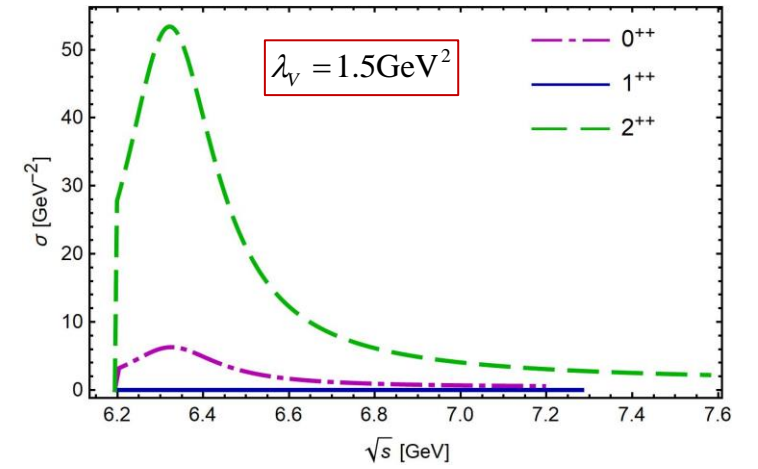
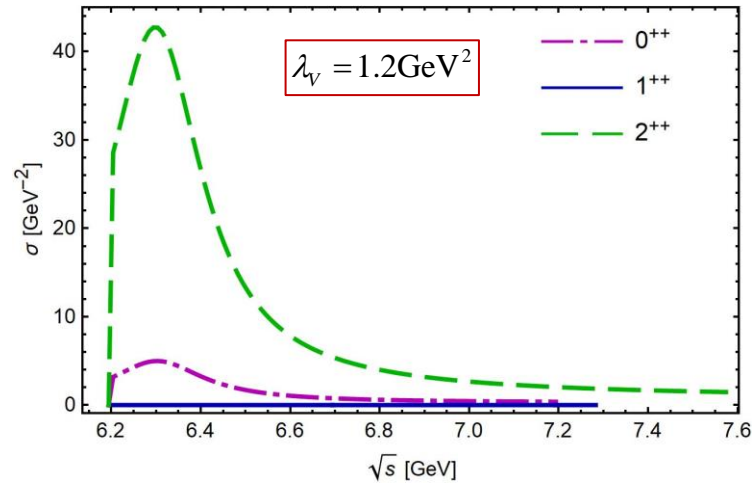
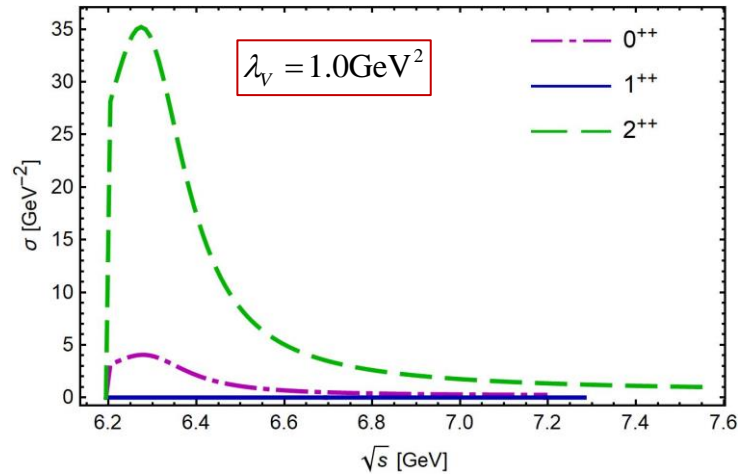
$$G(E) = \frac{1}{16\pi^2} \left\{ a(\mu) + \log \frac{m_1^2}{\mu^2} + \frac{m_2^2 - m_1^2 + s}{2s} \log \frac{m_2^2}{m_1^2} + \frac{k}{E} \log \frac{(2kE + s)^2 - m_1^2 + m_2^2}{(2kE - s)^2 - m_1^2 + m_2^2} \right\}$$

$$G(E) = \int \frac{l^2 dl}{4\pi^2} \frac{\omega_1 + \omega_2}{\omega_1 \omega_2} \frac{e^{-2l^2/\Lambda^2}}{E^2 - (\omega_1 + \omega_2)^2 + i\epsilon} \quad \text{with } \omega_i = \sqrt{m_i^2 + l^2}$$

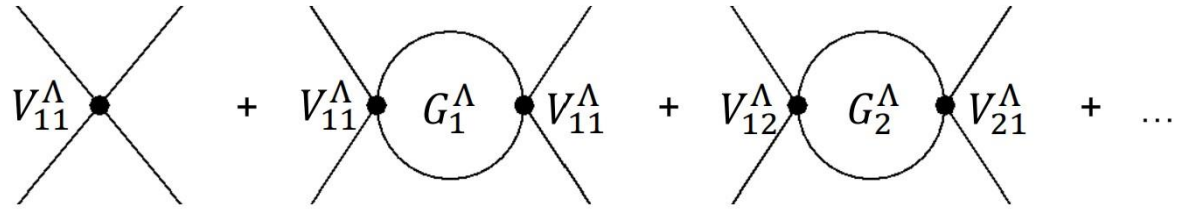
Diffraction scattering between J/ψ - $\psi(2S)$ via the Pomeron exchange



$\psi(2S)J/\psi \rightarrow \psi(2S)J/\psi$



Diffractive scattering between $J/\psi - \psi(2S)$ via the Pomeron exchange



$$G = \begin{pmatrix} G_{11} & 0 & 0 \\ 0 & G_{22} & 0 \\ 0 & 0 & G_{33} \end{pmatrix}$$

$$T(E) = V + VG(E)V + VG(E)VG(E)V + \dots = \frac{1}{V^{-1} - G(E)}$$

$$V = \begin{pmatrix} V_{11} & V_{12} & V_{13} \\ V_{21} & V_{22} & V_{23} \\ V_{31} & V_{32} & V_{33} \end{pmatrix}$$

$$G(E) = \frac{1}{16\pi^2} \left\{ a(\mu) + \log \frac{m_1^2}{\mu^2} + \frac{m_2^2 - m_1^2 + s}{2s} \log \frac{m_2^2}{m_1^2} + \frac{k}{E} \log \frac{(2kE + s)^2 - m_1^2 + m_2^2}{(2kE - s)^2 - m_1^2 + m_2^2} \right\}$$

$$G(E) = \int \frac{l^2 dl}{4\pi^2} \frac{\omega_1 + \omega_2}{\omega_1 \omega_2} \frac{e^{-2l^2/\Lambda^2}}{E^2 - (\omega_1 + \omega_2)^2 + i\epsilon} \quad \text{with } \omega_i = \sqrt{m_i^2 + l^2}$$

Diffractive scattering between J/ψ - $\psi(2S)$ via the Pomeron exchange

- We consider three coupled channel J/ψ - J/ψ , J/ψ - $\psi(2S)$ and $\psi(2S)$ - $\psi(2S)$

M. L. Du, V. Baru, F. K. Guo, C. Hanhart, U. G. Meißner, A. Nefediev and I. Strakovsky, Phys.Rev.Lett. 126, (2021) 132001

$$T = V \cdot (1 - GV)^{-1}$$

$$V = \begin{pmatrix} V_{11} & V_{12} & V_{13} \\ V_{21} & V_{22} & V_{23} \\ V_{31} & V_{32} & V_{33} \end{pmatrix} \quad G = \begin{pmatrix} G_{11} & 0 & 0 \\ 0 & G_{22} & 0 \\ 0 & 0 & G_{33} \end{pmatrix}$$

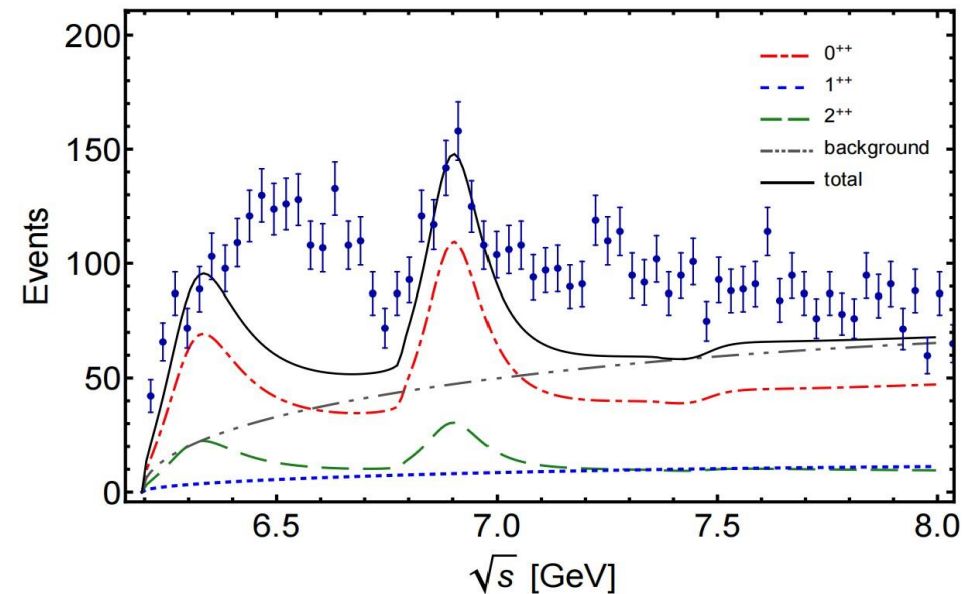
$$M_1 = P(\sqrt{s}) \left(1 + \sum r_i G_i(s) T_{i1} \right)$$

$$\Gamma(s) = \frac{|\vec{P}_{J/\psi}|}{8\pi s} |M_1|^2$$

- The phase difference between different channel.

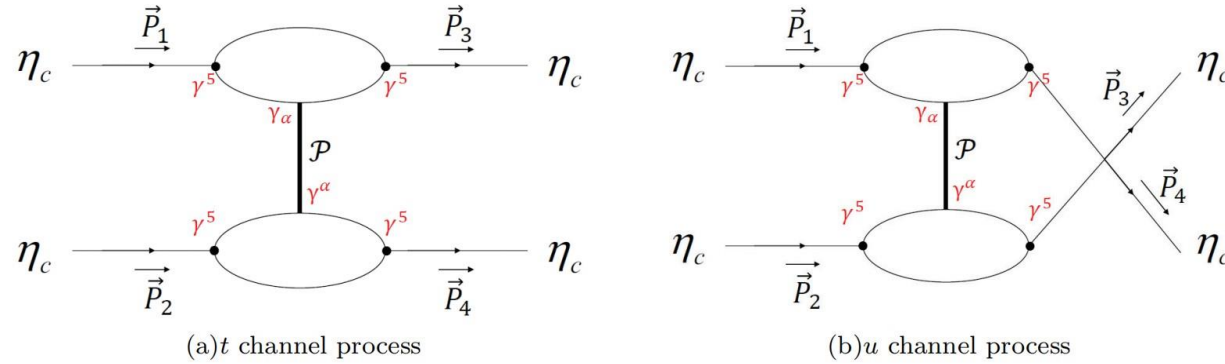
$$r_1:r_2:r_3 = 1: 2e^{-i\pi/2}: 1$$

- The X(6900) state favors $J^{PC} = 0^{++}, 2^{++}$



Chang Gong, Meng-Chuan Du, Bin Zhou, Qiang Zhao, Xian-Hui Zhong[arXiv:2011.11374[hep-ph]]

Diffractive scattering between η_c - η_c via the Pomeron exchange

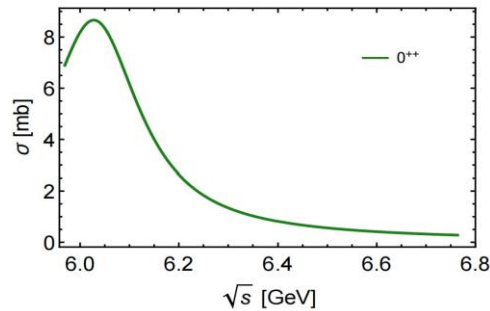


$$\Gamma^\alpha = \left(\frac{4m_c^2 + p_3^2}{p_3^2}\right)p_1^\alpha + \left(\frac{4m_c^2 - p_3^2}{p_3^2}\right)p_3^\alpha$$

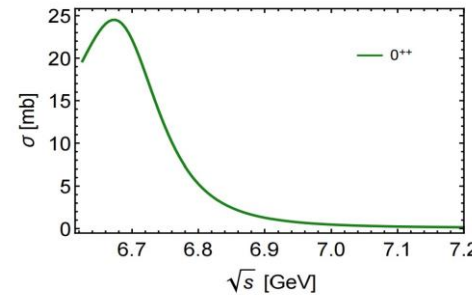
$$\approx 2p_1^\alpha,$$

- The S-wave couplings between two pseudo-scalar charmonia:

$$J^{PC} = 0^{++}$$



Scattering cross section for $\eta_c \eta_c \rightarrow \eta_c \eta_c$ via the Pomeron exchange.



Scattering cross section for $\eta_c \eta_c(2S) \rightarrow \eta_c \eta_c(2S)$ via the Pomeron exchange.

- In the case that the X(6900) being an tetra-quark state with quantum number 0^{++} , the X(6900) can also be observed in the η_c - η_c spectrum.
- However, the Pomeron exchange mechanism for the fully-heavy quark system gives different pole positions for different channel.

Summary

- For fully heavy charmonium system, the pomeron exchange potential offers a short-distance interaction mechanism
- The S-wave $J/\psi - \psi(2S)$ interaction via the Pomeron exchange can reproduce the X(6900) state.
- The di-pseudoscalar charmonia spectrum is a good tool to reveal the underlying dynamics in the fully heavy charmonium system.

Thanks For Your Attention.